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Secondhand market and the lifetime of durable goods*

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Abstract

This paper examines how a scrappage subsidy affects the lifetime of durable goods when secondhand markets are present. To this end, we propose a vintage model to analyze the replacement decision in an economy in which high and low income agents trade on a durable good. Thus, the existence of a secondhand market for this durable good has influence both on the price of used durables as well as on durables replacement. The value of the durable is decreasing with age so it is the price in the secondary market. We also discuss the implications of this model specification for the effects of second hand markets on the average age of the stock of durable goods.
Key words: Vintage capital, Durable goods, Secondhand market.

1 Introduction

Agents' expenditure on durable goods is an important component of aggregate consumption and one that considerably fluctuates over the business cycle. Among other reasons, this might be due to the fact that durable goods consumption involves fundamentally a replacement problem that can be governed by echoes. On the other hand, used durable goods compete with new goods, and the secondhand market is an important factor in durable goods purchases. In this paper we want to investigate the effects that the explicit consideration of a secondhand market for a durable good has for replacement activities focusing on the steady state. To this purpose we specify the durable goods replacement problem in line with vintage capital investment models and where secondhand trade arises from heterogeneous consumers.

Several authors have documented the presence of active secondhand markets for used durable goods and equipment. Porter and Sattler (1999) focus on the rapid growth of the used car market in the recent years because of fleet sales, leasing and improved reliability of used vehicles. Goolsbee (1998) documents mass sales of Boeing 707's to the developing world after the plane was effectively retired from use by US airlines. Fernández (2000) observes that a large fraction of appliances sales are accounted for by replacement that usually takes place prior to irreparable failure. While these pieces of evidence may correspond to somewhat unrelated phenomena all of them raise questions on the role secondhand markets in understanding the behavior of durable consumption expenditures as well as some forms of equipment purchases.

A substantial body of related theoretical literature focuses on the effects of durability and secondhand markets on equilibrium firm behavior, notably in the car market. Also, existing literature has not ignored the dynamics of consumers' car demand.[1] Unlike this research, we abstract from firm's production decisions and the demand for new durable goods is not explicitly modelled. Our emphasis is on the consumers' replacement decisions and the endogenous determination of steady state secondhand market's prices as a function of durable goods ages. From the household perspective, the stock of durable goods is only occasionally adjusted and aggregate observations come from the aggregation over heterogeneous consumers. Therefore, the potential gains from trade between heterogeneous consumers should be taken into account as can have influence on the secondhand market prices and quantities. Here, we explore these questions, focusing on the steady state.

Finally we also explore the response of aggregate car sales to the introduction of a scrapping subsidy. Fleet (cars, planes,...) renewal incentives and scrappage schemes that may have important implications for environmental, support the industry or lifting economic growth purposes. Adda and Cooper (2000) have studied the effects of tax credits on durable goods markets in a dynamic stochastic discrete choice model of car ownership at the household level. They find positive effects for the industry in the short-run followed by subsequent

[1] Indeed, much attention has been paid to the replacement problem in the optimal control literature since Rust's (1985) seminal contribution.

low activity. All these works abstract from secondhand trade considerations notwithstanding. Here we find the same conclusions in an autarchy model and show that the introduction of secondhand markets for durables can reduce the positive effect of a scrapping subsidy.

The rest of the paper is organized as follows. Section 2 analyzes the steady state used goods reservation value for a consumer with concave utility both in durable and non–durable goods. Section 3 studies an economy with two different types of consumers, both in autarchy and with a secondhand market for durable goods. In Section 4 we make some computational experiments to show the consequences of a secondhand market for steady state durables sales and average age of the introduction of a replacement scheme. Finally, Section 5 presents conclusions.

2 A simple illustration: identical agents

Let us assume that there exists a continuum of agents of measure one, all with the same instantaneous preferences defined over the consumption of a non-durable good and services of a durable good, let say a car. Services of a particular car are decreasing with age at the rate $\delta > 0$. This can be interpreted as reflecting depreciation or maintenance.

At any time t all agents have the same instantaneous exogenous income, $y > 0$. Further, we assume that there is always the same number of consumers and durables. This assumption combined with considering car services decreasing with age guarantees the occurrence of both scrappage and replacement. Since we are only interested in the replacement of durable goods, we don't specify the demand for new durables. At the initial time, there is a distribution of cars by age, which is given by past purchases, $\bar{m}(t)$ for all $t \in [-T(0), 0[$, where $T(0)$ satisfies

$$\int_{-T(0)}^{0} \bar{m}(z)\ \mathrm{d}z = 1.$$

The measure of operative durables is equal to the measure of the population, which allows to define the initial lifetime of durables $T(0)$. The function $\bar{m}(t)$ is supposed to be strictly positive and continuously differentiable. To produce one unit of the durable good we need $\frac{1}{\alpha}$ units of the non-durable good, $\alpha > 0$. Additionally, the scrapping value of a car is $\beta \in [0, \alpha[$, such that the replacement cost of an old for a new durable is $\alpha - \beta$. An optimal solution for this economy is a set of paths $\{c(t), m(t), T(t), s(t)\}_{t=0}^{\infty}$ which solve the following maximization problem,

$$\max \int_{0}^{\infty} \left(\frac{c(t)^{1-\sigma}}{1-\sigma} + \gamma \frac{s(t)^{1-\sigma}}{1-\sigma} \right)\ \mathrm{e}^{-\rho t}\ \mathrm{d}t$$

s.t.

$$s(t) = \int_{t-T(t)}^{t} m(z)\ \mathrm{e}^{-\delta(t-z)}\ \mathrm{d}z$$

$$\int_{t-T(t)}^{t} m(z)\ \mathrm{d}z = 1 \tag{1}$$

$$c(t) + (\alpha - \beta)\, m(t) = y \tag{2}$$

given $m(t) = \bar{m}(t)$ for all $t \in [-T(0), 0[$. $c(t)$ denotes the per-capita consumption of the non-durable good, $m(t)$ is the per-capita durable purchases, and $s(t)$ represents per-capita services of the durable stock. The parameter σ is strictly positive and different from unity and $\gamma > 0$ reflects the preference for durable services. Eq. (1) represents a saturation constraint, i.e., the number of durables in the economy is the same as the number of consumers. The last restriction is the feasibility constraint.

The optimal condition for this problem —see Appendix A.1 for a detailed discussion of the solution—, is

$$(\alpha - \beta)\, c(t)^{-\sigma} = \int_{t}^{t+J(t)} \gamma s(z)^{-\sigma} \left(\mathrm{e}^{-\delta(z-t)} - \mathrm{e}^{-\delta T(z)}\right) \mathrm{e}^{-\rho(z-t)}\ \mathrm{d}z, \tag{3}$$

where

$$J(t) = T(t + J(t)). \tag{4}$$

In terms of utility, the left hand side of (3) is the replacement cost, and the right hand side is the marginal utility of replacing, i.e., the discounted flow during the expected life time of the new durable, of durable services minus the discounted services of the oldest durable in the economy at each moment during the lifetime of this new durable, all weighted by the marginal utility of durable services.

2.1 Steady state

A steady state for this problem must verify $c(t) = c$, $m(t) = m$, $s(t) = s$ and $T(t) = J(t) = T$. Consequently,

$$\underbrace{\left(\frac{\alpha - \beta}{\gamma}\right)\left(y - \frac{\alpha - \beta}{T}\right)^{-\sigma}\left(\frac{1 - \mathrm{e}^{-\delta T}}{\delta T}\right)^{\sigma}}_{\equiv A(T)} = \underbrace{\frac{1 - \mathrm{e}^{-(\delta+\rho)T}}{\delta + \rho} - \mathrm{e}^{-\delta T}\frac{1 - \mathrm{e}^{-\rho T}}{\rho}}_{\equiv B(T)} \tag{5}$$

$$m = \frac{1}{T} \tag{6}$$

$$c = y - (\alpha - \beta)\, m. \tag{7}$$

$$s = \frac{1 - \mathrm{e}^{-\delta T}}{\delta T} \tag{8}$$

Proposition 1 *There exists a unique steady state with strictly positive T, m, s and c.*

Proof. The right hand side of (5) has the following properties:

1. $B(0) = 0$ and $\lim_{x\to\infty} B(x) = \frac{1}{\delta+\rho}$

2. $B'(x) = \delta e^{-\delta x} \frac{1- e^{-\rho x}}{\rho}$, with $B'(0) = 0$.

The left hand side has the following properties:

1. $\lim_{x\to 0} A(x) = 0$ and $\lim_{x\to\infty} A(x) = 0$

2. $A'(x) < 0$ all $T > \frac{\alpha-\beta}{y}$.

■

Therefore, as we can observe in Figure 1,[2] a unique steady state equilibrium for strictly positive T, m, s and c exists.

INSERT FIGURE 1 ABOUT HERE

It is interesting to notice that the value of a durable is decreasing with age, and the price in the secondhand market is also decreasing. Let us call $p(a)$ the steady state shadow price of a durable aged $a \in [0, T[$. From equation (3) we have

$$(\alpha - \beta) = \gamma(c/s)^{\sigma} \int_0^T \left(e^{-\delta z} - e^{-\delta T}\right) e^{-\rho z}\, dz,$$

Therefore, the shadow price of a durable held for $T - a$ periods —of age a— is

$$p(a) = \gamma(c/s)^{\sigma} \int_a^T \left(e^{-\delta z} - e^{-\delta T}\right) e^{-\rho(z-a)}\, dz + \beta$$

Let $u = z - a$. Then, the previous equation gives

$$p(a) = \gamma(c/s)^{\sigma} \int_0^{T-a} \left(e^{-\delta(u+a)} - e^{-\delta T}\right) e^{-\rho u}\, du + \beta, \tag{9}$$

with $p(0) = \alpha$, $p(T) = \beta$. It is very easy to show that the secondary market price is decreasing with durables age, for $a \in [0, T[$

$$p'(a) = -\delta\, \gamma(c/s)^{\sigma} \int_0^{T-a} e^{-\delta(u+a)}\, e^{-\rho u}\, du < 0$$

Figure 2, drawn with the same parameters as the previous one, represents the secondary market price for cars as a decreasing function of age, as well as the scrapping value —the horizontal line— as independent of age. This figure was drawn using equation (9).

INSERT FIGURE 2 ABOUT HERE

[2] In order to draw this figure, we assume that $\alpha = 1$, $\beta = .05$, $y = 1.75$, $\sigma = 4$, $\rho = .05$, $\gamma = 1.4$ and $\delta = .001$. The corresponding scrapping age is around 14 years and in average the weight of durable purchases in total income is around 4%.

3 An economy with two types of agents

Let us assume that the economy is populated by two types of agents, with different income flows, $y_1 > y_2$ which are exogenous. A fraction $n_1 \in]0,1[$ of total population is rich with income y_1. A fraction $n_2 = 1 - n_1$ is poor with income y_2 and always buys a used car when there are not secondhand market restrictions. Although the population structure and his market participation decision is taken as exogenous, below we will see that, under reasonable assumptions, it can not happen in equilibrium that rich people buy used and poor people buy new.

3.1 Secondary market imperfections

In order to illustrate the problem, let us assume that the secondary market is segmented by agent types. In this case, the steady state secondary market price is obtained from equation (9) and, as shown in Figure 3, up to scrapping this price is always larger for poor agents (dotted line) than for rich agents.[3] Notice that the marginal value of wealth is constant with homothetic preferences. Thus, the difference is given by the opportunity cost of holding a car of a given age. This opportunity cost is given by the difference between the services provided by that durable and by the oldest durable in use. As the oldest durable in use for poor people is older than the oldest of rich people, the value is higher for poor people. Indeed, if segmentation were eliminated, poor agents would be interested in buying durables in the secondary market from rich agents and rich agents would be interested in selling.

INSERT FIGURE 3 ABOUT HERE

3.2 Efficient secondary market

3.2.1 Rich agents

At the competitive equilibrium the behavior of rich agents is characterized by the solution of the maximization problem described in the previous section but for the feasibility constraint (2) which can be rewritten as

$$b_1'(t) = r(t)b_1(t) + y_1 - c_1(t) - (\alpha - p(t, T_1(t)))\, m_1(t)$$

First, notice that we index all variables in the problem solved by the rich agents with subscript 1. Second, we allow rich people to borrow from (or lend to) poor agents: $b_1(t)$ represents the stock of assets per-capita and $r(t)$ the interest rate. Finally, and more important, cars are now sold in a secondary market at the price $p(t, T_1(t)) > \beta$ which depends on car's age. Of course, for this problem to be completely specified we require $b_1(0)$ and $m_1(t) = \bar{m}_1(t)$ for all $t \in [-T_1(0), 0[$ to be given. The complete solution of this problem is given in Appendix A.2.

[3] Under the same assumptions on parameters than in the previous figure, we assume that a rich agent has $y_1 = 2$ and a poor agent has $y_2 = 1.5$. The corresponding scrapping ages are around 10 and 20 years, respectively.

3.2.2 Poor agents

To specify the problem solved by poor, we proceed correspondingly. Therefore, we have

$$b_2'(t) = r(t)b_2(t) + y_2 - c_2(t) - (p(t, A(t)) - \beta)\, m_2(t)$$

where now type 2 agents buy in the secondary market at the price $p(t, A(t)) < \alpha$ a durable aged $A(t)$.[4] Further, cars services for poor agents are given in this case by

$$s_2(t) = \int_{t-T_2(t)}^{t} m_2(z)\, \mathrm{e}^{-\delta(t-z+A(z))}\, \mathrm{d}z$$

It is worth noting that we are taking into account the acquisition age of a particular durable by poor agents in evaluating cars services, $s_2(t)$. Consequently, variable $T_2(t)$ measures poor's tenure period but not age. Finally, $A(t)$ is a control, and $b_2(0)$, $m_2(t) = \bar{m}_2(t)$ and $A_1(t) = \bar{A}_1(t)$ for all $t \in [-T_2(0), 0[$ have to be given.

The optimallity conditions for the maximization problems solved by poor agents are discussed in Appendix A.2 as well.

3.2.3 Equilibrium

In addition to the optimallity conditions, at an equilibrium total assets must be zero, $n_1 b_1(t) + n_2 b_2(t) = 0$. Also, the acquisition age for poor agents must be equal to the sale age for rich agents, $A(t) = T_1(t)$. Finally, the total number of new cars bought by rich agents must be equal to the total number of used cars bought by poor agents.[5] That is,

$$n_1 m_1(t) = n_2 m_2(t) \tag{10}$$

These conditions, together with the instantaneous budget constraints of the two types of agents imply the feasibility constraint:

$$y \equiv n_1 y_1 + n_2 y_2 = n_1 c_1(t) + n_2 c_2(t) + (\alpha - \beta)\, n_1 m_1(t).$$

[4] To clear the secondary market, it could arrive that poor agents buy durable goods with different ages, which does not make any problem if $A(t)$ is a continuous variable.

[5] Remember that creation and destruction must be equal for each group of agents. In particular, the number of car bought and sold by rich agents must be equal so that

$$m_1(t) = m_1(t - T_1(t))\left(1 - T_1'(t)\right)$$

Also, the number of durable goods bought by poor agents must be equal to the number of durables sold by rich agents,

$$m_2(t) n_2 = n_1 m_1(t - T_1(t))\left(1 - T_1'(t)\right).$$

Thus, (10) follows.

Steady state equilibrium price functions The crucial difference with respect to the solution in autarchy is the way in which the opportunity cost of holding durable-goods for each agent is modified. In the next proposition we obtain the steady state price functions.

Proposition 2 *At a stationary equilibrium the following properties apply.*

1. *Let $p_2(A+a)$ denote the value (shadow price) of a used car of age $A+a$, owned by poor consumers who scrap their cars at age T_2. A denotes the age at which the car was bought and $a \in [0, T_2]$ the tenure period. Therefore,*

$$p_2(A+a) = \gamma(c_2/s_2)^{\sigma} \int_0^{T_2-a} e^{-\delta A} \left(e^{-\delta(z+a)} - e^{-\delta T_2}\right) e^{-\rho z} \, dz + \beta \quad (11)$$

2. *Let $p_1(a)$ denote the value (shadow price) of a used car of age a, owned by rich consumers who sell their cars at age T_1 in the secondhand market. It is given by*

$$\begin{aligned} p_1(a) &= \gamma(\frac{c_1}{s_1})^{\sigma} \int_0^{T_1-a} \left(e^{-\delta(z+a)} - e^{-\delta T_1} - (1/\gamma)(\frac{s_1}{c_1})^{\sigma} p'(T_1)\right) e^{-\rho z} \, dz \\ &\quad + p(T_1) \end{aligned} \quad (12)$$

with $p'(T_1)$ given by

$$p'(T_1) = -\delta\gamma(\frac{c_2}{c_1})^{\sigma} e^{-\delta A} \frac{1 - e^{-(\delta+\rho)T_2}}{\delta + \rho} \quad (13)$$

Proof.

1. *For poor people, using equations (24) and (25) from Appendix A.2 in steady state, the following equation results:*

$$p(A) = \gamma(c_2/s_2)^{\sigma} \int_0^{T_2} e^{-\delta A} \left(e^{-\delta z} - e^{-\delta T_2}\right) e^{-\rho z} \, dz + \beta. \quad (14)$$

This expression equates the market price of a used good of age A on the left hand side, with the services of a durable of age A held for T_2 periods, plus the scrapping value. Therefore, the value of a durable held for $T_2 - a$ periods $p_2(A+a)$, is,

$$p_2(A+a) = \int_a^{T_2} e^{-\delta A} \left(e^{-\delta z} - e^{-\delta T_2}\right) e^{-\rho(z-a)} \, dz + \beta$$

Letting $u = z - a$, the previous equation gives equation (11).

2. *For rich people, stationary price function is given by (19) and (20) in Appendix A.2, taking all variables as constant. After doing this we have*

$$\alpha = \gamma(\frac{c_1}{s_1})^{\sigma} \int_0^{T_1} \left(e^{-\delta z} - e^{-\delta T_1} - (1/\gamma)(\frac{s_1}{c_1})^{\sigma} p'(T_1)\right) e^{-\rho z} \, dz + p(T_1), \quad (15)$$

On the left hand side we have the cost of a new car. On the right hand side we have the services of a new car held for T_1 periods, plus its secondhand market price when is sold at age T_1 in the secondhand market. Therefore, the value of a durable held for $T_1 - a$ periods $p_1(a)$, is,

$$\begin{aligned} p_1(a) &= \gamma(\frac{c_1}{s_1})^{\sigma} \int_a^{T_1} \left(e^{-\delta z} - \ e^{-\delta T_1} - (1/\gamma)(\frac{s_1}{c_1})^{\sigma} p'(T_1) \right) \ e^{-\rho(z-a)} \ dz \\ &+ p(T_1), \end{aligned}$$

As in equilibrium $T_1 = A$, inside the integral $p'(T_1)$ can be replaced by equation (26) in Appendix A.2 which, evaluated in steady state gives equation (13). Now, applying the same variable change as with poor people equation (12) results.

■

Corollary 3 *At the trading age $T_1 = A$, shadow prices of rich and poor are such that $\frac{\partial p_1(T_1)}{\partial a} = \frac{\partial p_2(A)}{\partial a}$.*

Proof. *Differentiating (12) with respect to a and evaluating it at $a = T_1$ gives*

$$\frac{\partial p_1(T_1)}{\partial a} = -\delta\gamma(\frac{c_2}{c_1})^{\sigma} e^{-\delta A} \frac{1 - \ e^{-(\delta+\rho)T_2}}{\delta + \rho}$$

which coincides with the derivative of equation of (11) evaluated at $a = 0$. ■

When the secondary market is open, rich agents buy new durables and sell them to poor agents after T_1 periods. The shadow price for age $a \in [0, T_1]$ is given by equation (12) with $p'(T_1)$ given by (13) and with $p(T_1)$ given by (14) as, in equilibrium, $A = T_1$. The shadow price for age $a \in [T_1, T_2]$ is given by (11). These functions are represented in Figure 4. Dashed and dotted lines represent, respectively, the durable reservation value for rich and poor consumers. As Corollary 3 showed, both shadow price functions meet tangentially at the price at which the used good is traded in the secondhand market.[6]

INSERT FIGURE 4 ABOUT HERE

In Figure 5, we combine Figure 3, where transaction between agents of different type were forbidden, with Figure 4.

INSERT FIGURE 5 ABOUT HERE

We see that price function for poor agents moves down (dotted lines), and for rich agents moves up (dashed lines). The consequence is that with secondhand markets, the lifetime of a durable is between the cars lifetime owned by poor and rich people under autarchy.

[6] In Figure 5 we use the same parameters' value as before giving $T_1 = 7.72$ years and the price of the used good, $p(A = T_1) = 0.23$.

Given the efficiency of our economy, the solution can also be easily found by solving the corresponding social planner problem. This may be a useful framework to study de dynamics of the model along the lines of Boucekkine, Licandro and Magnus (2001). However, as our focus here is on the steady state, we rest on equilibrium conditions.

4 Numerical experiments on the role of secondhand markets

To illustrate the main quantitative consequences of introducing secondhand markets, we evaluate the model response to an increase in the scrappage subsidy. The choice of parameter values used in the numerical experiments is given in Table 1. Most of these values are standard in the literature.

INSERT TABLE 1 ABOUT HERE

Thus, we choose $\beta = 0.05$ as our benchmark value for the subsidy with α normalized to 1. Also, we fix the discount rate, $\rho = 0.05$, and $\sigma = 4$ and $\delta = 0.001$ are chosen to select the length of ownership tenure of cars. Finally, the preference parameter $\gamma = 1.4$ fixes the ratio of cars over consumption expenditures. We restrict to $n_1 = 1/2$ in the numerical experiment, and we fix $y_1 = 2$ and $y_2 = 1.5$, to match the ratio of consumption expenditures between households in each market to $\frac{c_1}{c_2} = 1.30$.

4.1 Steady state effects

First, we explore the effects on steady state of opening up the secondhand market. Table 2 presents first, the steady state values in autarchy. Under autarchy, we solve the system[7] of equations (5)–(8), with the parameters of Table 1 and $y_1 = 2$ for rich people and $y_2 = 1.5$ for poor people. With secondhand markets we solve, in steady state, the system of equations (16)–(26) in Appendix A.2.[8] The results are shown in Table 2.

INSERT TABLE 2 ABOUT HERE

Two comments can be made to this table. First, it goes without saying that secondhand market improves efficiency. Second, new durable purchases are reduced by nearly a 9%. Two effects are influencing new durable purchases. On the one hand, opening the secondhand market reduces new cars purchases from poor people that now buy used goods. On the other hand, it increases the purchases of rich people that now sell their durable after 7.7 years. The final

[7] Numerical solutions are done using Mathematica.

[8] Concerning debt note that, as intertemporal preferences are identical for both type of consumers, $b_1(t) \equiv b_2(t)$. This joint with the asset market equilibrium condition, $n_1 b_1(t) + n_2 b_2(t) = 0$, implies $b_1(t) = b_2(t) = 0$.

effect on new durables purchases depends on which effect dominates. In this example the first effect dominates, so that cars purchases are slightly reduced.

A relevant dimension for comparative statistics is the change in the average age of the stock of cars which can be associated with quality of cars on the road. We make a numerical exercise to analyze the influence of the secondary market on the average age of the stock. In autarchy, the average age of the stock, denoted as $\bar{A}$, is given by

$$\bar{A} = n_1 m_1 \int_0^{T_1} a \text{ d}a + n_2 m_2 \int_0^{T_2} a \text{ d}a = n_1 \frac{T_1}{2} + n_2 \frac{T_2}{2}$$

With secondhand markets the average age, denoted as $\tilde{A}$ is:[9]

$$\tilde{A} = n_1 m \left(\int_0^{T_1} a \text{ d}a + \int_{T_1}^{T_1+T_2} a \text{ d}a \right) = n_1 \frac{T_1}{2} + n_1 \frac{T_2}{T_1} \frac{2T_1 + T_2}{2} = n_1 \frac{T_1}{2} + n_2 \frac{2T_1 + T_2}{2}$$

Figure 6 shows the change in average age, induced by the introduction of a secondhand market, as a function of the rate of rich people, for different values of income differences:

INSERT FIGURE 6 ABOUT HERE

Although we present only the results with three different values of income differences, simulations with several different values show that the secondhand market increases the average stock age for low income differences and reduces the average stock age when the difference is high. For intermediate values of income differences, the effect of secondhand markets on the average age of the stock depends on the relative size of rich to poor population.

These examples illustrate that the explicit modeling of durable goods purchases, in a vintage framework with transactions in the secondary market, is useful to understand the potential role of tax credits that promote replacement of some capital goods. Evaluating whether the predictions of our model are consistent with some of the observed patterns of car purchases after the implementation of the aforementioned tax credits, as well as transitional dynamics associated to these policies is left for further research.

5 Conclusions

In this paper we present a vintage model to analyze the replacement of durable goods. The model has the enhanced feature that a secondhand market for these goods is explicitly modeled. Also, consumers have concave and separable utility, both in durables and non–durable goods.

[9] Note that T_1 and T_2 in $\bar{A}$ and $\tilde{A}$ not only take different values, but also represent different things. In the first case, represent both, the lifetime of durables and the tenure period of durables of rich and poor people, respectively. Under second hand markets, T_1 and T_2 are tenure period of those who buy new (rich people) and used (poor people), respectively. But in this case the durable lifetime is $T_1 + T_2$.

We characterized the equilibrium properties of the steady state of the economy, focusing on the patterns of secondhand market prices. We found that the introduction of a secondhand market increases the value of used durable goods, both for the seller and for the buyer, modifying secondhand market prices and reducing the age at which durable goods are transacted in the secondhand market.

We also studied the effects of secondhand markets for the aggregate sales and the average age of the stock. These effects depend both on the inequality of income and the relative size of population buying used or new. For higher values of inequality, secondhand markets reduce the average age of the stock and the contrary happens when inequality is small. For intermediate values of income inequality, the results depend on the relative size of population. Although our numerical results show that secondhand markets improve efficiency based on the utility of private consumption, they also suggest that if high pollution and low quality —which are usually associated to high values car's age—, were taken into account to analyze efficiency, the benefits of secondhand markets are not clear.

References

[1] Adda, J. and Cooper, R. (2000). Baladurette and Juppette: A Discrete Analysis of Scrapping Subsidies. *Journal of Political Economy*, vol. 108 (4): 778–806.

[2] Boucekkine, R., M. Germain and Licandro, O. (1997). Replacement Echoes in the Vintage Capital Growth Model. *Journal of Economic Theory*, 74(2): 333–348.

[3] Boucekkine, R., Licandro, O and A. Magnus (2001). Numerical solution by iterative methods of a class of vintage capital growth models. *Journal of Economic Dynamics and Control,* 25: 255-269.

[4] Fernández, V. (2000). Observable and Unobservable Determinants of Replacement of Home Appliances. *Energy Economics*, 23(3): 305-323

[5] Goolsbee, A. (1998). The Business Cycle, Financial Performance, and the Retirement of Capital Goods. *Review of Economic Dynamics*, 1(2): 474–496.

[6] Malcomsom, J. (1975). Replacement and the Rental Value of Capital Equipment Subject to Obsolescence. *Journal of Economic Theory*, 10: 24–41.

[7] Rust, J. (1985). Stationary Equilibrium in a Market for Durable Assets. *Econometrica*, 53: 783-806.

[8] Porter, R. and Sattler, P. (1999). Patterns of Trade in the Market for Used Durables: Theory and Evidence. NBER working paper 7149.

A Appendix

A.1 The replacement problem when consumers are identical

A representative consumer solves the following problem

$$\max \int_0^{\infty} \left(\frac{c(t)^{1-\sigma}}{1-\sigma} + \gamma \frac{s(t)^{1-\sigma}}{1-\sigma} \right) \mathrm{e}^{-\rho t} \, \mathrm{d}t$$

s.t.

$$s(t) = \int_{t-T(t)}^{t} m(z) \, \mathrm{e}^{-\delta(t-z)} \, \mathrm{d}z$$

$$\int_{t-T(t)}^{t} m(z) \, \mathrm{d}z = 1$$

$$c(t) + (\alpha - \beta) \, m(t) = y$$

The Lagrangian for this problem is:

$$\begin{aligned}
\mathcal{L}(t) &= \int_0^{\infty} \left\{ \left(\frac{c(t)^{1-\sigma}}{1-\sigma} + \gamma \frac{s(t)^{1-\sigma}}{1-\sigma} \right) + \mu(t) - \varphi(t) s(t) \right\} \mathrm{e}^{-\rho t} \, \mathrm{d}t \\
&\quad + \int_0^{\infty} \left\{ \varphi(t) \int_{t-T(t)}^{t} m(z) \, \mathrm{e}^{-\delta(t-z)} \, \mathrm{d}z - \mu(t) \int_{t-T(t)}^{t} m(z) \, \mathrm{d}z \right\} \mathrm{e}^{-\rho t} \, \mathrm{d}t \\
&\quad - \int_0^{\infty} \left\{ \phi(t) \left(c(t) + (\alpha - \beta) \, m(t) - y \right) \right\} \mathrm{e}^{-\rho t} \, \mathrm{d}t
\end{aligned}$$

where $\phi(t)$ and $\mu(t)$ are, respectively, the Lagrangian multipliers associated to the instantaneous feasibility constraint and the saturation constraint, and $\varphi(t)$ is the multiplier associated with the services flow constraint. To obtain the first order conditions we adapt the approach used in vintage growth models in Boucekkine, Germain and Licandro (1997), first used in Malcomsom (1975). We change the order of integration using the identity $J(t) = T(t + J(t))$, which states that, under perfect foresight, the optimal life of a car bought at t, given by $J(t)$, is exactly the replacement age at $t + J(t)$, where $J(t)$ is endogenous. Applying this transformation and computing first order conditions, we get:

For $c(t), s(t), m(t)$, for all $t \geq 0$,

$$c(t)^{-\sigma} = \phi(t)$$

$$\gamma s(t)^{-\sigma} = \varphi(t)$$

$$(\alpha - \beta) \, \phi(t) = \int_t^{t+J(t)} \left(\varphi(z) \, \mathrm{e}^{-\delta(z-t)} - \mu(z) \right) \mathrm{e}^{-\rho(z-t)} \, \mathrm{d}z$$

For $J(t)$, for all $t \geq -T(0)$,

$$\mu(t + J(t)) = \varphi(t + J(t))\mathrm{e}^{-\delta J(t)}$$

$$J(t) = T(t + J(t))$$

Using the change of variable $z = t + J_1(t)$, and after some substitutions, the previous first order conditions are, for all $t \geq 0$,

$$(\alpha - \beta)\, c(t)^{-\sigma} = \gamma \int_t^{t+J(t)} s(z)^{-\sigma} \left(\mathrm{e}^{-\delta(z-t)} - \mathrm{e}^{-\delta T(t)}\right) \mathrm{e}^{-\rho(z-t)}\, \mathrm{d}z$$

$$J(t) = T(t + J(t))$$

which are equations (3) and (4) in the main text.

A.2 The replacement problem with secondhand markets

The problem solved by rich agents in Section 3.2 is the following:

$$\max \int_0^\infty \left(\frac{c_1(t)^{1-\sigma}}{1-\sigma} + \gamma\frac{s_1(t)^{1-\sigma}}{1-\sigma}\right) \mathrm{e}^{-\rho t}\, \mathrm{d}t$$

s.t.

$$s_1(t) = \int_{t-T_1(t)}^{t} m_1(z)\, \mathrm{e}^{-\delta(t-z)}\, \mathrm{d}z \tag{16}$$

$$\int_{t-T_1(t)}^{t} m_1(z)\, \mathrm{d}z = 1 \tag{17}$$

$$\dot{b}_1(t) = r(t)b_1(t) + y_1 - c_1(t) - (\alpha - p(t, T_1(t)))\, m_1(t) \tag{18}$$

given $b_1(0)$ and $m_1(t) = \bar{m}_1(t)$ for all $t \in [-T_1(0), 0[$.

The Lagrangian for this problem is:

$$\begin{aligned}
\mathcal{L}(t) \quad = \quad & \int_0^\infty \left\{\left(\frac{c_1(t)^{1-\sigma}}{1-\sigma} + \gamma\frac{s_1(t)^{1-\sigma}}{1-\sigma}\right) + \mu_1(t) - \varphi_1(t)s_1(t)\right\} \mathrm{e}^{-\rho t}\, \mathrm{d}t \\
& + \int_0^\infty \left\{\varphi_1(t)\int_{t-T_1(t)}^{t} m_1(z)\, \mathrm{e}^{-\delta(t-z)}\, \mathrm{d}z - \mu_1(t)\int_{t-T_1(t)}^{t} m_1(z)\, \mathrm{d}z\right\} \mathrm{e}^{-\rho t}\, \mathrm{d}t \\
& - \int_0^\infty \left\{\phi_1(t)\left(\dot{b}_1(t) - r(t)b_1(t) - y_1 + c_1(t) + (\alpha - p(t, T_1(t)))\, m_1(t)\right)\right\} \mathrm{e}^{-\rho t}\, \mathrm{d}t
\end{aligned}$$

where $\phi_1(t)$ and $\mu_1(t)$ are, respectively, the Lagrangian multipliers associated to the instantaneous budget constraint and the saturation constraint, and $\varphi_1(t)$ is the multiplier associated with the services flow constraint. Proceeding as in the previous section of this appendix, we transform the Lagrangian and obtain the following first order conditions:

For $b(t), c_1(t), s_1(t), m_1(t)$, for all $t \geq 0$,

$$\frac{\dot{\phi}_1(t)}{\phi_1(t)} = \rho - r(t)$$

$$c_1(t)^{-\sigma} = \phi_1(t)$$

$$\gamma s_1(t)^{-\sigma} = \varphi_1(t)$$

$$(\alpha - p(t, T_1(t)))\, \phi_1(t) = \int_t^{t+J_1(t)} \left(\varphi_1(z)\mathrm{e}^{-\delta(z-t)} - \mu_1(z)\right) \mathrm{e}^{-\rho(z-t)}\, \mathrm{d}z$$

For $J_1(t)$, for all $t \geq -T_1(0)$,

$$\begin{aligned} \mu_1(t + J_1(t))m_1(t) &= m_1(t)\varphi_1(t + J_1(t))\mathrm{e}^{-\delta J_1(t)} \\ &\quad + \phi_1(t + J_1(t))p'(t, J_1(t))m_1(t + J_1(t)) \end{aligned}$$

$$J_1(t) = T_1(t + J_1(t))$$

Using the change of variable $z = t + J_1(t)$, and after some substitutions, the previous first order conditions are, for all $t \geq 0$,

$$\frac{\dot{\phi}_1(t)}{\phi_1(t)} = \rho - r(t)$$

$$(\alpha - p(t, T_1(t)))\, c_1(t)^{-\sigma} = \int_t^{t+J_1(t)} \left(\gamma s_1(z)^{-\sigma}\mathrm{e}^{-\delta(z-t)} - \mu_1(z)\right) \mathrm{e}^{-\rho(z-t)}\, \mathrm{d}z \tag{19}$$

$$\mu_1(t) = \gamma s_1(t)^{-\sigma}\mathrm{e}^{-\delta T_1(t)} + \phi_1(t)p'(t, T_1(t))\frac{m_1(t)}{m_1(t - T_1(t))} \tag{20}$$

$$J_1(t) = T_1(t + J_1(t)).$$

In Section 3.2 poor agents buying a used durable, solve the following optimal control problem:

$$\max_{c_2(t), s_2(t), T_2(t), A(t), m_2(t)} \int_0^\infty \left(\frac{c_2(t)^{1-\sigma}}{1-\sigma} + \gamma\frac{s_2(t)^{1-\sigma}}{1-\sigma}\right) \mathrm{e}^{-\rho t}\, \mathrm{d}t$$

s.t.

$$s_2(t) = \int_{t-T_2(t)}^t m_2(z)\, \mathrm{e}^{-\delta(t-z+A(z))}\, \mathrm{d}z \tag{21}$$

$$\int_{t-T_2(t)}^t m_2(z)\, \mathrm{d}z = 1 \tag{22}$$

$$\dot{b}_2(t) = r(t)b_2(t) + y_2 - c_2(t) - (p(t, A(t)) - \beta)\, m_2(t) \tag{23}$$

given $b_2(0)$, $m_2(t) = \bar{m}_2(t)$ and $A_1(t) = \bar{A}_1(t)$ for all $t \in [-T_2(0), 0[$. Using the same methodology as for rich agents, the optimal conditions for this problem are

$$\frac{\dot{\phi}_2(t)}{\phi_2(t)} = \rho - r(t)$$

$$(p(t, A(t)) - \beta)\, c_2(t)^{-\sigma} = \int_t^{t+J_2(t)} \left(\gamma s_2(z)^{-\sigma} \mathrm{e}^{-\delta(z-t+A(t))} - \mu_2(z)\right) \mathrm{e}^{-\rho(z-t)}\, \mathrm{d}z \tag{24}$$

$$\mu_2(t) = \gamma s_2(t)^{-\sigma}\ \mathrm{e}^{-\delta(T_2(t)+A(t-T_2(t)))} \tag{25}$$

$$-c_2(t)^{-\sigma} p'(t, A(t)) = \delta\ \mathrm{e}^{-\delta A(t)} \int_t^{t+J_2(t)} \gamma s_2(z)^{-\sigma} \mathrm{e}^{-(\delta+\rho)(z-t)}\, \mathrm{d}z \tag{26}$$

$$J_2(t) = T_2(t + J_2(t)).$$

Important comments:

1. $\phi_1(t)$ and $\phi_2(t)$ grow at the same rate, $\rho - r(t)$, implying that, for all $t \geq 0$,

$$\frac{c_1(t)}{c_2(t)} = \left(\frac{\phi_2(0)}{\phi_1(0)}\right)^{\frac{1}{\sigma}},$$

   Since both type of agents have the same preferences and face the same interest rate, the ratio of type 1 to type 2 non–durable consumptions must be constant.

2. After some substitutions, the marginal value of the *market maturity* constraint for rich agents becomes

$$\begin{aligned}\mu_1(t) &= \gamma s_1(t)\, \mathrm{e}^{-\delta T_1(t)} \\ &\quad -\delta\gamma \left(\frac{c_1(t)}{c_2(t)}\right)^{-\sigma} \mathrm{e}^{-\delta T_1(t)} \frac{m(t)}{m(t - T_1(t))} \int_t^{t+J_2(t)} s_2(z)^{-\sigma} \mathrm{e}^{-(\delta+\rho)(z-t)}\, \mathrm{d}z\end{aligned}$$

3. Adding both optimal conditions for durable purchases, after multiplying for $\bar{\lambda}_i$ in each equation, we obtain:

$$\begin{aligned}(\alpha - \beta) &= c_1(t)^{\sigma} \int_t^{t+J_1(t)} \left(\gamma s_1(z)^{-\sigma} \mathrm{e}^{-\delta(z-t)} - \mu_1(z)\right) \mathrm{e}^{-\rho(z-t)}\, \mathrm{d}z \\ &\quad + c_2(t)^{\sigma} \int_t^{t+J_2(t)} \left(\gamma s_2(z)^{-\sigma} \mathrm{e}^{-\delta(z-t+A(t))} - \mu_2(z)\right) \mathrm{e}^{-\rho(z-t)}\, \mathrm{d}z\end{aligned}$$

Table 1: Parameter's values

β	α	σ	ρ	δ	γ	y_1	y_2	n_1
0.05	1.0	4.0	0.05	0.001	1.4	2	1.5	0.5

Table 2: Steady state values in autarchy

	m_1	m_2	T_1	T_2	c_1	c_2	s_1	s_2	$p(A = T_1)$
Autarchy	0.0917	0.0497	10.9	20	1.912	1.452	0.994	0.9900	–
Secondhand market	0.129	–	7.723	7.723	1.9	1.476	0.9961	0.988	0.23

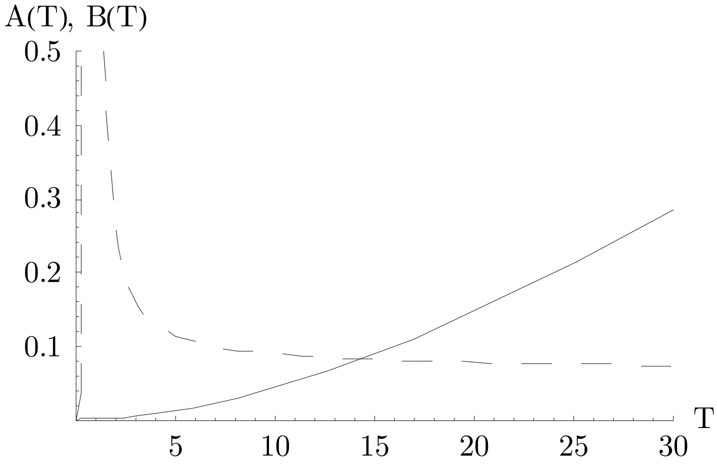

Figure 1: Existence and Uniqueness of T.

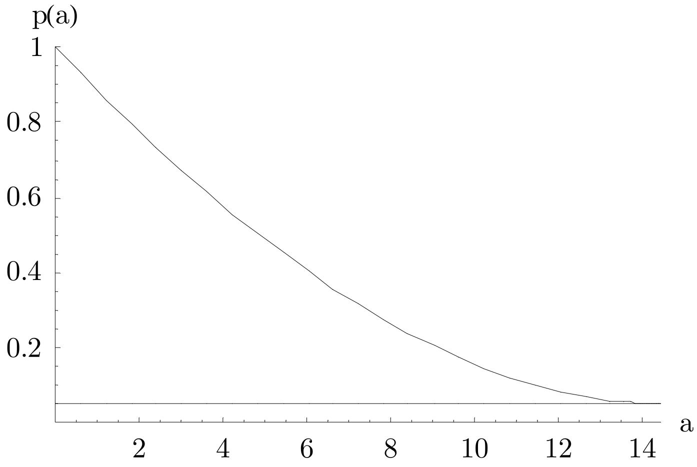

Figure 2: Secondary market prices.

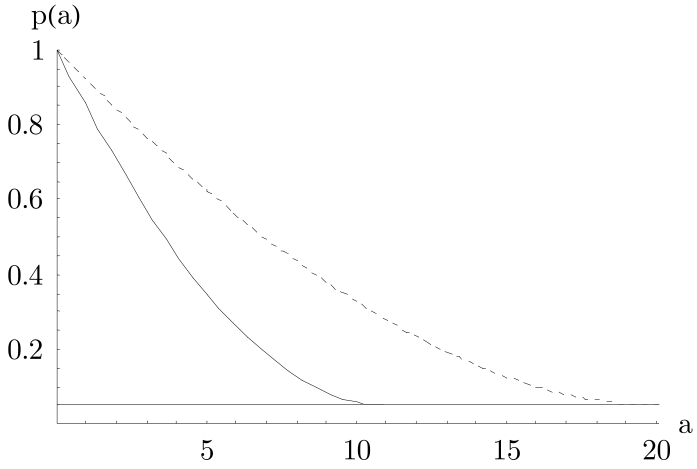

Figure 3: Secondhand market prices for rich (dashed line) and poor people (dotted line).

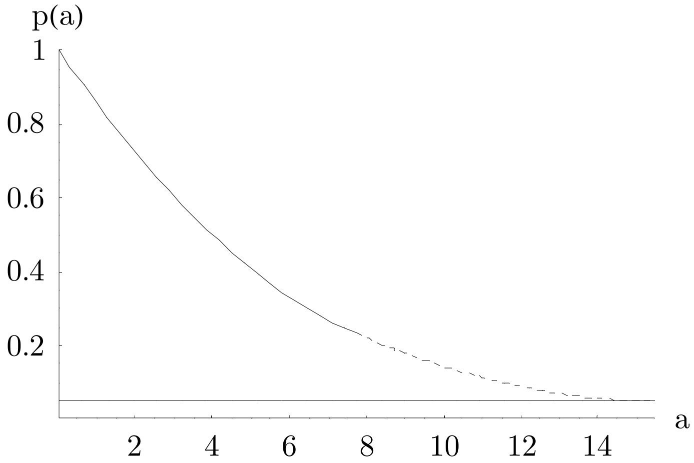

Figure 4: Durables price with open secondhand market. Dashed and dotted lines represent the rich and poor tenure periods respectively.

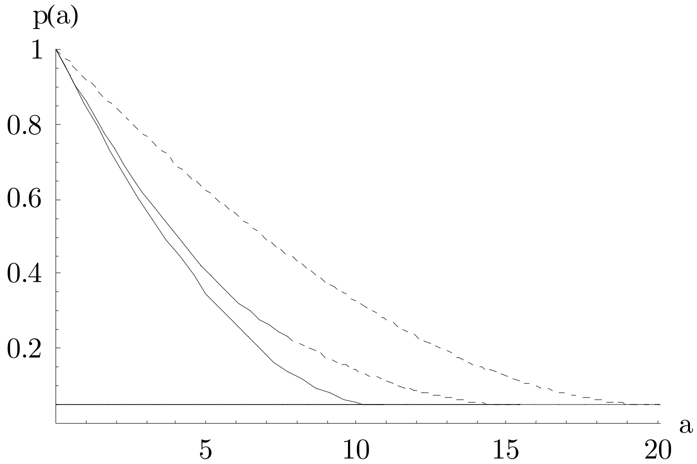

Figure 5: Prices of used goods when the secondary market is open: comparison with autarchy.

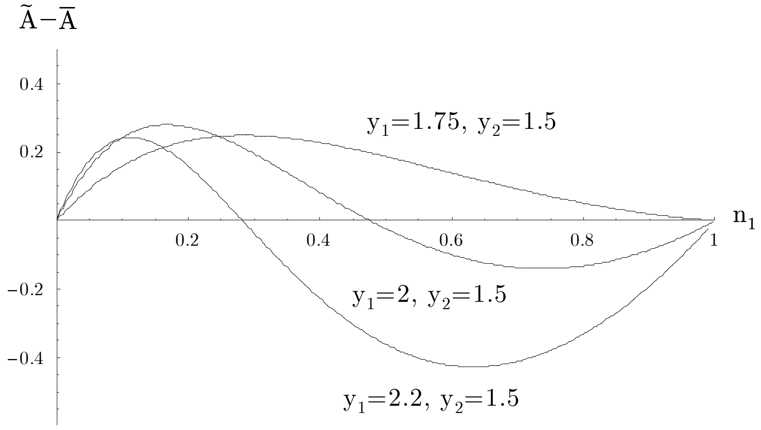

Figure 6: Average stock age as a function of relative size of both group of buyers.

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