Non-Linear Forecasting Methods: Some Applications to the Analysis of Financial Series

by
Oscar Bajo-Rubio*
Simón Sosvilla-Rivero**
Fernando Fernández-Rodríguez***

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* Universidad de Castilla-La Mancha.
** FEDEA and Universidad Complutense de Madrid.
*** Universidad de Las Palmas de Gran Canaria.

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Abstract

The evolution of financial data shows a high degree of volatility of the series, coupled with increasing difficulties of forecasting the shorter is the time horizon, when using standard (i.e., based on linear models) forecasting methods. Some alternative forecasting methods for non-linear time series, based on the literature on complex dynamic systems, have been recently developed, which can be particularly useful in the analysis of financial time series. In this paper we present a summary of some of these new techniques, and then show some applications to the analysis of several financial series (i.e., exchange rates, stock prices, and interest rates), which illustrate the usefulness of the approach. Since non-linear forecasting methods require the usage of very long time series, the availability of high-frequency data for these variables make them the best candidates among economic time series for the application of this methodology.
1. Introduction

A large body of literature has been accumulated over many years concerning the validity of the Efficient Market Hypothesis (EMH) with respect to stock markets. In its weak form, the EMH postulates that current prices fully reflect all the information contained in past prices and, therefore, the (log) prices should resemble a random walk (Fama, 1970). An important implication of the EMH is that no investor would be in a position to make unexploited profit opportunities by forecasting futures prices on the basis of past prices.

However, many aspects of economic behavior may not be linear and a number of recent studies cast some doubt on the relevance of the weak form of efficiency and the random walk model in providing reasonable descriptions of the movement of asset prices. In this sense, some new models and methods have been proposed that are able to capture non-linearities in financial time series. Important advances in non-linear time-series analysis with the ARCH and GARCH non-linear stochastic processes (Engle, 1982; Bollerslev, 1986), non-linear deterministic models such as chaotic dynamics (Trippi, 1995), non-parametric analysis (Diebold and Nason, 1990), multivariate adaptive regression splines (Lewis, Ray and Stevens, 1994) and artificial neural networks (van Eyden, 1995) have fuelled the recent interest in non-linearities in financial data, opening new possibilities in forecasting asset returns.

As Campbell, Lo and Mackinlay (1997, p. 80) point out, “[r]ecent econometric advances and empirical evidence seem to suggest that financial assets returns are predictable to some degree. Thirty years ago this would have been tantamount to an outright rejection of market efficiency. However, modern financial economics teaches us that other, perfectly rational, factors may account for such predictability. The fine structure of securities markets and frictions in the trading process can generate predictability. Time-varying expected returns due to changing business conditions can generate predictability. A certain degree of predictability may be necessary to reward investors for bearing certain dynamic risks”. In line with these considerations, special attention has been paid to testing for predictable components in stock prices (see, e.g., Fama and French, 1988, Lo and Mackinlay, 1988, and Fama, 1991).

In this regard, a field that has received an increasing attention in last years is the literature on complex dynamic systems and deterministic chaos, which can provide a useful tool in forecasting financial series. The main purpose of non-linear dynamics is to deal with complex processes using deterministic models. The interest in this area has been renewed during the last decades due to the surprising finding that very simple deterministic models of dynamic systems
can yield a very complex motion that exhibits the characteristics of random behavior.

Statistics has always been concerned with complex phenomena, providing successful stochastic models that are capable of describing such behavior. The key concept in stochastic models is randomness, assuming that the process under study is governed by chance and probability laws. Based on a philosophy opposite to randomness, non-linear dynamic systems and chaos offer the possibility of describing complex phenomena as the result of a non-linear deterministic process. The origin of these deterministic complex processes dates back to Poincaré (1908), who investigated planet dynamics and, in particular, the three bodies problem and its unpredictable dynamics. Even though the work on deterministic complex dynamics remained isolated of the main body of science during many years, the publication of Lorenz’s (1963) work on weather prediction was followed by an outburst of new research on the study of deterministic, non-linear, systems with an irregular behavior, which will be called chaotic behavior.

A chaotic system is one in which long-term prediction of the system’s trajectories is impossible because any uncertainty on its initial state grows exponentially fast along time. This characteristic property is called sensitive dependence on initial conditions, and is the reason of the rapid loss of predictive power in chaotic systems. However, chaotic processes are deterministic and show a crucial difference with random processes. Even though chaos puts a fundamental limit to long-term prediction, it also suggests the possibility of short-term prediction based on the fact that random-looking data may contain simple deterministic relationships, involving only a few irreducible degrees of freedom.

Starting with the seminal work of Packard et al. (1980) and Takens (1981), the development of short-term forecasting techniques for chaotic time series was initiated by Farmer and Sidorowich (1987). These authors proposed a forecasting technique based on embedding a time series in a state space using delayed coordinates, and looking for past patterns using the nearest neighbor (NN hereafter) to a given reconstructed state. In this way, the NN method is a prediction technique where segments with a similar dynamic behavior are detected in the series and then used to define a next term at the end of the series, which is computed as some average of the actually observed terms next to the segments involved. Notice that the philosophy behind the NN approach is quite different from that of the Box-Jenkins methodology. In contrast to Box-Jenkins models, where extrapolation of past values into the immediate future is based on the correlation among lagged observations and error terms, NN methods select
relevant prior observations based on their levels and geometric trajectories, not their location in time.

The NN approach to forecasting financial time series is attractive, because it means a certain mixture of technical analysis and chaotic behavior. The chaos paradigm assumes that non-linear behavior is able to produce deterministic, apparently random, series that are predictable in the short term; whereas chartism assumes that pieces of financial series in the past can have a resemblance to pieces in the future. In this sense, Clyde and Osler (1997) show that non-linear forecasting techniques, based on the literature of complex dynamic systems, can be viewed as a generalization of these chartist graphical methods; that is, the NN prediction method can be considered as a developed and sophisticated chartism inspired in chaotic dynamics, where, in order to yield predictions, present patterns of a time series are compared with past patterns. So, the NN method can be thought as a sort of bridge between chaos theory and technical analysis.

However, even though several theoretical models on complex economic dynamics suggest the possibility of chaos (Pesaran and Potter, 1993), detecting chaotic behavior in financial time series is an elusive task at least for several reasons. One is the low power of the tests for detecting chaos in time series of short length, like those usually available in economics (Barnett et al., 1998). But the main reason is the fact that a small noise contamination might mask the dynamics of chaotic behavior. In order to avoid the shortcomings of the available procedures for detecting chaos, Hsieh (1991) has proposed the use of non-parametric locally weighted regressions to forecast returns, in order to detect deterministic chaos in financial markets. If stock returns were governed by chaos of low complexity, we should be able to make short-term NN predictions much better than using simple methods, such as the random walk model.

The NN approach has been used in forecasting several financial time series, but the results are rather inconclusive. So, for instance, Diebold and Nason (1990), Hsieh (1991), Meese and Rose (1991) or Mizrach (1992), concluded that there was little gain in predictive accuracy over a simple random walk. However, more promising results are reported in Bajo-Rubio et al. (1992), Lisi and Medio (1997), Fernández-Rodríguez et al. (1999), or Cao and Soofi (1999).

In this paper we will survey our contribution to this programme of research on predictability in financial markets, addressing the question of whether NN prediction methods can improve out-of-sample forecasting for several financial time series. The paper is organized as follows. The NN
approach to forecasting is described in Section 2, while Section 3 presents some economic examples from its application to predict the evolution of several financial variables, namely, exchange rates, stock markets, and interest rates. Some concluding remarks are provided in Section 4.

2. The nearest-neighbor approach to non-linear forecasting

As mentioned above, the NN method works by selecting some geometric segments in the past of the time series, similar to the last segment available before the observation we want to forecast [see Farmer and Sidorowich (1987)]. In this section we will describe succinctly the NN forecasting technique as follows [see Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) for a more detailed account):

1. We first transform the scalar series \( x_t \) (\( t=1,...,T \)) into a series of \( m \)-dimensional vectors, \( x^m_t \), \( t=m,...,T \):
   \[
   x^m_t = (x_t, x_{t-1}, ..., x_{t-m+1})
   \]
   with \( m \) referred to as the embedding dimension. These \( m \)-dimensional vectors are often called \( m \)-histories.

2. As a second step, we select the \( k \) \( m \)-histories
   \[
   x^m_{12}, x^m_{13}, ..., x^m_{1k},
   \]
   most similar to the last available vector
   \[
   x^m_T = (x_T, x_{T-1}, x_{T-2}, ..., x_{T-m+1}),
   \]
   where \( k=\text{int}(\lambda T) \) (0<\( \lambda \)<1), with \( \text{int}(\cdot) \) standing for the integer value of the argument in brackets, and where the subscript \( i_j \) (\( j=1,2,...,k \)) is used to denote each of the \( k \) chosen \( m \)-histories.

   To that end, we look for the closest \( k \) vectors in the phase space \( \mathbb{R}^m \), in the sense that they maximize the function:
   \[
   \rho(x^m_i, x^m_T)
   \]
   (i.e., looking for the highest serial correlation of all \( m \)-histories, \( x^m_i \), with the last one, \( x^m_T \)).

3. Finally, to obtain a predictor for \( x_{T+1} \), we consider the following local regression model:
   \[
   \hat{x}_{T+1} = \hat{\alpha}_0 x_T + \hat{\alpha}_1 x_{T-1} + ... + \hat{\alpha}_{m-1} x_{T-m+1} + \hat{\alpha}_m
   \]
   whose coefficients have been fitted by a linear regression of \( x_{i+1} \) on \( x^m_i = (x_i, x_{i-1}, ..., x_{i-m+1}) \) (\( r=1,...k \)). Therefore, the \( \hat{\alpha}_i \) are the values of \( \alpha_i \) that minimize
\[
\sum_{i=1}^{k} (x_{i+1} - \alpha_0 x_i - \alpha_1 x_{i-1} - \ldots - \alpha_{m-1} x_{i-m+1} - \alpha_m)^2
\]

Note that the NN predictors depend on the values of embedding dimension \(m\) and the number of closest \(k\) points in the phase space \(\mathbb{R}^m\). In the empirical applications reviewed in the next section, they are chosen according to Casdagli’s (1991) algorithm.

On the other hand, when we have a set of simultaneous time series, the NN prediction can be extended to a multivariate case using the *simultaneous nearest neighbor* predictors (SNN hereafter). To simplify, let us consider a set of two time series:

\[
x_t (t=1,...,T), y_t (t=1,...,T)
\]

We are interested in making predictions of an observation of one of these series (e.g., \(x_{T+1}\)), by simultaneously considering nearest neighbors in both series. To this end, we embed each of these series in the vectorial space \(\mathbb{R}^{2m}\), paying attention to the following vector:

\[
(x_{mT}, y_{mT}) \in \mathbb{R}^m \times \mathbb{R}^m
\]

which gives us the last available \(m\)-history for each time series.

In order to establish nearest neighbors to the last \(m\)-histories \((x_{mT}, y_{mT})\), we can look for the closest \(k\) points that maximize the function:

\[
\rho(x_{mT}, x_{mT}) + \rho(y_{mT}, y_{mT})
\]

The predictor for \(x_{T+1}\) and \(y_{T+1}\) can be obtained from a linear autoregressive predictor with varying coefficients estimated by ordinary least squares:

\[
\hat{x}_{T+1} = \hat{\alpha}_0 x_T + \hat{\alpha}_1 x_{T-1} + \ldots + \hat{\alpha}_{m-1} x_{T-m+1} + \hat{\alpha}_m
\]

\[
\hat{y}_{T+1} = \hat{\beta}_0 y_T + \hat{\beta}_1 y_{T-1} + \ldots + \hat{\beta}_{m-1} y_{T-m+1} + \hat{\beta}_m
\]

The difference between this SNN predictor and the NN predictor is that now the nearest neighbors are established using criteria in which information on both series is used.

### 3. The nearest-neighbor forecasting method: Some economic examples

#### 3.1. Exchange rates

Following the adoption of floating exchange rates in the early 1970s, economic literature made a remarkable effort in order to model exchange rates in terms of some fundamental macroeconomic variables, simply named “fundamentals”, that might affect their evolution. However, the empirical
performance of these exchange rate models in terms of “fundamentals” has been quite poor and, as exemplified in the now classical paper by Meese and Rogoff (1983), their predictive ability usually proved even worse than that provided by a simple random walk. In turn, this has led to the appearance of some new alternative explanations, based on the role of expectations, the possibility of multiple equilibria, and the non-linear behavior of exchange rates; see Frankel and Rose (1995) for a survey.

On the other hand, concerns about excessive exchange rate volatility and its possible adverse effects on the process of European integration prompted the establishment of the European Monetary System (EMS) in March 1979. A main element of the EMS was the Exchange Rate Mechanism (ERM), an adjustable peg system in which each currency had a central rate expressed in the European Currency Unit (ECU). These central rates determined a grid of bilateral central rates vis-à-vis all other participating currencies, and defined a band around these central rates within which the exchange rates could fluctuate freely. In order to keep these bilateral rates within the margins, the participating countries were obliged to intervene in the foreign exchange market if a currency approached the limits of its band, for which some special credit facilities were established. If the participating countries decided by mutual agreement that a particular parity cannot be defended, realignments of the central rates were permitted.

The ERM was the most prominent example of a target zone exchange-rate system. In the 1990s an extensive literature appeared, building on the seminal paper by Krugman (1991), which studied the behavior of exchange rates in target zones. The main result of the simple target zone model was that, with perfect credibility, the zone would exert a stabilizing effect (the so-called “honeymoon” effect), reducing the exchange rate sensitivity to a given change in “fundamentals”. However, in a target zone with credibility problems, expectations of future interventions would tend to destabilize the exchange rate, making it less stable than the underlying fundamentals (Bertola and Caballero, 1992). Therefore, credibility (i.e., the degree of confidence that the economic agents assign to the announcements made by policymakers) becomes a key variable. In the context of an exchange-rate target zone, credibility refers to the perception of economic agents with respect to the commitment to maintain the exchange rate around a central parity. Therefore, the possibility for the official authorities to change the central parity could be anticipated by the economic agents, triggering expectations of future changes in the exchange rate that can act as a destabilizing element of the system.

A first contribution applying the methods reviewed in the previous section is Bajo-Rubio, Fernández-Rodríguez and Sosvilla-Rivero (1992a), where daily data for the Spanish peseta-US dollar, spot and one- and three-month forward exchange rates, during the period January 1985-May 1991, were
used in the empirical application; a deeper discussion of the methods used in the paper can be found in Bajo-Rubio, Fernández-Rodríguez and Sosvilla-Rivero (1992b). In that paper, several predictors based on the NN methodology were computed, and their performance was compared with that of a simple random walk, by calculating their respective forecasting errors, as measured by the root mean square error (RMSE). In general, the non-linear predictors outperformed the random walk in all cases for the forward rates, whereas for the spot rate this only occurred in four over nine cases.

The objective of Sosvilla-Rivero, Fernández-Rodríguez and Bajo-Rubio (1999) was to compute an indicator of volatility, defined as the (absolute value of the) forecast error, derived from the NN predictors, weighted by the standard deviation of the original series. This indicator was applied to six EMS currencies experiencing different evolutions after the crisis that affected the system after the summer of 1992, that lead to the broadening of the fluctuation bands in August 1993: two of them “temporarily” leaving the ERM (Italian lira and British sterling), two others forced to devalue (Spanish peseta and Portuguese escudo), and the remaining two not devaluing (French franc and Dutch guilder), with the sample period running from January 1974 to April 1995. The volatility indicators showed an initial low degree of exchange rate volatility, with a sudden increase from September 1992 on. Then, volatility remained high for the currencies that abandoned the ERM; however, for the rest of the currencies, the broadening of the bands after August 1993 would have led to a decrease in volatility to levels comparable with those prevailing before the crisis. These results were interpreted, rather than in terms of an unexpected loss of credibility, as being a consequence of the fragility of the EMS in a world of very high international capital mobility, which became evident with the problems associated with German reunification at the end of 1989 and the effects of self-fulfilling speculative attacks (Eichengreen and Wyplosz, 1993).

The above papers compute predictions for each variable using information from the own series, i.e., in a univariate (NN) context. In a later contribution, Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) applied the SNN predictors, i.e., using the information content of a wider set of time series, to nine currencies participating in the ERM. The use of SNN predictions in this context can be seen as an attempt to incorporate structural information into the non-parametric analysis. The data set includes daily observations of nine exchange rates (Belgian franc, Danish crown, Portuguese escudo, French franc, Dutch guilder, Irish pound, Italian lira, Spanish peseta, and British sterling) vis-à-vis the German mark, covering the period January 1978-December 1994. When choosing the related exchange-rate series in order to establish occurring analogues for the SNN predictor, three groups of currencies were considered, according with the credibility with respect to the commitment to maintain the exchange rate around the central parity (see
Ledesma-Rodríguez et al., 2001). It is interesting to note that these groups roughly corresponded to those found in Jacquemin and Sapir (1996), by applying principal components and cluster analyses to a wide set of structural and macroeconomic indicators, in order to form a homogeneous group of countries.

After finding evidence of non-linear dependence in the series using the well-known BDS test statistic (see Brock et al., 1996), hence supporting their approach to forecasting, the authors evaluated the forecasting performance by means of Theil’s U statistic, i.e., the ratio of the RMSE from the NN predictors to the RMSE from the random walk, so that a value of U less than one indicates better performance than the random walk specification. Table 1 shows the forecasting performance, relative to the random walk, from both the SNN predictors and the traditional ARIMA(1,1,0) models. As can be seen, the U statistics were, for the SNN predictors, above one only in three of the nine cases, suggesting that the non-linear predictors marginally outperformed the random walk, despite the forecasting period being very long and heterogeneous, with the best SNN predictor presenting an improvement of 18.9%. We can also see that the predictors from an ARIMA(1,1,0) model always show U statistics below one, the best one showing an improvement of 12.2% out of sample. Nevertheless, in six out of nine cases, the SNN predictors show lower U statistics than the ARIMA(1,1,0) model.

**Table 1: Forecast accuracy (U statistic)**

<table>
<thead>
<tr>
<th></th>
<th>SNN predictor</th>
<th>ARIMA (1,1,0) predictor</th>
</tr>
</thead>
<tbody>
<tr>
<td>BFR&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.984</td>
<td>0.995</td>
</tr>
<tr>
<td>DKR&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.939</td>
<td>0.954</td>
</tr>
<tr>
<td>ESC&lt;sup&gt;c&lt;/sup&gt;</td>
<td>1.016</td>
<td>0.997</td>
</tr>
<tr>
<td>FF&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.908</td>
<td>0.952</td>
</tr>
<tr>
<td>HFL&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.811</td>
<td>0.878</td>
</tr>
<tr>
<td>IRL&lt;sup&gt;d&lt;/sup&gt;</td>
<td>1.014</td>
<td>0.997</td>
</tr>
<tr>
<td>LIT&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.973</td>
<td>0.981</td>
</tr>
<tr>
<td>PTA&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.995</td>
<td>0.999</td>
</tr>
<tr>
<td>UKL&lt;sup&gt;d&lt;/sup&gt;</td>
<td>1.022</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Notes: <sup>a</sup> BFR, DKR, ESC, FF, HFL, IRL, LIT, PTA and UKL denote, respectively, the Belgian franc, the Danish crown, the Portuguese escudo, the French franc, the Dutch guilder, the Irish pound, the Spanish peseta and the British sterling.  
<sup>b</sup> Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF, and HFL.  
<sup>c</sup> Time series used in establishing occurring analogues in the SNN predictor: ESC, LIT and PTA.  
<sup>d</sup> Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.

As a further test of forecasting performance, the percentage of correct predictions was also computed. According to the results in Table 2, in eight of nine cases, the SNN predictors showed a value higher than 50% (taken as the usual benchmark), clearly outperforming the random walk directional forecast; and, in seven out of the nine cases, the SNN predictors presented higher values than the ARIMA model. In addition, the forecast accuracy of the SNN predictors was formally assessed by means of the test proposed by Diebold and Mariano (1995), on the (corrected) value of the differential between two forecasting errors; as well as the Pesaran and Timmerman (1992) non-parametric test, on the proportion of correctly predicted signs. Overall, the evidence presented in Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (1999) suggest that, when predicting exchange-rate time series, some forecast accuracy can be gained by considering the information content of other related exchange rates through SNN predictors.

<table>
<thead>
<tr>
<th></th>
<th>SNN predictor</th>
<th>ARIMA (1,1,0) predictor</th>
</tr>
</thead>
<tbody>
<tr>
<td>BFR</td>
<td>63.23</td>
<td>52.19</td>
</tr>
<tr>
<td>DKR</td>
<td>67.35</td>
<td>63.73</td>
</tr>
<tr>
<td>ESC</td>
<td>55.95</td>
<td>53.86</td>
</tr>
<tr>
<td>FF</td>
<td>67.43</td>
<td>62.02</td>
</tr>
<tr>
<td>HFL</td>
<td>69.19</td>
<td>65.34</td>
</tr>
<tr>
<td>IRL</td>
<td>57.73</td>
<td>51.77</td>
</tr>
<tr>
<td>LIT</td>
<td>57.25</td>
<td>54.83</td>
</tr>
<tr>
<td>PTA</td>
<td>50.64</td>
<td>55.33</td>
</tr>
<tr>
<td>UKL</td>
<td>47.58</td>
<td>51.21</td>
</tr>
</tbody>
</table>

Notes:  
\(^a\) Percentage of correct forecast direction.  
\(^b\) Time series used in establishing occurring analogues in the SNN predictor: BFR, DKR, FF, and HFL.  
\(^c\) Time series used in establishing occurring analogues in the SNN predictor: ESC, LIT and PTA.  
\(^d\) Time series used in establishing occurring analogues in the SNN predictor: IRL and UKL.


Finally, Fernández-Rodríguez, Sosvilla-Rivero and Andrada-Félix (2002) assessed the economic significance of the predictability of EMS exchange rates by transforming the SNN predictions into a technical trading rule, whose profitability was evaluated against the traditional moving average trading rules, taking into account both interest rates and transaction costs. Their results suggested that, in most cases, a trading rule based on a non-linear predictor outperformed the moving average, both in terms of returns and in terms of the ideal profit and the Sharpe ratio profitability indicators.
3.2. Stock markets

During the last decades, special attention has been paid to testing for predictable components in stock prices [see, e.g., Breen, Glosten and Jagannathan (1990) or Pesaran and Timmerman (1995, 2000), and Fama (1991) for a review], despite its implication for market efficiency. The existence of patterns in stock markets can be exploited to improve stock-market predictability.

Many empirical studies have uncovered significant non-linearities in stock prices [see, for example, Hsieh (1991), Abhyankar, Copeland and Wong (1995), and Ryden, Teräsvirta and Asbrink (1998)]. Economic theory highlights a number of potential sources for the presence of non-linearities in stock prices. The main explanations include diversity in agents’ beliefs (e.g., Brock and Hommes, 1998), heterogeneity in investors’ objectives arising from varying investment horizons and risk profiles (e.g., Peters, 1994) or herd behaviour (Lux, 1995) [see Hommes (2001) for a survey].

Fernández-Rodríguez, Sosvilla-Rivero and García-Artiles (1997) used daily closing prices of the General Index of the Madrid Stock Market (IGM) and the Standard & Poors 500 Index of the New York Stock Market (SP500), covering the period January 1968-January 1994. After finding evidence of non-linearity that supports this approach to forecasting, they assessed the forecasting performance of the NN predictors for the IGM in both versions: univariate (NN) and multivariate (SNN). In the latter case, the SP500 series was used for establishing nearest neighbors.

When evaluating the forecasting performance by means of Theil’s U statistic, the results for the NN case, shown in the upper part of Table 3, were greater than one only in two of the 36 cases, and less than 0.99 in 30 of the 36 cases, suggesting that the NN predictors marginally outperformed the random walk. Note that the best NN predictor presented an improvement of 2.98%. On the other hand, from the lower part of Table 3, we see that in all 36 cases, SNN predictors offered lower U statistics than the NN case, the best SNN predictor showing an improvement of 5.95% out-of-sample.
Table 3: Forecast accuracy (U statistic)

(A) NN predictor

<table>
<thead>
<tr>
<th></th>
<th>m=2</th>
<th>m=3</th>
<th>m=4</th>
<th>m=5</th>
<th>m=6</th>
<th>m=7</th>
</tr>
</thead>
<tbody>
<tr>
<td>k=90</td>
<td>0.9898</td>
<td>1.0004</td>
<td>0.9796</td>
<td>0.9771</td>
<td>0.9705</td>
<td>0.9786</td>
</tr>
<tr>
<td>k=100</td>
<td>0.9894</td>
<td>1.0004</td>
<td>0.9771</td>
<td>0.9753</td>
<td>0.9739</td>
<td>0.9706</td>
</tr>
<tr>
<td>k=110</td>
<td>0.9860</td>
<td>0.9989</td>
<td>0.9799</td>
<td>0.9788</td>
<td>0.9760</td>
<td>0.9706</td>
</tr>
<tr>
<td>k=120</td>
<td>0.9893</td>
<td>0.9949</td>
<td>0.9801</td>
<td>0.9791</td>
<td>0.9757</td>
<td>0.9702</td>
</tr>
<tr>
<td>k=130</td>
<td>0.9883</td>
<td>0.9946</td>
<td>0.9807</td>
<td>0.9727</td>
<td>0.9730</td>
<td>0.9734</td>
</tr>
<tr>
<td>k=140</td>
<td>0.9862</td>
<td>0.9950</td>
<td>0.9824</td>
<td>0.9727</td>
<td>0.9763</td>
<td>0.9701</td>
</tr>
</tbody>
</table>

(B) SNN predictor

<table>
<thead>
<tr>
<th></th>
<th>m=2</th>
<th>m=3</th>
<th>M=4</th>
<th>m=5</th>
<th>m=6</th>
<th>m=7</th>
</tr>
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<td>k=90</td>
<td>0.9725</td>
<td>0.9786</td>
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As a further test of the forecasting performance of the NN predictors relative to the forecasts of a random walk, accuracy in predicting the direction of IGM movements was evaluated by computing the percentage of correct predictions. As can be seen in Table 4, these percentages were higher than 50 for both NN and SNN predictors, clearly outperforming the random walk directional forecast. It is interesting to note that in 30 out of the 36 cases, the SNN predictors offered higher values than the NN case.

Table 4: Directional forecast

(A) NN predictor

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(B) SNN predictor

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Finally, the authors considered two tests of forecast accuracy. The Diebold-Mariano test suggested that both the NN and SNN predictors were not statistically significantly better predictors than the random walk. However, the Williams-Kloot test (Williams, 1959) showed that the SNN predictor was superior to the random walk in 16 out of 36 cases, and that in only 7 out of 36 cases the SNN predictors were superior to the NN case.

Therefore, the evidence presented in Fernández-Rodríguez, Sosvilla-Rivero and García-Artiles (1997) suggested that in predicting the IGM time series, some forecast accuracy can be gained by considering the information content of other related stock prices (in this case, SP500). In addition, when assessing the economic value of the local predictors, the results of using non-linear predictions in a filter technique were always superior to a buy-and-hold strategy.

In a further paper, Fernández-Rodríguez, Sosvilla-Rivero and García-Artiles (1999) applied the NN predictors to the Nikkei 225 Index of the Tokyo Stock Market for the period 1 January 1986-5 June 1997. When forecasting performance was measured by Theil’s U statistic, the NN predictors performed worse than a random walk, outperforming the random walk directional forecast. When formally testing for forecast accuracy, the results suggested that predictions from a random walk were statistically significantly better than the NN predictors for the entire forecasting period, as well as for one of the subperiods (a “bull” market episode). Finally, when assessing the economic value of the NN predictors in absence of trading costs, the results of using them as a filter technique were superior to a buy-and-hold strategy for both the entire forecasting period and for “bear” market subperiods, where tests of “forecast conditional efficiency” (or “forecast encompassing”) detected that the NN predictors contained useful information for forecasting the Nikkei Index that was not contained in the random walk.

Finally, Andrada-Félix et al. (2001) investigated the profitability of non-linear trading rules based on NN predictors. Applying this investment strategy to the New York Stock Exchange, their results suggested that, taking into account transaction costs, the non-linear trading rule was superior to a risk-adjusted buy-and-hold strategy (both in terms of returns and of Sharpe ratios) for the 1998 and 1999 periods of upward trend. In contrast, for the relatively “stable” market period of 2000, both strategies were found to generate equal returns, although the risk-adjusted buy-and-hold strategy yielded a higher Sharpe ratio.
3.3. Interest rates

A recurrent line of research within international macroeconomics is the so-called $n-1$ problem faced by fixed exchange rate systems: there are $n$ countries pegging their exchange rates but only $n-1$ exchange rates among them, which gives the system one degree of freedom when setting money supply and the interest rate. This degree of freedom can be used either in an asymmetric (i.e., hegemonic) way, by enabling one country to become the leader and settle monetary policy independently, with the other countries following its stance; or, alternatively, in a symmetric (i.e., cooperative) way, so that all countries are allowed to decide jointly over the implementation of monetary policy (De Grauwe, 2000).

The EMS has been no exception to this problem. However, and despite the initial objectives of the founders of the EMS, a general consensus has emerged that the system worked in an asymmetric way, with Germany assuming the leading role and the remaining countries passively adjusting to German monetary policy actions. In its turn, the follower countries may find beneficial to behave in such a way, since they can take advantage of the firmly established anti-inflation credibility of the German Bundesbank (Giavazzi and Pagano, 1988). On the other hand, the launching of the Economic and Monetary Union (EMU) on January 1st 1999, which superseded the EMS with the adoption of a common currency (the euro) and the starting of the European Central Bank’s operations, has meant the loss of monetary independence of the now 12 participating countries. Therefore, a careful examination of the German dominance hypothesis into the EMS would be a way to assess whether the loss of monetary autonomy in Europe associated with EMU had been significant, which, in turn, could be taken as an argument in favor of EMU itself.

In this way, several studies have tested this hypothesis, i.e., the German dominance into the EMS, most of them by performing Granger-causality tests between German and other countries’ interest rates; a non-exhaustive list would include, among others, Cohen and Wyplosz (1989), von Hagen and Fratianni (1990), Koedijk and Kool (1992), Henry and Weidmann (1995), or Bajo-Rubio and Montávez-Garcés (1999).

In Bajo-Rubio, Sosvilla-Rivero and Fernández-Rodríguez (2001) some additional evidence on the hypothesis of German leadership and asymmetric performance in the EMS was provided, using high frequency (i.e., daily) data for the period March 1979-April 1998. Unlike previous studies, the empirical methodology made use of non-linear forecasting methods based on the literature on complex dynamic systems, extended to allow testing for Granger-causality. Also, they extended the analysis to nine EMS countries using daily three-month
interbank interest rates, and included in the sample the more recent events in the EMS history, such as the German reunification, the monetary turmoil at the end of 1992, and the broadening of fluctuation bands in 1993.

The traditional method for testing causality in economic time series is based on the Granger definition of causality (Granger, 1969). Given two variables, $x$ and $y$, $x$ is said to Granger-cause $y$ if the latter can be predicted better by past values of $x$ and $y$, rather than by past values of $y$ alone. The variable under analysis was the three-month interbank interest rate of the seven countries participating at the ERM from its start in 1979: Belgium, Denmark, France, Germany, Ireland, Italy, and the Netherlands, as well as the three newcomers to the ERM (in 1989, 1990 and 1992, respectively): Spain, the United Kingdom, and Portugal. From the above definition, the test proceeds by using $x$ and $y$ as the independent variables, so that four results are possible: $x$ Granger-causes $y$, $y$ Granger-causes $x$, two-way Granger-causality, and no Granger-causality. In practice, the criterion for assessing Granger-causality consists of comparing the prediction errors (PE) from both information sets. Formally, denoting by $y'_t$, the prediction of $y_t$, if

$$\text{PE}(y'_t \mid Y_{t-1} \cup X_{t-1}) < \text{PE}(y'_t \mid Y_{t-1})$$

then $x$ Granger-causes $y$, where $X_{t-1}$ and $Y_{t-1}$ are, respectively, the set of all past information on variables $x$ and $y$ existing at time $t$.

As an alternative to this traditional approach, Bajo-Rubio, Sosvilla-Rivero and Fernández-Rodríguez (2001) proposed to compute NN predictors based on past information of a variable and compared its prediction error with that from a SNN predictor on the same variable using the information content of other related variable. After finding evidence of non-linearity in the time series examined by means of the BDS test statistic, the NN and SNN predictors were computed. Table 5 shows the forecasting performance, measured by the RMSE, of the predictors in both versions (univariate, NN, and bivariate, SNN), for the whole period. In the bivariate case, the interest rate of Germany was used for establishing occurring analogues for each of the remaining countries, and *vice versa*. Then, by comparing the RMSEs, the last column reports the result of the causality test, so that if the RMSE in the bivariate case were lower (higher) than the RMSE in the univariate case, there would be (there would not be) causality from the first country to the second.
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**Note:** The forecasting period is 13-1-87 to 30-4-98 for Belgium, Denmark, France, Ireland, Italy, and the Netherlands, 19-6-89 to 30-4-98 for Spain, 08-10-90 to 30-4-98 for the United Kingdom, and 9-4-92 to 30-4-98 for Portugal.

**Source:** Bajo-Rubio, Sosvilla-Rivero and Fernández-Rodríguez (2001).

As can be seen in the table, the interest rates in all the countries considered could be predicted better by adding German interest rates to the past values of the interest rates in every country, rather than by past values of national interest rates alone. On the other hand, causality was also found running from interest rates in Belgium, Denmark and the Netherlands to those in Germany. Note also that, when two-way causality was found, the reduction in RMSEs was greater for forecasts of national interest rates using information
about German interest rates than in the cases of German interest rates using information about other national rates. This could be taken as a first indication that, in these cases, the German influence on the other country was stronger than the other way round.

In order to evaluate forecasting accuracy, in the sense of testing whether the differences between RMSEs obtained above were statistically significant or not, the Diebold-Mariano test was used. The results showed that SNN predictors would be statistically significantly better predictors than NN predictors, when predicting interest rates in all the countries considered by adding the information content of the German interest rates; and that, except for Belgium, Denmark and the Netherlands, SNN local predictors would not be statistically significantly better predictors than NN predictors. Overall, the results from the Diebold-Mariano test reinforced the earlier conclusion from the table above.

As a further check of the robustness of these results, the sample was divided into two parts, before and after 29 November 1990, in order to investigate the possible consequences of the German reunification on the working of the EMS. In general, the results obtained, both from the computation of the RMSEs and the Diebold-Mariano test, although similar for the first subperiod, changed strikingly after the German reunification. Then, causality was only found running from Germany to all other countries, except for the Netherlands, where two-way causality was still detected. Accordingly, it would seem that German leadership in the EMS would have increased after reunification.

Finally, a comparison of the previous results with those obtained using standard linear econometric techniques was performed. Although the standard approach led to somewhat different results, when computing Diebold-Mariano tests to assess the forecasting accuracy of both predictors, the non-linear predictors clearly outperformed in all cases the standard linear predictors. This, in turn, would suggest that inference on causality based on the non-linear predictors would be more appropriate for the issues analyzed in that paper, and preferable to that based on the standard linear approach.

As a general conclusion, the results of the paper could be taken as an indication of the special role played by Germany within the EMS, even though “dominance” in a strict sense was not found. In addition, these results would suggest a relatively low cost of giving up monetary sovereignty by the non-German EMS countries, once EMU is in force.
4. Concluding remarks

The purpose of this paper has been to contribute to the debate on the relevance of non-linear forecasting methods for high-frequency data in financial markets. To that end, we first presented a brief description of one of this techniques, namely, the nearest neighbor (NN) approach to forecasting, and then showed some economic examples from its application to the prediction of several financial variables: exchange rates, stock markets, and interest rates.

The basic idea behind these predictors is that pieces of time series sometime in the past might have a resemblance to pieces in the future. In order to generate predictions, similar patterns of behavior are located in terms of nearest neighbors. The time evolution of these nearest neighbors is exploited to yield the desired prediction. Therefore, the procedure only uses information local to the points to be predicted and does not try to fit a function to the whole time series at once.

The economic examples reviewed in the paper illustrate the usefulness of the approach. In particular, we observed in all cases a generally better performance, in terms of forecasting accuracy, of the NN predictors, so that the usage of the non-linear approach might reveal preferable as compared to other more standard techniques, when forecasting and making economic inferences from the behavior of financial variables.
References


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2001-20: “Son relevantes el capital humano y el mercado de trabajo en los modelos de contabilidad generacional? Un estudio sobre el caso español”, Javier Alonso Meseguer.

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