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On gender gaps and self-fulfilling expectations:
Theory, policies and some empirical evidence*

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ABSTRACT

This paper considers a simple model of self-fulfilling expectations that leads to a multiple equilibrium of gender gaps in wages and participation rates. Rather than resorting to moral hazard problems related to unobservable effort, like in most of the related literature, our model fully relies on statistical discrimination. If firms believe that women will quit their jobs more often than equally productive men when shocks affecting household chores take place, our model predicts that this belief will increase the wage gap in favour of men which, in turn, will increase the female share of housework and exacerbate lower female participation in the labour market. Hence, both effects lead to a gendered equilibrium with large gaps, even though an ungendered equilibrium with no gaps is feasible. We examine the effects of gender-based and gender-neutral subsidies and find that the latter are more effective in removing the gendered equilibrium. Empirical analysis based on a time use survey for Spain is provided to test most of the implications of the model.

JEL Classification: J16 and J71.

Keywords: gender wage gap, participation, multiple equilibria.

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1. Introduction

There is a growing literature that studies the joint determination of gender differentials in earnings and in the household division of labour.¹ Most of these studies depart from Becker’s (1991) observation that a small initial comparative advantage of women in household production (e.g., in bearing and nurturing children) can lead to full specialization if, due to learning-by-doing and to household work increasing the marginal disutility of market work, the comparative advantage can grow over time. However, as it has been pointed out (see, e.g., Albanesi and Olivetti, 2007), huge improvements in medical and household technologies (plus less need of physical strength in most jobs) have rendered this comparative advantage unimportant and yet gender differences in the division of labour persist (see Bassanini and Saint Martin, 2008 for a recent review).

To tackle this puzzle, several explanations have been proposed without relying on the existence of active discrimination. They rely instead upon incentive problems in the labour market leading to self-fulfilling prophecies even when comparative advantages are absent. The idea is that the expectation by firms that women spend more time than men undertaking home tasks leads to earnings differentials in favour of men. Hence, since the expected opportunity cost is lower for women, they devote more time to household work, validating in this way firms’ beliefs. For example, Albanesi and Olivetti (2006) propose a model that generates this feature where firms are subject to incentive compatible constraints stemming from lack of observability of both effort (a moral hazard problem) and home hours which, as in Becker (1991), affect the marginal utility of effort (an adverse selection problem). Lommerud and Vagstad (2007) follow Lazear and Rosen’s (1990) model of job ladders in assuming that there are two types of jobs: fast track and slow track jobs. Workers are placed in the fast track jobs if the firm pays a fixed investment. Since effort is not observable, firms will only place workers in the fast track jobs if their expected output is large enough to recoup the investment cost. If women have been traditionally the ones exerting primary major responsibility at home and wages are non-contractible, in equilibrium they will predominantly follow a “mommy track”. They analyze the stability properties of symmetric and asymmetric equilibria and characterize the situations where anti-discrimination policies (e.g.,

affirmative action) are effective depending on both the initial equilibrium and the (permanent or transitory) nature of these policies.

Our paper falls into this stream of the literature and makes two contributions. First, we propose a simple model which yields self-fulfilling prophecies without using a moral hazard problem originated by the difficulty of perfectly monitoring effort or by the substitutability of effort at home and in the market, as the previous papers do. While moral hazard motivations are plausible, they are also somewhat restrictive. For example, wage gaps should be negligible for routine tasks by low-skilled employees for whom effort and output should be easily observable but, nevertheless, we still observe substantial gaps in those categories (see, e.g., de la Rica et al. 2008). Likewise, one could also think that household work, akin to running a “small firm”, leads to better organization skills therefore increasing female labour productivity instead of reducing it.

Instead of stressing incentive problems, we let statistical discrimination play the key role. Specifically, statistical discrimination arises from different expectations about the distributions of disutility shocks (unexpected need of irregular hours at work or events that require parental leave, etc.) affecting equally productive men and women once they have been trained for a job. If future wages are predetermined with respect to these shocks, then job quit rate will differ across genders, inducing differences in labour market participation to depend on expected wage gaps, an issue that is not tackled by models relying on the moral hazard problem. Thus, in this fashion we are able to generate predictions about the relation between gender participation and wage gaps broadly in line with the available empirical evidence (see below). Further, the structure of our model is much simpler than in the above-mentioned literature since we do not need to analyse the design of incentive-compatible contracts in order to generate similar predictions. Secondly, in contrast to most existing work (see, e.g., the discussion in Lommenrud and Vagstad, 2007), our framework implies that, when multiple equilibria exist, welfare may be higher in the symmetric than in the asymmetric equilibrium. The reason why welfare tends to be greater with discrimination is that it promotes some form of “efficient specialization” in the labour market, an effect which is also present in our model. However, we also allow for a direct disutility of housework which is minimized –i.e. efficiency maximized- when there is equal split of housework.

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2 See Altonji and Black (1999) and Donohue (2005) for recent surveys of the literature on “statistical discrimination”.

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Under some scenarios, the second effect will dominate, implying that welfare is higher in the symmetric equilibrium.

In our framework, under some assumptions about the distribution of shocks and the degree of diminishing returns to training, there are two types of equilibria: (i) an *ungendered* one, where there are no participation and wage gaps and the division of household work is fully egalitarian, and (ii) a *gendered* equilibrium where both gaps are favourable to men, so that women specialize in household tasks. We examine the effect of alternative policies in an economy that is initially in a *gendered* equilibrium despite the fact that men do not enjoy any comparative advantage in productivity or bargaining. We find that family aid subsidies targeted at women not only do not move the economy to the *ungendered* equilibrium but also make the equilibrium allocation of housework and the distribution of wages more unequal. In contrast, when family aid is received by both men and women, the *gendered* equilibrium may disappear.

A first look at cross-country evidence provides some preliminary support to our motivation. Figure 1 shows the relationship between the male-female employment rate gap (horizontal axis) and the raw hourly wage gap (vertical axis) for workers aged 15-64 with an educational attainment of upper secondary education or less in several OECD countries. The data correspond to 2001, the latest year for which comparable data on a large number of OECD countries is available (see Bassanini and Saint Martin, 2008). The choice of lower-educated workers is due to the fact that higher educated women are less prone to quit their jobs when faced with a household shock because of their larger investment in human capital and the affordability of formal household/child-care arrangements (see Altonji and Blank, 1999, and de la Rica et al., 2008). The figure shows a slight positive correlation (0.22) between both variables, which remains positive (0.16) once the two clear outliers of Korea and Italy are omitted. That is, without controlling for observable characteristics, higher labour market participation gaps are associated with higher wage gaps, as our model predicts. Regarding training by gender, Arulampanam et al. (2004) report evidence for Europe finding that although, on

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3 Similar graphs for the overall population of workers aged between 15 and 54, working at least 15 hours per week, can be found in Bassanini and Saint Martin (2008). The correlation coefficient is -0.13.

4 The low wage gender gap in Italy has been analyzed by Olivetti and Petrongolo (2006) where it is claimed that, in countries with low female participation, women with relatively low return to paid job would choose to stay out of the market, thus narrowing the wage gap. However, why the gap is much lower in Italy than in other low female participation countries, like Greece or Spain, remains a puzzle. The high wage gap in Korea is probably due to the prevalence of low-paid jobs in the high-growth East-Asian economies.
average, women are no less likely than men to get training, there are significant differences in favour of men in the South-Mediterranean countries (see also the detailed discussion in de la Rica et al., 2008 for the Spanish case). \(^5\)

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**Figure 1: Relationship between wage and employment gaps**

![Graph showing the relationship between wage and employment gaps](image1)

**Figure 2: Relationship between family aid expenditure and employment gaps**

![Graph showing the relationship between family aid expenditure and employment gaps](image2)

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\(^5\) Note, however, that, although training is the key channel which leads to wage differentials in our model, a similar interpretation could be obtained using the job-ladder allocation of workers analysed in Lazear and Rosen (1990) and even, albeit more loosely, in terms of the high concentration of women in college degrees with lower market returns (see Machin and Puhani, 2003)
Next, Figure 2 shows the relation between the participation gaps and the proportion of GDP devoted to family aid expenditure.\textsuperscript{6} The correlation is clearly negative implying that more generous family-aid policies give rise to lower participation gaps and, in view of Figure 1, to lower wage gaps. As it is well known, Northern European countries are the ones with lower gaps and higher family-aid expenditure shares whereas the South-Mediterranean ones are the ones with higher gaps and lower expenditure.

The rest of the paper is structured as follows. Section 2 lays out the model, first with homogeneous individuals and later allowing for gender differences. Section 3 discusses the properties of the different equilibria. Section 4 deals with welfare analysis. Section 5 studies the effects of different policies to eliminate the asymmetric equilibrium. Section 6 provides some detailed empirical evidence on some of the predictions of the model using micro data from a survey on time use by members of Spanish households. Finally, section 7 concludes.

2. Modelling gender gaps

2.1 The basic setup: A training model

To account for the presence of both gender participation and wage gaps, we use a simple model inspired by Acemoglu and Pischke (1998)’s analysis of the financing of training in frictional labour markets, adapted to our framework where there are no search frictions but exogenous disutility shocks are allowed to induce quits.\textsuperscript{7}

The basic setup is as follows. Men and women live for two periods and are endowed with identical time in each period and ability. They are both normalized to 1. The overall population is also normalized to 1 and each gender represents half of it. In period 1, firms are randomly matched with just one worker of either gender who is single. All individuals receive an amount of (specific) training $\tau$ offered by the firm, which may differ across genders. For simplicity, workers do not receive a wage during the first period. Finally, there is free entry of firms in this period, which bear a linear training cost, $c(\tau) = \tau$.

At the start of period 2, individuals of each gender form couples (exogenously) and decide on how to split the household chores on the basis of expected relative wages.

\textsuperscript{6} It includes cash transfers, services and tax breaks towards family; see www.oecd.org/els/social/family/database.

\textsuperscript{7} See also de la Rica et al. (2008).
Once this decision is taken, workers (ready to start producing after being trained) are made a wage offer, \( W \), by the firm. After the wage has been announced, individuals suffer a disutility shock, \( \omega \), related to household tasks, which may force them to quit the job. The \( \omega \) shock is an \( i.i.d. \) random variable with c.d.f. \( F(\omega) \). The support of the distribution function may differ between men and women, and will depend on the way in which household chores are split. Individuals then decide whether to work or to quit. In the first case, production takes place and wages are subsequently paid. Output depends on the level of training received in period 1 with a production technology given by \( a(\tau) \), where \( a(\tau) = \beta \tau^{\alpha/2}, 0 < \alpha < 1 \), so that \( a'(\cdot) > 0 \) and \( a''(\cdot) < 0 \).

Since workers’ productivity depends on the amount of received training, firms will decide this amount in period 1 and the corresponding wage in period 2, taking as given the division of homework by couples, and hence the distribution of the shock for each gender. Conversely, couples bargain over the division of housework at the start of period 2 before the disutility shock is realized, taking as given the wages offered by firms to each gender. Accordingly, workers will always get trained in period 1 and will produce in period 2 as long as \( W - \omega \geq 0 \).

Summing up, the timing of decisions can be graphically represented as follows:

\[
\begin{array}{ccccccc}
\text{t=1} & \text{t=2} \\
\hline
\text{Training} & \text{Household decision} & \text{Wage offer} & \text{Disutility shock} & \text{Production} \\
\uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
\end{array}
\]

2.2 Homogenous workers

To solve for both wages and the amount of training, we proceed backwards in time. In order to simplify the derivations, we will assume that the distribution of shocks is uniform, i.e., \( \omega \sim U[0,\epsilon] \) with \( \epsilon \leq 1 \), since the shock can never exceed the unit of time each individual has in period 2. To understand the choice of wages and training by firms, we start with a situation in which all workers are identical. We refer to this the setup as the one with homogenous workers. Later, gender differences will be considered in the analysis.

Under the assumption that the wage in period 2 is announced by the firm before the shock \( \omega \) is realized, firms will choose \( W \) in order to maximize expected gross profit in this period, denoted by \( \Pi \), leading to the following optimization problem:
\[
\max_{W} \prod = \max_{W} \int_{0}^{\epsilon} [a(\tau) - W]\frac{1}{\epsilon} d\omega = \max_{W} \frac{a(\tau)W - W^2}{\epsilon}. \tag{1}
\]

Hence, the first-order condition (henceforth, f.o.c.) with respect to \( W \) implies that the wage paid in equilibrium, \( W^* \), satisfies: \(^8\)

\[
W^*(\tau) = \frac{a(\tau)}{2}, \tag{2}
\]

so that expected gross profits in period 2 are:

\[
\prod(\tau) = [a(\tau) - a(\tau)/2] \frac{W^*}{\epsilon} = \frac{a(\tau)^2}{4\epsilon}, \tag{3}
\]

where the term \( W^*/\epsilon \) captures the probability of not quitting, i.e., that \( W - \omega \geq 0 \), and hence that the individual participates in the labour market.

Free-entry of firms implies that expected profits at the beginning of period 1 are driven down to zero. The zero-profit condition then pins down the optimal level of training in period 1, \( \tau^* \), that is:

\[
\prod(\tau^*) - \tau^* = 0. \tag{4}
\]

Given the functional form of \( a(\tau) \), \( \tau^* \) is chosen to be:

\[
\tau^* = \left( \frac{\beta^2}{4\epsilon} \right)^{\frac{1}{\alpha}}. \tag{5}
\]

Next, replacing (5) in (2) yields the optimal wage:

\[
W^* = \frac{\beta}{2} \left( \frac{\beta^2}{4\epsilon} \right)^{\frac{\alpha}{2}}. \tag{6}
\]

These expressions imply that, when the support of the shock is larger (i.e., \( \epsilon \) is higher), the worker gets less training and a lower wage. The intuition is that a greater expected shock implies a higher probability of quitting in period 2 and hence, since this reduces expected profits, firms respond by lowering the amount of training. Note that our assumption that \( \alpha < 1 \) plays a crucial role. If \( \alpha \geq 1 \) - that is, if there were weak diminishing returns in training - then the firm would respond to a higher probability of quitting by increasing the amount of training, using the resulting wage rise to offset the higher expected value of the shock. We assume that diminishing returns in training are

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8 This is just the average of the worker’s productivity and the outside wage, which is assumed to be zero. The weight \( \frac{1}{2} \) in the wage is due to the choice of the uniform distribution in the illustration. Alternative distributions will give rise to a weighted average with unequal weights.
sufficiently strong to prevent this counterintuitive result.

From equation (5) we can compute the probability that an individual works, which corresponds to the participation rate since the population equals unity. This probability is given by $P^* = \Pr (\omega \leq W^*) = W^* \epsilon$, while the expected wage is $P^*W^* = W^* \epsilon^2$. Using (5), we have:

$$P^* = \left( \frac{\beta}{2} \right)^{1-a} \left( \frac{1}{\epsilon} \right)^{2(1-a)},$$

(7)

$$P^*W^* = \left( \frac{\beta^2}{4\epsilon} \right)^{1-a},$$

(8)

which, under the assumption that $\alpha < 1$, implies that a greater value of $\epsilon$ results in a lower probability of employment/participation rate and a lower expected wage. We further suppose:

**Assumption 1**: The following inequality holds: $\left( \frac{\beta^2}{4\epsilon} \right)^{1-a} \leq \epsilon$.

Assumption 1 ensures that $P^* \leq 1$. The assumption simply requires the productivity of training, $\beta$, not to be too large since, otherwise, the resulting wage would be sufficiently high, relative to the shock, to lead to full participation. In what follows, we will denote $\left( \frac{\beta^2}{4\epsilon} \right)^{1-a} \epsilon$ by the parameter $b_1$, which, by Assumption 1 and $\epsilon \leq 1$, satisfies $b_1 \leq 1$. Furthermore, we can express the equilibrium wage as $W^* = \sqrt{\epsilon b_1}$, implying that $W^* \leq 1$.

### 2.3 Allowing for different distributions of shocks

Once the basic expressions for training and the wage have been derived for homogenous workers, we introduce gender differences by allowing for different distributions of disutility shocks for men and women. Although at this stage we take this differential feature to be exogenous, it will be endogenised later on when we model the decision process within the household. The key difference across genders is that the c.d.f. for men, $F_m(\omega)$, is assumed to be stochastically dominated by the c.d.f. for women, $F_f(\omega)$, namely, $F_m(\omega) \geq F_f(\omega)$ for $\omega > 0$. As earlier, we assume that the corresponding distributions are uniform, so that $\omega_i \sim U[0, \epsilon_i]$ ($i=m, f$) with $\epsilon_f \geq \epsilon_m$. 


From (5) and (6), the optimal amount of training received by men and women and their corresponding wages are given by:

$$\tau_i = \left(\frac{\beta^2}{4\epsilon_i}\right)^{\frac{1}{2\alpha}}, \quad W_i = \frac{\beta}{2}(\tau_i)^{\frac{\alpha}{2}}, \quad i = f, m. \quad (9)$$

Clearly, if \(\epsilon_m < \epsilon_f\), then \(\tau_f < \tau_m\) and \(W_f < W_m\). That is, since women have a higher probability of quitting, they receive less training and therefore a lower wage than men.

As before, we can also derive the corresponding participation rates in the labour market which are given by \(P_i = \Pr(\omega_i \leq W_i)\), yielding:

$$P_m = W_m / \epsilon_m, \quad P_f = W_f / \epsilon_f. \quad (10)$$

Thus, it follows that \(P_f < P_m\). To ensure that participation rates are bounded above by unity, we modify Assumption 1 accordingly to:

**Assumption 2:** The following inequality holds: \(\left(\frac{\beta^2}{4\epsilon_i}\right)^{\frac{1}{1-\alpha}} \leq \epsilon_i \) for \(i = m, f\).

### 2.4 Gender gaps

From (9) and (10), it is straightforward to compute the gender participation and wage gaps, denoted by \(p\) and \(w\), respectively, which are defined as the ratio of the relevant variables for men and for women. That is, \(p = P_m / P_f\) and \(w = W_m / W_f\), yielding:

$$p = \left(\frac{\epsilon_f}{\epsilon_m}\right)^{\frac{1}{2(1-\alpha)}}, \quad w = \left(\frac{\epsilon_f}{\epsilon_m}\right)^{\frac{\alpha}{2(1-\alpha)}} \quad (11)$$

Since \(0 < \alpha < 1\), then the following proposition holds:

**Proposition 1:** Gender gaps depend on the difference in the distribution of the shock. Whenever \(\epsilon_m < \epsilon_f\), it is verified that \(p > 1\) and \(w > 1\). For \(\epsilon_m = \epsilon_f\), then \(p = w = 1\).

The intuition for this result is again simple. If \(\epsilon_m < \epsilon_f\), then women will have a higher probability of quitting in period 2 than men. Hence, they are offered less training in period 1 and a lower wage in period 2. Their lower wage is obviously the result of lower training, while their lower participation is a combination of a direct effect due to the actual distribution of the shock and of an indirect effect as statistical discrimination.
leads to a lower wage. Identical distributions of the shock, on the other hand, would imply no statistical discrimination and hence the absence of gaps.

3. Household division of labour and equilibria

3.1 Household division of labour

The next step is to endogenise the couple’s decision of how to split household chores at the beginning of period 2. We assume that there is a household good to be produced by the members of the household, and that this good provides a fixed utility level, $u$. The couple jointly decides how to split the responsibility for production of this good by choosing the fraction $s \in [0,1]$ of the household chores allocated to the woman.

The production of the good has two disutility costs. Part of the cost is known \textit{ex ante}, while another part is random and depends on the shock received by the household in period 2. To give an example, suppose that the household good consists of raising children. Children have to be collected from school and ferried to their after-school activities every day, imposing a (known) disutility cost to the member of the couple doing it. Then, there are shocks, such as a child being sick and needing to stay home with a parent, which are uncertain. These shocks, however, have an opportunity cost only if the parent is working since they imply a reduction in the (monetary) utility derived from the job.

The certain disutility costs of producing the household good for the man and woman are assumed to be $(s^{-1} - 1)$ and $((1-s)^{-1} - 1)$, respectively, which are increasing and convex. Thus, if $s = 1$, the man gets no disutility, and vice versa. As before, there is a single shock affecting the household, $\omega$, whose distribution is $U[0,\varepsilon]$. Since a fraction $s$ of this shock is borne by the woman and $(1-s)$ by the man, this implies $\varepsilon_f = s\varepsilon$ and $\varepsilon_m = (1-s)\varepsilon$. Further assumptions are that there is full income sharing within the household and that the partners maximize the sum of utilities with respect to $s$ taking their wages as given.

Thus, expected household utility accruing to the household, net of the cost of the shock, denoted by $V^H$, is given by:

$$V^H = u + \left[ \int_0^{W_f/(1-s)} (W_m - (1-s)\omega) \frac{1}{\varepsilon} d\omega + \int_0^{W_f/s} (W_f - s\omega) \frac{1}{\varepsilon} d\omega \right] - \left[ \frac{1}{s} + \frac{1}{1-s} - 2 \right],$$

which can be expressed as:
The household members jointly maximize household utility as in the collective decision making models of, e.g., Chiappori (1988, 1997). There are two elements that the household takes into account when choosing \( s \). On the one hand, there are the convex costs of housework - captured by the last bracketed term in (12) - which have an equalizing effect as total disutility is minimized when housework is equally split. On the other, there is a participation effect. To understand this last effect, note that expected household income - the first bracketed term in (12) - is maximized when the member of the couple with the lower wage bears all the shock, the reason being that this ensures labour market participation of at least one of the household members. The choice of \( s \) is then driven by this trade-off: specialization in market and household production by the two household members maximizes income, but full sharing of housework minimizes the disutility associated with producing the household good.

The, maximizing (12) with respect to \( s \) yields the f. o. c.:

\[
\frac{\partial V^H}{\partial s} = 1 + \frac{W_m^2}{2\epsilon} - \frac{W_f^2}{2\epsilon} - \frac{1}{s} + \frac{1}{1-s} - 2 = 0,
\]

such that the equilibrium allocation of household work, denoted by \( s^* \), satisfies:

\[
\left(1 - \frac{s^*}{s}\right)^2 = \frac{W_m^2}{2\epsilon} - \frac{W_f^2}{2\epsilon}.
\]

Under the assumption that participation rates are less than 1, i.e. \( P_i = W_i / \epsilon_i \leq 1 \) for \( i = m, f \), the term \( W_i^2 / 2\epsilon \) is also less than 1.\(^9\) Then \( ds^* / dW_f < 0 \) and \( ds^* / dW_m > 0 \), implying that a higher female (male) wage results in a reduction (increase) in the share of household work allocated to women. Moreover, when wages are equalised, i.e., \( W_f = W_m \), then the allocation is even among genders, with \( s^* = 1 - s^* = 0.5 \). Note that this is due to the symmetry assumption in the way in which we model the costs of housework (no comparative advantage of either gender), together with the fact that the convex cost function implies that the total disutility cost is minimized when chores are evenly split.

\(^9\) The fact that \( W_i^2 / 2\epsilon \leq 1 \) also ensures that the second-order condition for a maximum is satisfied.
3.2 Multiple equilibria

Firms’ and households’ decisions are given by equations (9) and (14). In equilibrium expectations are fulfilled, and hence the equilibrium is determined by the solution of the following system of equations:

\[ W_f = \sqrt{b_1 e \cdot s^{\frac{\alpha}{\alpha-1}}}, \quad (E.1) \]
\[ W_m = \sqrt{b_1 e \cdot (1-s)^{\frac{\alpha}{\alpha-1}}}, \quad (E.2) \]
\[ \left( \frac{1-s^*}{s^*} \right)^2 = \frac{1-W_m^2}{2e} \cdot \frac{W_f^2}{1-W_f^2}, \quad (E.3) \]

which jointly determine the division of housework across household members and their respective wages. To analyse the equilibrium configurations, substitute (E.1) and (E.2) into (E.3) so that the f.o.c. (14) becomes:

\[ \left( \frac{1-s^*}{s^*} \right)^2 = \frac{1-b_2 (1-s^*)^{\frac{\alpha}{1-\alpha}}}{1-b_2 (s^*)^{\frac{\alpha}{1-\alpha}}}, \quad (15) \]

with \( b_2 \equiv b_1 / 2 \), such that \( b_2 \leq 1. \)

To examine the solutions of equation (15) it is useful to define the following functions of \( s \):

\[ f(s) \equiv \left( \frac{1-s}{s} \right)^2, \]
\[ g(s) \equiv \frac{1-b_2 (1-s)^{\frac{-\alpha}{1-\alpha}}}{1-b_2 s^{\frac{-\alpha}{1-\alpha}}}, \]

whose intersection results in the equilibrium allocation of housework. On the one hand, \( f(s) \) is decreasing and convex with a vertical asymptote at \( s = 0 \), with \( f(1) = 0 \) and \( f(0.5) = 1 \); moreover, from (11), it can be seen that \( f(s) \) is inversely related to the gender wage gap. On the other hand, \( g(s) \) has two vertical asymptotes at \( s = b_2^{\frac{1-\alpha}{\alpha}} \), and \( s = 1 \); further, \( g(0) = 0, g(0.5) = 1, g(1-b_2^{\frac{1-\alpha}{\alpha}}) = 0 \), and it is increasing in the range \( s \in [0, b_2^{\frac{1-\alpha}{\alpha}}] \) and decreasing when \( s \in (b_2^{\frac{1-\alpha}{\alpha}}, 1) \). Lastly, under Assumption 2, it can be

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\(^{10}\) Given Assumption 1, it follows that \( b_2 \leq 0.5. \)
checked that it has an inflection point in the interval $s \in \left( b_2 \frac{1-a}{\alpha}, 1 - b_2 \frac{1-a}{\alpha} \right)$.

Figure 3 depicts the intersections of $f(s)$ and $g(s)$. There are three values of $s$ that satisfy equation (15). In one of them, $s_1^* = 0.5$, while in the other two solutions we have $s_2^* \in (0.5, 1 - b_2 \frac{1-a}{\alpha})$ and $s_3^* \in (0, b_2 \frac{1-a}{\alpha})$. Note that corner solutions are ruled out because our assumption about the functional form of the cost function implies that, if one member of the couple does all household work, her/his marginal disutility becomes infinite and hence such a solution is never optimal.

**Figure 3: Gendered and ungendered equilibria**

Because we have assumed symmetry across genders in all respects, we have two possible asymmetric equilibria: one in which women bear a greater share of housework and get a lower wage (point G), and another in which men do more than half of the household chores and receive a lower wage (point G’). In the sequel, we focus on the more realistic case where women carry out a disproportionate share of household chores, and hence ignore that part of the domain of the $g(s)$ function defined by $[0, b_2 \frac{1-a}{\alpha})$ so that the permitted range is $s \in (b_2 \frac{1-a}{\alpha}, 1) \equiv S$. In other words, we restrict the analysis to two possible equilibria, labelled respectively as the *gendered* equilibrium (denoted by G), where $s_G^* > 0.5$, and the *ungendered* equilibrium (U) where $s_U^* = 0.5$. Thus, by replacing $\varepsilon_f = s^* \varepsilon$ and $\varepsilon_m = (1 - s^*)\varepsilon$ in (11), the following result holds:
**Proposition 2:** Under Assumption 2 and $s \in S$, there are two equilibrium solutions for the female share of household work: (i) an ungendered solution with $s^*_U = 0.5$, $w^*_U = 1$ and $p^*_U = 1$ and (ii) a gendered solution with $s^*_G \in (0.5, 1 - b_2^{(l-a)/\alpha})$, $w^*_G > 1$ and $p^*_G > 1$.

To illustrate these equilibria, consider the following numerical example. Using the values of the parameters: $\alpha = 0.5$, $\varepsilon = 1$ and $\beta = 2^{3/4}$, which satisfy Assumption 2, the roots of equation (15) are: $s^*_G = 0.7236$ and $s^*_U = 0.5$. In the G-solution, the wage and participation gaps are $w^*_G = 1.62$ and $p^*_G = 4.22$. In spite of yielding rather extreme gaps, this example illustrates that the asymmetries can be very large in the G-equilibrium. Indeed, (raw) wage gaps of 62 per cent can be found in those countries with the largest gender wage differences; see Blau and Kahn (2000).

### 3.3 Comparative statics

Inspection of (15) and Figure 3 indicates that the system in (E.1)-(E.3) may have only one equilibrium. In effect, consider a higher value of $\beta$ which increases $b_2$. Then as the range $s \in (b_2^{(l-a)/\alpha}, 1 - b_2^{(l-a)/\alpha})$ gets narrower, the $g(s)$ function becomes steeper, moving the G-equilibrium to the left. That is, as $b_2$ increases, the G-equilibrium exhibits a less unequal division of housework and hence lower wage and participation gaps. Indeed, as shown in Figure 4, for sufficiently high values of $\beta$, there could be a unique U-equilibrium. Hence:

**Proposition 3:** Under Assumption 2 and $s \in S$,

(i) The higher the value of $\beta$, the less unequal the division of household labour is in the gendered equilibrium (i.e. the lower $s^*_G$ is) and hence the lower the wage and participation gaps are.

(ii) Economies with a sufficiently high value of $\beta$ will exhibit a unique ungendered equilibrium.

The intuition for Proposition 3 is as follows. Recall that the household faces a trade-off between expected income and housework disutility: the former effect implies that full specialization is optimal ($s=1$), while the latter tends to induce an even
allocation of housework across genders ($s=1/2$). When wages are low ($\beta$ is small), the household is less willing to forgo expected income in order to reduce the disutility cost. Hence, if firms offer equal wages there will be an even division of household chores but, if they offer different wages, housework will be unevenly allocated. By contrast, when wages are high ($\beta$ is large), the household is more willing to forgo expected income in order to reduce the disutility cost, leading to a lower $s_G^*$. If wages are sufficiently high, the disutility effect dominates, making the household division of work (almost) even when wages are different across genders.\footnote{To see this simply let $b_2 \to \infty$ in equation (15), which makes its RHS equal to 1, implying that $s=0.5$.} But if $s$ is (close to) 0.5, then firms will pay similar wages to men and women. Hence the G-equilibrium cannot exist.

Figure 4: The effect of productivity on equilibria

To the extent that a higher value of $\beta$ corresponds to more productive economies, Proposition 3 implies that the gender gaps will be lower in these economies. This result on its own would explain the lower wage and participation gaps in the Nordic (richer) countries than in the Southern countries reported in Figure 1, either because their gendered equilibrium is less unequal or because it does not exist.

4. Welfare analysis

In order to analyze the welfare implications of the above-mentioned equilibria, let us consider the problem faced by a social planner who chooses the allocation of housework.

\footnote{To see this simply let $b_2 \to \infty$ in equation (15), which makes its RHS equal to 1, implying that $s=0.5$.}
taking into account its effect on wages. Since firms make zero expected profits due to the free-entry assumption, aggregate welfare is simply equal to the welfare of the representative household. Thus, substituting (9) into (12), the welfare function becomes:

\[
V^S(s) = u + \left[ b_2(1-s) \frac{1}{1-\alpha} + b_2 s \frac{1}{1-\alpha} \right] - \left[ \frac{1}{s} \frac{1}{1-s} - 2 \right]. \tag{16}
\]

We can now examine which of the two equilibria results in a higher level of welfare by substituting the f.o.c. (15) of the household into this expression, yielding:

\[
V^S(s^*) = u - 2 - \frac{1 - b_2 s^* \frac{1}{1-\alpha}}{(s^*)^2}. \tag{17}
\]

Differentiating (17) yields:

\[
\frac{dV^S(s^*)}{ds^*} = \frac{1}{s^3} \left[ 2 - \frac{2 - \alpha}{1-\alpha} b_2(s^*)^\alpha \right],
\]

which may be positive or negative, implying that it is ambiguous whether welfare is higher in the G- or in the U-equilibrium. The reason for this ambiguity is again the trade-off between maximizing market income, which occurs when there is full specialisation, and minimizing the disutility from household work, which requires equal sharing of housework.

Once more, the level of productivity \( \beta \) is the key parameter determining which effect dominates. Recall that \( b_2 \) is increasing in \( \beta \), while \( s^*_G \) decreases with the productivity level. Hence, \( dV^S(s^*)/ds^* > 0 \) for sufficiently high values of \( \beta \), leading to higher welfare in the G-equilibrium. To illustrate this result consider again our previous numerical example where \( \alpha = 0.5 \), \( \epsilon = 1 \), \( \beta = 2^{3/4} \), leading to the two equilibria \( s^*_G = 0.7236 \) and \( s^*_U = 0.5 \). If we further assume that \( u=10 \), and substitute these values in (17) we obtain that the level of welfare in the G-equilibrium, \( V^S(s^*_G) = 3.2 \), is lower than that obtained in the U-equilibrium, \( V^S(s^*_U) = 4.1 \). Lower values of \( \beta \) would yield the opposite result.

This finding contrasts with the results in existing work on this type of multiple equilibria, where it has generally been found that specialization results in higher welfare.\(^\text{12}\) The difference lies in the symmetric way we have modelled the disutility cost

\(^{12}\) An exception is the model model proposed by Lang et al. (2005) about racial discrimination in labour markets due to posted wage offers, where it is shown that discrimination creates economic inefficiency.
associated with housework. Moreover, our analysis has the implication that the nature of the efficient equilibrium may change. Suppose that the productivity parameter grows exogenously over time. Initially, when \( \beta \) is low, specialization delivers higher welfare. But as productivity grows, the opportunity cost of sharing housework falls and the U-equilibrium becomes the more efficient one.\(^{13}\)

5. Policies: Family aid and affirmative action

In this section we discuss which type of policies could shift the economy from the G-equilibrium to the U-equilibrium. Following an extensive literature on this issue, we examine two specific gender policies: (i) subsidised family aid, and (ii) affirmative action.

5.1 Subsidised family aid

5.1.1 Gender-based vs. Gender-neutral family aid

Consider the introduction of government-funded family aid. To start with, suppose that only working women receive the subsidy, and that this subsidy is proportional to their wage in period 2 so that, if working, they will receive an income equal to \( W_f (1 + \kappa) \), where \( 0 < \kappa < 1 \). Hence, women remain at work in period 2 if \( W_f (1 + \kappa) - \omega \geq 0 \). Men do not receive the subsidy and therefore work if \( W_m - \omega \geq 0 \). For the time being, we will ignore the financing of the subsidy to concentrate on the partial equilibrium effect. At the end of the section this issue will be dealt with.

Following the same reasoning as in section 2.2 about firms’ behaviour, but this time with the upper limit of the integral for women in (1) changed from \( W_f \) to \( W_f (1 + \kappa) \), we have that firms choose the following amount of training and wages:

\[
\tau_f^{\kappa*} = \left( \frac{(1 + \kappa)\beta^2}{4e_f} \right)^{1/\alpha}, \quad W_f^{\kappa*} = \frac{\beta}{2} (\tau_f^{\kappa*})^{\alpha/2}, \quad (18)
\]

where the superscript \( \kappa \) denotes the case with subsidies. Male workers are offered the training level and wage derived in (9). Note that the total income of women in period 2, however, their setup is quite different from our model since they assume that the posted wage offers are binding on the employers.

\(^{13}\) For analyses of how exogenous changes in productivity affect gender differences in the labour market, see Olivetti (2006) and Albanesi and Olivetti (2007).
\( Y_f^{*\kappa} \), is now given by:

\[
Y_f^{*\kappa} = (1 + \kappa)W_f^{*\kappa} = \frac{\beta(1 + \kappa)}{2}(\tau_f^{*\kappa})^{a/2}.
\]

(19)

Clearly, \( \tau_f^{*\kappa} > \tau_f^{*} \) and \( W_f^{*\kappa} > W_f^{*} \). For \( \kappa < (\epsilon_f - \epsilon_m)/\epsilon_m \), i.e. if the subsidy is not too large, it is also the case that women receive less training and lower wages than men, that is, \( \tau_f^{*\kappa} < \tau_m^{*} \) and \( W_f^{*\kappa} < W_m^{*} \).\(^{14}\) Thus, not surprisingly, women fare better in the labour market when they are subsidised to stay in the job. They may even get higher wages than men if the subsidy is sufficiently high, a possibility that we ignore in the sequel. Likewise, from (18) and (19), a straightforward result would be that, abstracting from the household decision, the corresponding participation and wage gaps with family benefits, \( p^\kappa \) and \( w^\kappa \), are lower than without subsidies. This result, however, will not hold once we take into account the endogeneity of the division of housework.

Consider the household’s decision in this case. Each household chooses \( s \) to maximize the expected net utility which is now given by:

\[
V^{*\kappa} = u - 2 + \frac{1}{2s} \left[ \frac{W_m^2}{(1-s)} + \frac{Y_f^{*\kappa^2}}{s} \right] - \left[ \frac{1}{s} + \frac{1}{1-s} \right].
\]

(20)

The resulting f. o. c., once we have substituted for wages, yields the new equilibrium relationship:

\[
\left( \frac{1-s^{*\kappa}}{s^{*\kappa}} \right)^2 = \frac{1-b_2(1-s^{*\kappa})^{2-a}}{1-b_2s^{*\kappa 1-a}},
\]

(21)

where \( b_2 = b_2(1 + \kappa)^{2-a} > b_2 \). The LHS of equation (21) is the same as in (15), while the RHS tilts upwards and takes a value greater than 1 when \( s=0.5 \). The new equilibrium is depicted in Figure 5 and can be summarised as follows:

**Proposition 4:** Under Assumption 2 and \( s \in S \), a wage subsidy to female workers leads to a gendered equilibrium with \( s^{*\kappa} \in (0.5,1) \). The equilibrium division of household work implies a higher share for women, and hence a greater wage and participation gap than in the absence of the subsidy.

\(^{14}\) In equilibrium, since \( \epsilon_f = s\epsilon \) and \( \epsilon_m = (1-s)\epsilon \), this condition becomes \( \kappa < 2s - 1 \).
A surprising feature of (21) is that, with the subsidy in place, the U-equilibrium with $s = 0.5$ does not exist any longer. In other words, a gender-based subsidy policy only yields the G-equilibrium since the asymmetry in income due to the subsidy prevents a symmetric equilibrium. In effect, suppose that households set $s = 0.5$. Then women have a greater probability of staying in the job than men (the combination of the same shock plus the subsidy), implying that firms will offer women more training and a higher wage. But if female wages are higher than men’s, then $s = 0.5$ cannot be a solution to the household’s problem. Hence, the U-equilibrium disappears. Moreover, from Figure 5 we can see that the new G-equilibrium lies to the right of the initial one, implying a higher equilibrium value of $s$. For example, in our previous example, with $\kappa = 0.1$, we get $s^* = 0.7299 > s^*_G = 0.7236$. To understand this result, recall the trade-off faced by the household between increasing expected income and reducing the disutility of housework. Because of the increase in the probability of female participation due to the subsidy, the household can now afford to raise the probability of male participation by reducing their housework share. This result shares the spirit of the analysis of affirmative action policies in Coate and Loury (1993) where it is argued that an exogenous increase in the hiring probability faced by a minority would reduce their educational effort and hence increase the educational gap. Similarly, in our framework the exogenous increase in the probability of participation of women reduces their commitment to the labour market.
Consider now the alternative policy of giving the same subsidy to men and women. Following the same reasoning as above, this would yield the equilibrium relationship:

\[
\left(1 - s_{s}^{*}\right)^{2} = \frac{1 - b_{1}(1 - s_{s}^{*})^{1-a}}{1 - b_{1}s_{s}^{*1-a}},
\]  

(22)

that will again narrow the range of values of \(s\) for which the RHS of (22) is positive, since \(b_{3} > b_{2}\). The first implication is that the subsidy shifts the G-equilibrium to the left, reducing the value of \(s_{G}^{*}\). Moreover, if the subsidy is high enough (i.e. if \(b_{3}\) is sufficiently large), equation (22) would have a unique U-equilibrium, as in Figure 4, with higher participation rates and wages of both genders than under laissez-faire, although the corresponding relative gaps remain the same as without subsidies. Once more this is the result of the trade-off between higher expected income due to specialization and lower disutility due to the sharing of housework. The subsidy effectively increases expected income and hence reduces the opportunity cost of sharing housework. If the increase in income is large enough, the household will simply minimize the disutility associated with housework and choose equal sharing of domestic tasks. This reasoning also echoes some of Saint-Paul (2007)’s recent arguments in against gender-based taxation.

5.1.2 Financing the subsidy

We next turn to the financing of the subsidy. It is clear from the earlier discussion that a subsidy for women financed by taxing men will lead to an asymmetry in the RHS of (21) eliminating therefore the U-equilibrium. Instead, we suppose that firms are taxed for their training expenditures at a proportional rate \(t\) in period 1.\(^1\) Under a balanced government budget, this implies that \(t(\tau_{f} + \tau_{m}) = \kappa(W_{f}P_{f} + W_{m}P_{m})\).

In this tax-subsidy scheme, denoted by the superscript \(T\), participation is given by \(P_{i}^{T} = (1 + \kappa)W_{i}^{T} / \varepsilon_{i}\), and firms set the wage to be \(W_{i}^{T} = a(\tau_{f}) / 2\), implying that their gross profits are

\[
\Pi(\tau_{f}) = \frac{a(\tau_{f})^{2}}{4\varepsilon_{i}}(1 + \kappa),
\]  

(23)

\(^1\) We have also examined the case where the tax is lump-sum in the first period. This case yields similar results though the calculations are somewhat more complex.
Whereas the zero-profit condition for firms is now given by

\[ \Pi(\tau_i) - (1 + t)\tau_i = 0. \]  

(24)

Noting that we can write \( \Pi(\tau_i) = P_i W_i \), this condition is simply equivalent to \( \tau_i(1 + t) = P_i W_i \), which can then be replaced into the budget constraint to obtain the equilibrium relation between the tax and the subsidy rates, i.e., \( t = \kappa/(1 - \kappa) \). The zero-profit condition, together with the expression for \( t \) just obtained, yields the optimal level of training

\[ \tau^*_i = \left[ \frac{\beta^2 (1 + \kappa)}{4\varepsilon_i (1 + t)} \right]^{1/\alpha} \]  

(25)

Equation (25) implies lower training and wages than without subsidies as a result of the labour tax paid by firms. Participation, given by \( P_i^r = (1 + \kappa)W_i^r / \varepsilon_i \), may be higher or lower than under laissez-faire due to the opposite effects of the subsidy that tends to increase it and the lower wage that tends to reduce it.

As regards the household decision on \( s \), a similar argument as before yields the following f.o.c.:

\[ \left( \frac{1 - s^r}{s^r} \right)^2 = \frac{1 - b_4 (1 - s^r)^{1/\alpha}}{1 - b_4 s^r^{1/\alpha}}, \]  

(26)

where \( b_4 = b_2 h(\kappa) \) with \( h(\kappa) = [(1 - \kappa)^\alpha (1 + \kappa)]^{1/\alpha} \). Then, \( \kappa(0) = 1 \) and \( h'(\kappa) > 0 \) iff \( \kappa < (1 - \alpha)/(1 + \alpha) \), that is, for not too high values of \( \kappa \), it holds that \( h(\kappa) \) is increasing and therefore \( b_4 > b_2 \). Hence, this tax-subsidy scheme increases the slope of the \( g(s) \) function, implying that the equilibrium value of \( s \) will fall and, potentially, become a unique U-equilibrium. Indeed, for the G-equilibrium to disappear we also need that there is a unique intersection, which will be the case if \( 1 - b_4^{(1 - \alpha)/\alpha} \ll 0.5 \), i.e. if \((1 + \kappa)(1 - \kappa)^\alpha \geq 2^{3-2\alpha} \varepsilon / \beta^2 \). Thus, the following result holds:

**Proposition 5:** Under Assumption 2 and \( s \in S \), if \( \kappa \) is not too large, an equal wage subsidy to male and female workers financed with a proportional tax on training expenditures by firms in period 1 will reduce the degree of gender segregation and may lead to an ungendered equilibrium with \( s^r = 0.5 \).

22
The intuition for this result again relies on the two opposite effects affecting participation: a direct one from the subsidy which tends to increase participation, and an indirect one operating through the reduction of training induced by the tax paid by firms, which tends to reduce participation. The condition \((1-\alpha)/(1+\alpha) > \kappa\) is easy to interpret since, from (26), a low value of \(\alpha\) implies a low elasticity of training with respect to the subsidy. This means that the wage does not fall by much and the direct effect dominates, leading to higher participation and higher expected income for any given division of housework. As in section 3.3, a higher income implies that couples can afford to reduce the disutility cost of housework thereby choosing an equal split.

5.2. Affirmative action

An alternative policy is to impose by law an affirmative action rule that prevents firms from engaging in statistical discrimination. In our setup this type of discrimination appears because firms offer different amounts of training to men and women. Consider then a policy that forces firms to post the amount of training they will provide before they are matched with a candidate. Recalling that all individuals get trained in period 1 and that the population is equally split across genders, the expected profit of a firm is given by:

\[
\Pi(\tau) = \frac{1}{2} \left( \frac{a(\tau)^2}{4\epsilon_f} + \frac{a(\tau)^2}{4\epsilon_m} \right). \tag{3'}
\]

Then, the zero-profit condition \(\Pi(\tau^a) - \tau^a = 0\) pins down the optimal level of training in period 1 under affirmative action, \(\tau^a\), yielding:

\[
\tau^a = \frac{\beta^2}{8} \left( \frac{1}{\epsilon_f} + \frac{1}{\epsilon_m} \right)^{\frac{1}{\epsilon^a}}. \tag{5'}
\]

Since now men and women receive the same amount of training, equation (2) implies that they also receive the same wage.

Turning to the couple’s division of household work, from the f.o.c. in (E.3), equal wages imply equal sharing of domestic tasks. Hence, the only possible equilibrium is \(s = 0.5\). Note that this implies that it is optimal for the firm to offer the same amount of training to both genders. In other words, since the reason for the existence of the U- equilibrium is a coordination problem, affirmative action will coordinate firms and households on the U- equilibrium in which firms would choose not
to discriminate across genders even if they could.

That affirmative action policies reduce gender gaps is not surprising. A recurrent question in the literature is whether or not such policy needs to be permanent or whether a one-shot policy can permanently reduce gender gaps; see Coate and Loury (1993). The fact that, in our setup, multiplicity is due to a coordination problem implies that a one-period policy will change the equilibrium permanently. Imposing equal amounts of training for men and women changes the household decision problem and results in equal sharing of household tasks. Once firms observe $s=0.5$ and couples are paid equal wages, there is no reason for either of them to expect a different outcome even if the affirmative action policy were removed. Hence, the economy will remain in the U-equilibrium.

5.3. Asymmetric economies

Our framework makes the strong assumption of complete symmetry between men and women, and it is precisely this assumption that allows for the existence of U-equilibria. In this section we briefly examine how results get modified when we assume that there is an (exogenous) asymmetry associated to gender.

There are many ways of allowing for asymmetries, ranging from differences in comparative advantage in home/market production to the structure of intra-household bargaining. For simplicity, we focus on the latter and assume that men have higher bargaining power in the household decision-making process, measured by $\eta$, so that household utility can be expressed as:

$$V'' = u + \frac{1}{2\epsilon} \left[ \frac{(1+\eta)W_m^2}{(1-s)} + \frac{(1-\eta)W_f^2}{s} \right] - \left[ \frac{1}{s} + \frac{1-\eta}{1-s} - 2 \right],$$

with $\eta \in (0,1)$. The resulting f. o. c. together with the expressions for wages in (9) imply that equilibrium is given by:

$$\left( \frac{1-s^*}{s^*} \right)^2 = \frac{1-\xi b_2(1-s^*)}{\xi - b_2(s^*)} = \frac{\alpha}{1-\alpha}, \quad (30)$$

where we denote the relative bargaining power $(1+\eta)/(1-\eta)$ by $\xi > 1$. The LHS of this expression is the same as in the symmetric case, while the RHS, i.e. the $g(s)$ function, shifts upwards when compared to equation (15). As a result, when $s=0.5$, $g(s)$ takes a value greater than 1, implying that the U-equilibrium cannot exist.
Because the household gives greater weight to the disutility of the man, even when wages are the same across genders, women will end up doing more than half of the housework. But as women are bearing a greater fraction of the shock, firms will pay lower wages to them. Hence only the G-equilibrium exists.

Under this asymmetric case, a wage subsidy targeted to women can work. In effect, a subsidy equal to $\kappa W_j$ paid to participating women implies that equations (21) and (30) yield the following f. o. c.:

$$\left(1 - \frac{s^{\kappa,\eta^*}}{s^{\kappa,\eta^*}}\right)^2 = \frac{1 - \xi b_2 (1 - s^{\kappa,\eta^*})^\frac{\alpha}{1-\alpha}}{\xi - b_2 (s^{\kappa,\eta^*})^\frac{\alpha}{1-\alpha}} \tag{31}$$

where the superscript $(\kappa, \eta)$ denotes the case with asymmetric power and subsidies, and $b_1 = b_2 (1 + \kappa)^{\frac{2}{1-\alpha}}$. Thus, one could choose $\kappa$ so as to make the right-hand-side of (31) equal to 1 when $s=0.5$, yielding:

$$(1 + \kappa) = \left(\xi + \frac{\xi - 1}{\alpha} b_2 2^{\frac{1}{1-\alpha}}\right)^{\frac{1-\alpha}{2-\alpha}},$$

implying that the f. o. c. in (28) becomes:

$$\left(1 - \frac{s^{\kappa,\eta^*}}{s^{\kappa,\eta^*}}\right)^2 = \frac{1 - \xi b_2 (1 - s^{\kappa,\eta^*})^\frac{\alpha}{1-\alpha}}{1 - \xi b_2 (s^{\kappa,\eta^*})^\frac{\alpha}{1-\alpha} + (\xi - 1) \left(1 - (2s^{\kappa,\eta^*})^\frac{\alpha}{1-\alpha}\right)}. \tag{32}$$

Comparison of (32) and (15), using the same reasoning as in (29), implies that the U-equilibrium becomes more likely. Whether it is a unique equilibrium or not hinges on the sizes of $\xi$ and $b_2$, which in turn depend upon $\eta$ and $\beta$ (for given values of $\alpha$ and $\varepsilon$). This result somewhat mimics the argument made by Alesina et al. (2007) in their proposal of different taxation for men and women. In their reasoning, the asymmetry across genders is that women have higher elasticity of labour supply than men and, therefore, in line with to the Ramsey principle of optimal taxation, the former should have lower taxes than the latter. In our setup, the asymmetry arises from different bargaining power but the policy implication is similar. Notice, however, that (29) also implies the novel result that, for given $\eta$, this gender-based taxation scheme is bound to be more effective in achieving the U-equilibrium in more productive economies (those with higher $\beta$), than in less productive ones.
6. Empirical evidence

6.1. Data and descriptive statistics

The main data used for the empirical analysis is taken from the 2002-2003 Spanish Time Use Survey (STUS) where individual information is available to us. This dataset is part of the Harmonized European Time Use Survey (HETUS) carried out by Eurostat.

STUS contains a wide range of information about roughly 20,000 households (including household composition, region, domestic service, housing characteristics, household income, etc.) and on each of its members (demographic and labour market characteristics, including wages). The main novelty of this database is the availability of individual information on daily activities, through a personal diary, which all members of the household over ten years old must complete on a selected day. The diary records the activities undertaken by each respondent. For each 10 minutes interval (and during 24 hours), individuals must record which are the primary and secondary activities that have been undertaken. Activities are coded according to a list provided by Eurostat (for details, see Table A1 in the Appendix). Housework time is defined as the number of minutes reported in the diary that each individual devotes, as primary activity, to category 3 in Table A1, i.e. household and family (including cooking, cleaning, laundry, gardening and pets, maintenance and repairs, shopping, household management, childcare and other adults’ care). From this information, we can compute the shares in total housework of each partner and hence the ratio between women’s and men’s shares.

Our sample is restricted to employed couples living in the same household, aged 25-64 years, with valid information on wages and housework share. The resulting final sample consists of 1,578 households for which there is also full information on basic demographic and labour market characteristics of both the head and the spouse. Table 1 presents descriptive statistics (means and standard deviations) of wages, full-time/part-time status, age, education, availability of family aid and domestic service, and female housework share by different demographic characteristics of the couple.\textsuperscript{16}

\textsuperscript{16} Wages are measured on an hourly basis. However, the original wage variable is net monthly wages and it is reported in intervals of 500 euros. There are seven wage intervals: 0-500, 500-1000, 100-1500, 1500-2000, 2000-2500, 2500-3000 and >3000. We have taken the average value of each interval as the point value for net monthly wages. Then, we converted net monthly wages into hourly wages by using weekly working hours, which are available for each member of the household.
<table>
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<th>Variables</th>
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</tr>
<tr>
<td>Average Age Woman</td>
<td>40.1</td>
<td>8.07</td>
</tr>
<tr>
<td>Average Age Man</td>
<td>42.33</td>
<td>8.32</td>
</tr>
<tr>
<td><strong>Female Housework Share</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.65</td>
<td>0.18</td>
</tr>
<tr>
<td>By Education</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary</td>
<td>0.67</td>
<td>0.19</td>
</tr>
<tr>
<td>Secondary</td>
<td>0.66</td>
<td>0.18</td>
</tr>
<tr>
<td>University</td>
<td>0.62</td>
<td>0.18</td>
</tr>
<tr>
<td>By Age</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt; 30</td>
<td>0.64</td>
<td>0.21</td>
</tr>
<tr>
<td>31-40</td>
<td>0.64</td>
<td>0.17</td>
</tr>
<tr>
<td>41-50</td>
<td>0.66</td>
<td>0.18</td>
</tr>
<tr>
<td>&gt; 50</td>
<td>0.68</td>
<td>0.17</td>
</tr>
<tr>
<td><strong>Other Household variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Receiving family aid income</td>
<td>0.03</td>
<td>0.18</td>
</tr>
<tr>
<td>% Domestic service</td>
<td>0.22</td>
<td>0.41</td>
</tr>
<tr>
<td>Number of children</td>
<td>0.65</td>
<td>0.86</td>
</tr>
</tbody>
</table>


Inspection of Table 1 shows that men earn 19% on average more than their partners. Also, 97% of males in our sample work on full-time basis while around 15% of women work as part-timers. Regarding education, the percentage of college education is higher for women (32% compared to 26%), indicating that working wives are on average more educated than their spouses. In addition, they are on average two years younger. Finally, the burden of housework is vastly taken by the female partner: on average, the female housework share is 65%. Further, it is higher the lower the
female educational level is, but even for women with college education it averages 62%. Regarding age, younger women seem to share the housework with their partners more equally than older ones. Finally, 22% of the households have some form of domestic service but only 3% receive family aid.

6.2. Female share of household work and the gender wage gap

Proposition 2 above implies that, under the G-equilibrium, the female share of housework is larger than 0.5 and the wage gap favours men. In addition, the higher the gender wage gap, the higher the female housework share will be in equilibrium. Thus, we should observe a positive correlation between the female share and the gender wage gap within couples. This is the first empirical implication to be tested with our dataset.

We present two types of evidence to check whether Proposition 2 holds. The first one is just the correlation between aggregate gender wage gaps and aggregate female housework share across countries. The HETUS website allows users to get aggregate information for participating countries on the average time spent by men and women in housework tasks for specific demographic groups. Like in Figure 1, we have selected married and working men and women with less than tertiary education and computed the female share of housework. In Figure 6 this share is plotted against the gender wage gap in the HETUS countries. The correlation, excluding the Italian outlier (see footnote 4), is strongly positive and therefore consistent with Proposition 2.

Figure 6: Correlation between Gender Wage Gap and Female Housework Share

Notes: (X-axis) Female Housework Share is constructed as the relative (%) share of housework task that married working females exert relative to married working males (from HETUS). (Y-axis) Raw Gender wage gaps for workers with less than tertiary education – 2001.
Secondly, we make use of the micro data in STUS to compute the intra-household division of housework. Specifically, we select households of working couples as our unit of analysis and estimate the effect of the gender wage gap within the household on the female relative share of housework, i.e., \( (s/1-s) \). To achieve a support of the distribution of the dependent variable over the entire real line, we estimate a linear regression model between the log of \( (s/1-s) \) and the (logged) wage gap within the couple (log male hourly wage minus log female hourly wage). Other controls are the education levels and full-time job status, number of children and availability of domestic service and dummies for regions. Initially, we also included the age of both women and spouse, although we finally decided to exclude them as their estimated coefficients were highly insignificant. OLS results for the specification without regional dummies are presented in the first column of Table 2.17

The results in Table 2 reveal that the response of the female relative housework share with respect to the (logged) wage gap is such that an increase of 10 percentage points in the wage gap raises the female share by 0.45 percentage points, and this magnitude is statistically significant at 5% significance level.18 Other interesting results are the expected ones: female’s (male’s) higher educational attainment and working full-time (part-time) lead to a reduction (increase) in the female share of housework, whereas a larger number of children or the lack of domestic service increases it.

17 OLS is used because, at the individual level, wages are taken as given. However, it could be argued that the wage gap might be endogenous for the female housework share if unobserved individual characteristics are positively correlated with the wage gap, thus creating spurious correlation between this variable and the error term. We have tried to instrument the wage gap with second or third-order polynomials in the age gap and interactions between female education and the age gap, as in Mroz (1987), Fortin and Lacroix (1997) and Chiappori et al. (2002). However, somewhat surprisingly, the correlation between these variables and the logged wage gap is very weak, which prevents them from being used as suitable instruments. Unfortunately, STUS does not contain any other variables that can be used as adequate instruments for the gender wage gap.

18 The estimated coefficient (0.196) corresponds to \( \frac{\partial \log(s/1-s)}{\partial \omega} \). Thus, \( \frac{\partial s}{\partial \log \omega} = \frac{\partial s}{\partial \log(s/1-s)}0.196 = s(1-s)0.196 \). Using the average value of \( s \) in Table 1 (0.65) yields a response of 0.045 with a standard error of 0.021 using the delta method.
Table 2. Estimates of the Impact of the Gender Wage Gap on Relative Female Housework Share. Dependent Variable: log(s/(1-s))

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Logged Wage Gap</td>
<td>0.196 **</td>
<td>0.188 **</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.090)</td>
</tr>
<tr>
<td>Dummy of Rich Region</td>
<td>---</td>
<td>-0.086 *</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.050)</td>
</tr>
<tr>
<td>Secondary Educ. (woman)</td>
<td>0.032</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>Secondary Educ. (spouse)</td>
<td>-0.084</td>
<td>-0.103</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>University Educ (woman)</td>
<td>-0.102</td>
<td>-0.132 *</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
<td>(0.082)</td>
</tr>
<tr>
<td>University Educ. (spouse)</td>
<td>0.107</td>
<td>0.142 *</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Full-Time job (woman)</td>
<td>-0.413 ***</td>
<td>-0.406 ***</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Full-Time job (spouse)</td>
<td>0.534 ***</td>
<td>0.540 ***</td>
</tr>
<tr>
<td></td>
<td>(0.134)</td>
<td>(0.132)</td>
</tr>
<tr>
<td>Number of children</td>
<td>0.140 ***</td>
<td>0.143 ***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Domestic Service</td>
<td>-0.123 **</td>
<td>-0.114 *</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.063</td>
<td>0.065</td>
</tr>
<tr>
<td>N. observations</td>
<td>1578</td>
<td>1578</td>
</tr>
</tbody>
</table>

Note: Robust standard errors. *, **, *** mean significantly different from zero at 10%, 5% and 1% levels, respectively. In column [1], 17 controls for CCAA are also included. In column [2], instead of 17 regional controls, we have introduced a dummy for “Rich Regions”. These are those with GDP per capita above the Spanish average: Balearic Islands, Cataluña, Madrid, Navarra and the Basque Country.

6.3. Female Share of Household work and Productivity

Given the evidence shown above, we can describe Spain as exhibiting, if anything, a G-equilibrium. Proposition 3 above suggests that the G-equilibrium in economies with high productivity levels will exhibit a lower female housework share than in economies where productivity levels are smaller, a prediction that can be tested with our data.

As before, we present two types of evidence. First, simple aggregate correlations for those countries participating in HETUS are offered. Next, more elaborate intra-household evidence from STUS is presented. Regarding the former, Figure 7 displays GDP per capita and the female share of household work across countries. The correlation is strongly negative, and therefore consistent with this implication of the our model.
In order to measure the partial correlation between productivity and the within household female share, we can make use of the fact that we have information on the Spanish regions (*Comunidades Autónomas* or CC.AA) in which the household lives. Given that there are significant differences in productivity levels among regions, we can examine to what extent more productive regions exhibit a lower female share of housework, as Proposition 3 asserts. Regional productivity levels are measured in terms of regional GDP per capita (in 2002) and we compute the average female housework share by region from our sample of couples. Five regions have a GDP per capita clearly above the national average: the Balearic Islands, Cataluña, Madrid, Navarra and the Basque Country. We denote these as “rich regions”. At the aggregate level, Figure 8 depicts the correlation between regional labour productivity and the female share, which turns out to be negative as well. At the micro level, a dummy variable for rich regions is constructed and, again using OLS, we look at the impact of working a rich region on the female share of housework task. We also intersected this dummy variable with the logged wage gap but the coefficient turned out to be insignificant. Column 2 of Table 2 displays the results of this exercise. The impact of working in a rich region is negative and significant at the 10% level, which point out to the existence of a smaller female share in housework tasks in more productive regions, as Proposition 3 suggests.
6.4. **Female share of housework and policies**

The empirical predictions of the proposed model regarding the effects of a subsidy are not clear-cut: in asymmetric households where men have more bargaining power than women, a subsidy targeted at working women can shift the G-equilibrium to the U one. However, in a symmetric household, the U-equilibrium can only be achieved through a neutral-gender subsidy targeted at both members of the household. Testing accurately this empirical prediction is not feasible with the data at hand since the STUS lacks information on whether the male, female or both members of the household receive a family-aid subsidy. In these circumstances, all we can offer is some descriptive evidence on the correlation between the percentage of GDP spent on family aid expenditure and the female share of housework task across countries using the aggregate data from HETUS. Figure 9 displays such correlation. The relationship between the two variables is strongly negative: countries which devote a greater share of the GDP to family aid are those where the female share of housework is lower, and vice versa. This preliminary evidence may be interpreted in terms of our discussion in section 5.3 about asymmetric economies where a different bargaining power may exist. However, more research needs to be done in order to clarify the channels through which these subsidies affect household decisions.
Figure 9: Correlation between Expenditure on Family Aid and Female Share of Housework

Notes: (X-axis) Female Housework Share, constructed as the relative (%) share of housework task that married working females exert relative to married working males (from HETUS). (Y-axis) Family Aid Expenditures as a percentage of GDP (2001).

7. Conclusions

We have proposed a simple model of self-fulfilling prophecies in which statistical discrimination of women results in housework and labour market differences across genders despite men and women being ex-ante identical. In contrast with other models in this literature, our model does not rely either on moral hazard due to unobservable effort, efficiency wages in some sectors or adverse selection problems. Instead, we propose a framework in which employers train equally productive men and women, but have different expectations about the distribution of disutility shocks (unexpected need of household work) that each gender will face after they have been trained for a job. If future wages are predetermined with regard to these shocks, then job quit rates will differ across genders, causing labour market participation to depend on expected wages and vice versa.

Our model gives rise to two types of equilibrium -gendered and ungendered-leading to several policy implications which are harder to obtain in other models. First, in contrast to most of the literature, welfare in the symmetric equilibrium can be greater than in the asymmetric one. The reason for this result is that having one member of the household specializing in home production has two opposing effects: on the one hand, it leads to greater expected household income, as is standard in the existing literature; on the other, the disutility of housework is minimized when this task is evenly shared.
amongst household members. Which effect dominates depends crucially on the level of productivity: the ungendered equilibrium results in higher welfare in highly productive economies, while the opposite holds in less productive ones. The immediate implication of this result is that the desirability of policy intervention may not be the same in all economies. In particular, we have shown that a policy of family aid (e.g., wage subsidies) targeted to married women may not only fail to achieve a symmetric equilibrium but could also worsen the gender wage gap. In contrast, a gender-neutral subsidy (i.e., targeted to both members of the couple) could be more efficient at leading to an ungendered equilibrium, and such policy works better in more productive economies.

Preliminary evidence using European (macro data) and Spanish (micro data) time-use surveys yields support to some of our empirical predictions concerning the relationship between wages and the sharing of household tasks, as well as the role of productivity. However, more empirical work is needed in order to test other ones, notably the effect of alternative tax-subsidy policies whose effects we cannot identify with the datasets at hand. This remains in our future research agenda.
References


Data Appendix


<table>
<thead>
<tr>
<th>Group of Activities</th>
<th>Activities included</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. Personal Care</td>
<td>Sleep, eat, other personal care</td>
</tr>
<tr>
<td>1. Job</td>
<td>Main and second job, activities related to employment</td>
</tr>
<tr>
<td>2. Study</td>
<td>School and University, homework</td>
</tr>
<tr>
<td>3. Family and housework</td>
<td>Food preparation, dish washing, cleaning dwelling, other household upkeep, laundry, ironing, handicraft, gardening, tending domestic animals, caring for pets, walking the dog, construction and repairs, shopping and services, physical care, supervision of child, teaching, reading and talking with child, other domestic work</td>
</tr>
<tr>
<td>4. Voluntary work and meetings</td>
<td>Organisational work, informal help to other households, participatory activities</td>
</tr>
<tr>
<td>5. Social life and leisure</td>
<td>Visits and feasts, other social life, entertainment and culture, resting</td>
</tr>
<tr>
<td>6. Sports and outdoor activities</td>
<td>Walking and hiking, other sports or outdoor activities</td>
</tr>
<tr>
<td>7. Hobbies and computer games</td>
<td>Computer and video games, other computing, other hobbies and games</td>
</tr>
<tr>
<td>8. Lecture, radio and TV</td>
<td>Reading books, other reading, TV and video, radio and music, unspecified leisure</td>
</tr>
<tr>
<td>9. Travel related to work or others</td>
<td>Travel to/from work, study or shopping, other travel</td>
</tr>
</tbody>
</table>