Demographic Change and Pension Reform in Spain:  
An Assessment in a Two-Earner, OLG Model 
by 
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Abstract

Recent pension reforms in Spain have been guided by two opposite goals, achieving financial stability and improving redistributive aspirations. In particular, reforms implemented in 1997/2002 entailed a mixture of both through (i) changes in the pension formula, (ii) the extension in the entitlement to early retirement to all cohorts, and, finally, (iii) increases in survival pensions. This paper builds an Applied General Equilibrium OLG model that captures the fundamental non-stationarity of the Spanish reality (ageing population, education transition and increasing female’s attachment to the labour market) to assess the impact of those reforms. As a novel feature with respect to the literature, households in our model economy are made of two potential earners that make saving and labour supply decisions. Our main conclusions from the analysis are at three different levels. First, the Spanish pension system is clearly unsustainable, the pension expenditure will reach a figure of about 18% of the GDP in 2050, and the reforms have been clearly ineffective in improving the financial prospects of it. Second the reforms have had substantial redistributed effects, benefiting low educated groups against high educated and future cohorts against current cohorts. Finally we show that exploring the financial prospects with traditional single earner households models may result in underestimates of the future financial burden of the pension system.

JEL codes: D58, H55, J11
1 Introduction

The aging of the population and the imminent retirement of the large cohorts of “Baby Boomers” have generated widespread concern about the financial sustainability of PAYG pension systems. In response, most countries have engaged in reforms of their current systems, largely aimed at reducing pension generosity. Spain has not been an exception to this general trend, having introduced significant changes in its pension rules from 1997 to 2002. The Spanish reform, however, stands out as remarkably timid and unambitious, packing together a heterogeneous mix of measures with opposite financial consequences. This outcome is probably the result of two main forces. On the one hand, the Spanish pension system enshrines the principle of solidarity as one of its core foundational values. This reflects in the existence of important redistributive dispositions in a system whose basic structure is largely “Bismarkian” (i.e. individual pensions are proportional to previous contributions). Redistribution is essentially achieved by putting minimum guaranteed benefits for both old-age and survival pensions. Besides, survival pensions themselves are considered a key redistributive tool, by making possible to bridge the large gender gap in pension income currently existing in Spain (stemming from the extremely low participation rates of the females belonging to Spanish most senior cohorts). On the other hand, the recent economic performance of the Spanish economy (prior to the 2008 crisis) has been quite remarkable, featuring a particularly good behavior of the labor market and large flows of immigrants. Naturally, this has produced a marked improvement in the short-term financial indicators of the pension system, what may have mistaken the Spanish legislators about the real long-term health of the system. All in all, the political process has delivered a fragmentary set of reforms with clearly contradicting financial consequences. Fiscal consolidation has been pursued by increasing the length of the averaging period in the pension formula and by imposing stronger penalties for an insufficient record of contributions. At the same time and for the sake of enhanced solidarity, the generosity of survival pensions have been significantly increased and dispositions limiting the right to early retire (before 65) to some specific cohorts have been waived.

The goal of this paper is to assess the impact of the set of implemented reforms, both in financial and in redistributive terms (i.e. changes in welfare). To achieve this goal we develop a dynamic general equilibrium model specially designed to capture the specificities of the Spanish case. Individuals in the model differ in their gender, education and age of birth. They are grouped together upon entering in the labor market in households, which take lifecycle consumption/savings decisions and labor participation decisions. The household head is assumed to participate continuously, until an endogenously chosen retirement age. The spouse, in contrast, make a once and for all decision on labor participation at the beginning of her productive life. This setting is rich enough to accommodate most of the essential elements of the question in hand: a detailed reproduction of old-age pensions (including certain features that are contingent on household composition, as for instance, the size of the minimum pension) and survival pensions, the changing patterns in education and female labor participation by cohort and the endogenous reaction in both labor and savings to

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2 A concise review of recent reforms can be found in Whiteford and Whitehouse (2006) or (with much more detail) in Caseay et al. (2003).

3 See Objetivo 1 in MTAS (2005)

4 In the interval 2001-2007, 28% of old-age pensions and 37% of survival pensions were increased up to the annually legislated minimums. Note also that some redistribution takes place through the existence of “Special Regimes” (for eg. farmers, sailors, coal miners ...) but their are doom to progressively disappear according with the Pacto de Toledo political agreement.
changes in the institutional environment.

The literature on general equilibrium analysis of pensions in the context of ageing population has relied on standard OLG models in the Auerbach and Kotlikoff (1987) tradition. The literature is rather large, having explored issues like the intra-generational redistribution implied by the pension system (e.g. Cubeddu (1996) emphasizes differences in education, sex and race while Huggett and Ventura (1999) explore the role of minimum pensions); the insurance provided against survival risks or idiosyncratic income shocks (e.g. Storesletten et al. (1999) or Imrohoroglu et al. (1995)) or the strong inter-generational consequences of pension reform or even complete privatization (e.g. De Nardi et al. (1999)). This strand of the literature is particularly well represented in the Spanish case. Conesa and Garriga (2001) study the inter-generational properties of the Spanish pension system in a representative agent life-cycle model. Rojas (2005) extends the analysis to a framework in which productivity depends on age and Sánchez (2010) to a framework in which individuals are ex ante heterogeneous in their education. Finally, Díaz-Giménez and Díaz-Saavedra (2009) consider an economy in which individuals are heterogeneous due to both ex ante differences in education and also due to the different earnings shocks they suffer along their life. However, up to now, this literature has resorted to one-earners household decision makers, playing down the changing pattern of female labor participation and sometimes ignoring survival pensions altogether.

By reflecting gender differences and having households made of two potential earners as the economic units of our model we join a strand of the macroeconomic literature that initially focused on the determinants of the increase in female labour supply and then moved towards the analysis of its aggregate consequences. Some outstanding examples of the former are Caucutt et al. (2002), Greenwood et al. (2005), Olivetti (2006) or Attanasio et al. (2008). Among the latter, and more related to us, are those contributions that emphasizes the importance of two potential earner households for policy analysis. Guner et al. (2008) show the importance of adding the female labour supply margin for the analysis of tax reforms. They find that married women account for a disproportionately large fraction of the changes in labour supply (hours and participation) resulting from tax reforms. The aggregate implications of reforms can, then, vary wildly depending on whether a single-earner or household model is used.

Intrahousehold heterogeneity models have also made headway in pension analysis. There has been a steady progress in modeling the interaction of pension and fiscal rules and household decisions in partial equilibrium. For example, Scholz et al. (2006) focus on the optimal savings behaviour of US couples, taking explicit account of role of survival pension benefits. Gustman and Steinmeier (2000) and Casanova (2009) focus on the joint retirement decisions made by the household’s members. Finally, a two potential earner households framework has been used in a general equilibrium setting in Kaygusuz (2008). He studies the steady state implications of several reforms of the US pension system, like the elimination of the spousal’s benefit or changes in the progressive character of pension benefits. Our work differs from his in a number of dimensions (e.g. we reproduce population ageing and other non-stationary patterns of the economy, endogenize retirement and allow for gender differences in mortality risk). To the best of our knowledge, the present paper is the first application of household models in a general equilibrium analysis of pension reform with population aging.

Our two potential earner households model is carefully calibrated to reproduce the basic demographic and economic properties of the Spanish economy as of the beginning of the twenty-first century. In particular, we reproduce the institutional details of the old-age and survival pension system and the observed historical changes in education and female labor supply. The model unobservable parameters are selected to reproduce the aggregate macroe-
economic and labor-market performance in the interval 2001-2007. Once equipped with a fully specified model we undertake several experiments. We simulate the model in its current form to generate a precise quantitative projection of the finances of the Spanish pension system in the interval 2010-2050. We then study the financial and welfare consequences of the 1997/2002 reforms and, finally, emphasize the importance of including survival pensions and gender differences in the model by comparing with the predictions obtained from a model that only features old-age pensions. The most outstanding findings of our simulations can be summarized as follows:

- In its current form, the Spanish pension system is clearly unsustainable. This is so even in presence of strong immigration flows and a steady improvement in the employment rates of the economy (they increase from 64 to 79%). Pension expenditure as a proportion of GDP will more than double by 2050, reaching 18% from an initial value of 7%. Survival pensions also roughly double in the interval and so contribute 15% of the total increase. The total implicit unfunded liability of the system is calculated to add up to 105.4% of the tax base in 2001 (using a 5% discount factor). Under these circumstances, any feasible fiscal path will demand strong tax hikes. In particular, keeping the overall budget balanced year by year by adjusting the income tax would demand a tax rate jump from 13 to 21% in 2050.

- Figures above leave no doubt about the ineffectiveness of the implemented reforms in tackling the financial imbalances of the system. Notwithstanding, they have had some financial impact. If we exclude the extension of the early retirement entitlement, the implemented reforms have reduced the implicit unfunded liabilities (as a proportion of the tax base) by around 9 percentage points. Including the extension of early retirement, however, we find the opposite result: an increase of around 11 pp. Note, however, that the final total expenditure to GDP ratio would have been roughly the same even in the most favorable scenario.

- The reforms had quite appreciable welfare consequences, even after allowing for behavioral responses (which are appreciable, specially in terms of retirement). In comparison with the status quo, they redistribute the cost of ageing putting a larger burden on more educated vs less educated individuals and on the cohorts of current active workers vs future cohorts. Whether the household has a working spouse makes little difference if the legal retirement ages are left unaltered. Remarkably, the decision to extend the right to early retire to all cohorts generated an appreciable redistribution from all other groups to the current and future low-educated households (with a working female).

- Finally, we repeat our base simulation but in an economy without survival pensions. To make the experiment meaningful, we recalibrate the old-age pension (reducing the replacement rate and increasing its coverage) so that we roughly reproduce the observed total pension expenditure to GDP ratio in 2002/2007. The simulated equilibrium path obtained is surprising different, projecting significantly smaller aggregate pension expenditures in the long term. The underestimation peaks at around 2050 with a divergence of as much as 1.5 percentage points (or a 10% error in the pension expenditure to GDP ratio). The implicit unfunded liabilities of the system are assessed to be 7 percentage points smaller than in the benchmark case.

The rest of the paper is organized as follows. In Section 2 we describe the model that serves as the basic tool for our analysis. In Section 3.1 we explain the details of the demographic patterns included in the simulations, while Section 3.2 deals with the calibration of the unobservable parameters of the model and of the details of the pension system. In Section 3.3 we check the ability of the model to achieve its calibration targets and test its
performance in other non-calibrated dimensions. The main findings of the paper are reported in sections 4.1 (projection of the economy in its current form), 4.2 (projections under the pre 1997-2002 reforms system) and 4.3 (impact of omitting the survival pensions). The paper closes with some general comments in section 5. Some extra information is confined to the appendices.

2 The model

The model consists of overlapping generations of ex-ante heterogeneous agents that live up to \( I \) periods in an economy without aggregate uncertainty. However, at the micro level, individuals are uncertain about the length of their life. There is no insurance market for this risk and borrowing against future pension income is not allowed. Concerning the degree of openness of the model, we allow for migrant flows, but we abstract from international financial relationships. A period in the model stands for one year of real time, which we denote by \( t \) when referring to calendar time and by \( i \) when referring to age. The cohort the individual belongs to is denoted by \( u (u = t - i + 1) \). When use the subindex \( g \) to distinguish between the household members: \( g = 1 \) denotes the head of the household, while \( g = 2 \) refers to a second potential wage earner, that (for the sake of simplicity) we identify with the female of the house. After the age of entrance in the labor market (20 years), all individuals are grouped in households and start making economic decisions. At that time, households are classified according to their educational attainment into one of \( J \) possible categories (denoted by \( j \in J = \{1, ..., J\} \)). As a general notational convention, variables characterizing the household or its members are written in lower case and indexed by (depending on the context) education, gender, year of birth, age or calendar time. Aggregate variables are denoted with capital letters and are normally indexed by (depending on the context) education, gender, year of birth, age or calendar time.

2.1 Demography

Demography evolves according with a one sex population model, were individuals are classified according to their country of birth as “Natives”, \( N^t \), or “Migrants”, \( M^t \).\(^5\) The number of people born at \( t \) is determined by the profile of age-specific fertility rates \( \{\theta^t_i\} \) (assumed to vary between the threshold ages \( f_0 \) and \( f_1 \)):

\[
N^t_i = \sum_{i=f_0}^{f_1} \theta^t_i (N^{t-1}_{i+1} + M^{t-1}_{i+1})
\] (1)

After-birth population dynamics is given by:

\[
N^t_i = h s^t_i (N^{t-1}_{i-1} + M^{t-1}_{i-1}) \quad M^t_i = h s^t_i (N^{t-1}_{i-1} + M^{t-1}_{i-1}) + F^t_i \quad 1 < i \leq I
\] (2)

where \( F^t_i \) stands for immigrant flows and \( \{h s^u_i\}_{i=1}^{I} \) for the cohort-\( u \) vector of age-conditional survival probabilities. Equations (1) and (2) constitute the law of motion of the population in the interval \( t \in (t_0, t_1) \), characterized by a demographic transition in which the fertility and mortality parameters are changing in time (see section 3.1). After \( t_1 \) (set to 2050 in the simulations) the fertility and mortality patterns stay constant and immigration flows

\(^5\)The household formation process for migrants older that 20 years is assumed to reproduce the distribution in the population at large. I.e, for each possible education level and year of birth of a newly arrived immigrant we assume the same distribution of single and biparental families in the corresponding household as in the entire population. This assumption is not realistic, but has a negligible impact in our results.
progressively die out. After \( t_2 = t_1 + I \) the age-distribution of the population is constant (the population becomes stable), and we finally assume the convergence of the entire economy to a final balance growth path at time \( t_3 \). The complete time span of the simulation is represented by \( T \).

2.2 The public sector

The main role played by the Public Sector is to run a Defined-Benefit pension system, financed with the contributions made by active workers (ie, run on a PAYG basis). The social contributions paid in calendar time \( t \) by a worker of age \( i \), are a fixed proportion (\( \varsigma \)) of his/her covered earnings, \( cov_i^t \). Covered earnings, in turn, are a fixed proportion, \( 1 - \chi \), of the individual gross labor income, \( w^t \epsilon_{i,g} \), with an annually legislated maximum \( C_M^t \):

\[
 cov_i^t = \min \{(1 - \chi) w^t \epsilon_{i,g}^1, C_M^t\}
\]

\( \epsilon_{i,g} \) is the endowment of efficient labor units at age \( i \), and \( w^t \) is the market price of a unit of efficient labor.

Eligible workers (ie. those with a long enough, \( h \geq 15 \), contributive record) can claim an old-age pension at any time after the early retirement age, \( \tau_m \), an following a complete withdrawal from the labor force. The initial pension for an individual belonging to cohort \( u \) who retires at age \( \tau \) after \( h \) years of contributions is:

\[
b(\tau, h, u) = \alpha_E(\tau) \alpha_H(h) \left( \frac{\sum_{e=\tau-1}^{\tau-1} cov_{e+e-1}^w}{D} \right)
\]

The formula combines a moving average of covered earnings in the \( D \) years immediately preceding retirement (called benefit base) and two linear replacement rates:

- The replacement rate, \( \alpha_E(\tau) \) captures early retirement penalties. For each year that the individual anticipates retirement (from the Normal Retirement Age, \( \tau_N \), of 65), the final benefit is reduced by a \( \Delta \alpha_E \) percent. This penalty depends on the length of the contributive record (see section 3.2.1). There is also a annual bonus \( \Delta \alpha_{E+65} \) for staying employed after 65. Formally:

\[
\alpha_E(\tau) = \begin{cases} 
\alpha_E^0 & \text{if } \tau < 60 \\
\alpha_E^0 + \Delta \alpha_E(\tau - 60) & \text{if } \tau \in \{60, \ldots , 64\} \\
1.0 + \Delta \alpha_{E+65}(\tau - 65) & \text{if } \tau \geq 65
\end{cases}
\]

- The replacement rate \( \alpha_H(h) \) captures penalties associated with an insufficient record of social contributions. The length of the record is deemed to be sufficient only after 35 years:

\[
\alpha_H(h) = \begin{cases} 
\alpha_H^0 + \Delta \alpha_H(h - 15) & \text{if } h \in \{15, \ldots , 25\} \\
\alpha_H^0 + \Delta \alpha_2 H(h - 25) & \text{if } h \in \{25, \ldots , 35\} \\
1.0 & \text{if } h \geq 35
\end{cases}
\]

The final balance growth is only really reached asymptotically, but in the simulation it is assumed to be reached in period \( t_3=2220 \). We check that the particular value chosen for \( t_3 \) does not affect the performance of the economy in the interval of interest (\( t_0, t_1 \)).

The contributive “wedges” \( \chi \) stem form legal dispositions preventing some components of the overall remuneration (like travel expenses, food tokens and other subsistence expenses) from generating pension rights. As reviewed when discussing the calibration of the pension system in section 3.2.1, Spanish companies seem to pay a sizable part of the wages in the form of this type of expenses.
In the current formulation, the penalty is more severe for those with shorter records ($\Delta \alpha_1^H > \Delta \alpha_2^H$).

The *initial* pension $b()$ is indexed to price inflation and subject to annually legislated maximums, $bM^t$, and minimums, $bm^t$, (whose value changes in presence of a dependant spouse). Therefore, the *effective* pension income in year $t$ for the individual above would be:

$$ib^t_i(\tau) = \min\{bM^t, \max\{bm^t, b(\tau, h, t - i + 1)\}\}$$

(7)

Along with the old-age pensions, the system also provides survival pensions to the widows/widowers of deceased spouses. The initial value is obtained as a fraction $\alpha^V$ of the *benefit base* of the deceased. We simplify the complexity of the real world eligibility conditions by assuming that all surviving spouses older than a (advanced enough) age, $\tau_m^V$, qualify for the survival pension.\(^8\) There is also an specific guaranteed minimum for survival pensions, denoted by $bm^t_V$. All in all, the pension income of a survival spouse is:

$$iv^t_i = \max\{bm^t_V, \alpha^V b(\tau, t - i + 1, h)\}$$

(8)

In addition to running the pension system, the Public Sector performs two additional functions: it runs a fiscal system and it incurs in a certain amount of public expenditure, $CP^t$.\(^9\) Both tasks are extremely simplified in our model. Fiscal revenue is assumed to originate in the full taxation of involuntary bequests and from a proportional tax levied on both labor and capital income, while public expenditure is a fixed proportion of aggregate output.

### 2.3 The household problem

Households are the economic agents of the model. They are all made of two people (i.e. we assume no heterogeneity in the marital status) and a number of children that varies with the cohort. Households decide about their labour supply and about their optimal life cycle profiles of consumption and asset holdings ($c^t_i$ and $a^t_i$ respectively, with $t = u + i - 1$). In particular, labour supply decisions take two forms: a life-time female participation decision and a coordinated retirement decision by the active members of the household.\(^10\) Male participation along the life-cycle is, therefore, assumed exogenous, with the exception of the age of withdrawal from the labor force.\(^11\) The binary variable $E^u = \{0, 1\}$ describes the

\(^8\)The age requirement is a proxy for the real requirement in terms of number of years of contributions.

\(^9\)CP\(^t\) stands for both the running cost of the public administrations and for expenditure in the provision of public goods. For simplicity, we abstract from the welfare impact of the consumption of those goods.

\(^10\)Our assumption on female labor supply is supported by the evidence in Adam (1996), describing a dual participation behavior by Spanish women. Although most (young) women participate at early ages, the population clearly divides in two groups after childbirth, with those who fail to reenter shortly after the event ending up in a complete withdrawal from the labor market. This polar conduct has been confirmed by Gutiérrez-Domenech (2005) in the Spanish case, and contrasted with the patterns observed in other countries. The assumption of simultaneous retirement of husband and wife draws on the strong empirical correlation found between the two decisions both in Spain and in other countries (see eg. Jiménez et al. (1999) for the Spanish cases and Banks et al. (2007) for US and UK).

\(^11\)In our view, the two margins of the household labor supply that are more sensitive to economics incentives are the female participation decisions and the retirement decision of the head. The later is particularly important in our context because of its large sensitivity to the incentives provided by the pension system. Note, in particular, that the size of the pension benefits obtained by a household depends strongly on its composition and the labor participation of its members. The minimum pension and survivors pension rules are responsible for this dependence.
employment decision of the female, while the discrete variable \( \tau^u \in \{60, \ldots , 70\} \) represents the retirement age of the head (and of the working spouse).

We assume both spouses share the same age of birth (\( u \)) and educational attainment (\( j \)). \(^{12}\) Apart from the education and age of birth, we introduce as an additional source of heterogeneity across households the utility cost that they incur if the female participates in the labour market throughout her lifetime. \(^{13}\) We represent this extra heterogeneity with the distribution function \( \Phi \), which is assumed to be independent of the year of birth or the education of the spouses. Households are formed at the age of entrance in the labor market (20 years of age) and suffer no changes in their composition other than those derived from the effects of mortality. As the mortality hazard takes its toll on the progressively older family members widows and widowers arise in the economy. In order to account for the heterogeneity in household size that arises due to mortality we make the following simplification assumption. We assume that household income in a particular age, conditioning on education and age of birth, is a weighted sum of the first and second potential earners incomes. The weight for each g-earner is equal to the fraction of families in which the g-earner is still alive. Obviously, the income of the second potential earner is contingent on the female participation decision.

We model the female participation decision as a trade-off between the extra labor income generated for the household, the utility cost, \( z \), and a direct financial cost (representing the purchase in the market of services that would have otherwise been generated by the spouse). As a consequence, there is a fraction of couples with a working spouse for each cohort \( u \) and education level \( j \).

As usual, each household behavior is obtained from the solution of an intertemporal optimal control problem. Preferences are represented by a pure-time-impatience discount factor \( \beta < 1 \) and a period utility function \( U(c, E, \tau, z) \) that depends on consumption, on leisure (i.e. retirement age and female’s participation, \( E \)). The problem of the household belonging to cohort \( u \) (omitting the educational \( type \) to simplify notation) is:

\[
\begin{align*}
\max_{E^u, \tau^u, \{c_i^u, a_i^u\}_{i=1}^I} & \sum_{i=1}^I \beta^{i-1} s_i^u U(c_i^u, E_i^u, \tau_i^u, z) \\
\sum_{i=1}^I c_i^u - C^u E^u + a_{i+1}^u &= H I_i^u + (1 + \varphi^u) a_i^u \\
\forall \, i &
\end{align*}
\]

\[ a_1^u = 0, \quad a_i^{u+i-1} = 0, \quad a_i^{u+i-1} \geq 0 \quad \forall \, i \geq \tau \]  

where \( s_i^u \) stands for the survival probability (of at least one household member) to age \( i \), \( \varphi^u \) stands for the net-of-taxes return on capital (i.e. \( \varphi^u = r^u(1 - \phi^u) \), with income tax rate \( \phi^u \)). \( C^u \) capture the direct financial costs of female labor market participation, which varies across cohorts depending of the average number of children raised by the household. The household income \( (HI_i^u) \) is the sum of the contributions made by each of the potential family earners \( W_{i,g}^u \), with \( g = \{1,2\} \). Before retirement, the net labor income of member \( g \) takes the form:

\(^{12}\)This simplifying assumption implies and overstatement of the degree of assortative mating in the economy, which in Spain is around 70% for low educated workers.

\(^{13}\)The introduction of this type of heterogeneity to model participation decision is also used in Guner et al. (2008) and Kaygusuz (2009).
\[ W^t_{i,g} = (1 - \phi^t) \left[ w^t \varepsilon_{i,g} - \zeta \text{cov}^t_{i,g} \right] \]  

(10)

Gross labor income \( w^t \varepsilon_{i,g} \) is the product of the endowment of efficient labor units and their market price, while payroll taxes are a fixed proportion of covered earnings (defined in eq. (3)). All in all, the household income is:

\[ H^t_i = \pi_{i,1} W^t_{i,1} + \pi_{i,2} W^t_{i,2} E^t \]  

(11)

where \( \pi_{i,g} \) is the proportion of families that, despite the effects of mortality, still include the \( g \)-earner at age \( i \).

Similarly, for retirees we can write household income as:

\[ H^t_i = \pi_{i,1} \xi_{i,1} B^t_{i,1} + \pi_{i,2} \xi_{i,2} B^t_{i,2} \]  

(12)

where \( B^t_{i,g} \) is the sum of the old-age and survival pensions of member \( g \) at age \( i \) (computed according with expressions (7) and (8)) and \( \xi_{i,g} \) is the share of members of gender \( g \) that are entitled to get pensions according with legal dispositions (discussed in section 2.2).

2.4 Technology
The production side of the model is entirely neoclassical: we assume a standard technology, with a constant returns to scale production function, \( F(K, L) \), no adjustment costs and exogenous labor-augmenting technological progress, \( A^t \). The growth rate of labor productivity, \( \rho \), is exogenous and constant.

2.5 Equilibrium
An equilibrium path over the time interval \( T \) is a set of time series of population aggregates and distributions, household decisions (consumption and savings), aggregate inputs, prices and public policies (tax rates, public consumption and minimum and maximum pensions and contributions) with the standard properties: households are rational (ie solve problem 9) taking the environment as given, factor markets clear, prices are competitive, the public budget balances and an aggregate feasibility constraint is observed. Appendix A provides a formal definition of the equilibrium of the model economy. As in the standard Auerbach and Kotlikoff (1987) methodology, we assume the convergence of the equilibrium path to a final balance growth path in finite time. Notwithstanding, the initial equilibrium is non-stationary (as in eg (Kotlikoff et al. 2000)).

3 Calibration
We calibrate our model to mimic the observable economic and demographic characteristics of the Spanish economy, and in accordance with standard projections for future demographic and productivity trends. The next two sections review the details of the demographic and economic calibrations. Section 3.3 test the effectiveness of the calibration by comparing the model predictions and the empirical evidence in the interval 2001-2007.

3.1 Demography
A period in the model stands for one year of calendar time and we assume a maximum lifespan \( I \) of 100 years. The simulated equilibrium path starts in \( t_0 = 2001 \). Our demographic
model in section 2.1 is only used starting in 2008. For the interval 2001-2007 we reproduce
the observed population aggregates, distribution and age profiles of fertility and survival
probabilities.\textsuperscript{14} From 2008 to 2050 we simulate a changing pattern of fertility and mortality,
according with the main scenario ("hipótesis 1") by the Spanish Statistical Institute (INE).\textsuperscript{15}
The main patterns can be appreciated in figure 12 (confined to appendix B.1). The total fertility rate is assumed to recover from the extremely low values observed during the nineties
(in 1995, 1.2 children per women nationwide) to a final stationary value of 1.53 in 2050. We
also reproduce the trend towards lower mortality rates by assuming that life expectancy rises
from the 76.6 years observed in 2001 to 81.0 years in 2050 for males and from 83.4 to 87
years for females. In both cases, the bulk of the recovery is concentrated in the first decades
of the simulation. For the immigration flows, we reproduce the observed data between 2000
and 2005 (which registered flows of unprecedented size for the Spanish historical experience)
and follow INE ("hipótesis 1") for the projection in the interval 2005/2050.\textsuperscript{16}

\section*{3.2 Calibration of the Economic Model}

The model described in section 2 becomes an operative tool once we complete its calibration
by assigning specific values to all its parameters. This involves three different processes.
First, there is a subset of parameters with reasonably clear empirical counterparts. The
relatively straightforward assignments made in this case are reported in subsection 3.2.1.
Secondly, as our simulation directly starts with a non-stationary equilibrium (assimilated
to the state of the economy in 2001) we need initial values for a number of predetermined
variables. Again, clear candidates are available for this election (section 3.2.2). Finally, all the
other parameters (reflecting preferences, technology and the distribution of the heterogeneity
in the economy) are set to reproduce some specific properties of the observed behavior of the
Spanish economy in the interval 2001-2007 (section 3.2.3).

\subsection*{3.2.1 Parameters with empirical counterparts}

\textbf{Education, income and cost of the female participation decision}

\textbf{Education.} For compatibility with our income data, individuals are classified in three
educational groups. The lowest education level correspond to those who fail to complete
secondary education, the highest level is made of tertiary graduates and the medium level
includes all the rest. The recent historical evolution of this variable, as measured in the
\textit{Labour Force Survey}, has been truly remarkable. Low educated workers were almost 95\% of
the total among the individuals born in 1920/1930. This figured was reduced to 55\% for the
cohort born in 1970/80. We reproduce this historical trend and assume it to continue at a
progressively slower pace for the cohorts born after 1975. The resulting time series of the
evolution of the distribution by education are displayed in figure 13 in appendix B.2. It also
provides more details on its construction from the observable information.

\textsuperscript{14}Our source is the publicly available demographic information of the Spanish National Statistics Institute (INE). In particular, we use the 2001-Census, the “Explotación estadística del Padrón” database, and the mortality and fertility information in the “Movimiento Natural de la Población” records.

\textsuperscript{15}Available from INE web page: "INEbase/ Proyecciones de población calculadas a partir del Censo de Población de 2001". e-Link: http://www.ine.es/metodologia/120/12039251k.htm.

\textsuperscript{16}We use the 1997-2004 Encuesta de Variaciones Residenciales microdata to compute the net aggregate flows of incoming population and its breakdown by age. This breakdown is assumed constant throughout the simulation path 2005-2050. The initial stock of immigrants is taken from the 2001 Census data.
Life-cycle profiles of labor earnings. The individual profiles are estimated using the microdata of the European Community Household Panel (ECHP) 1994-2001. We use standard regression techniques to obtain earnings profiles for males and females and for each of our educational levels. Two remarks are in order here. First, as we do not formally include unemployment in the model, we make our measure of earnings reflects the observed income losses created by unemployment episodes along the workers lifetime. Second, we include a linear trend as a regressor for compatibility with our assumption of constant, labor-augmenting, productivity growth. The resulting profiles (figure 14 in appendix B.3) are increasing with age and the rate of growth over the life-cycle is increasing in education. Concerning the gender gap, female earnings are, on average, 28% lower than male earnings. The profiles are subsequently hold constant throughout the simulation. Needless to say, these profiles are of paramount importance in our simulation. For instance, they largely determine the size of the pension benefits and exert a strong influence on the timing of retirement and on the female participation decisions (see appendix C). The individual profiles are combined within the household (according to equation (11)).

Direct monetary costs of participation. The labor participation of the female inflicts monetary costs on the household that stem from the fixed costs of working, foregone home production, child care cost and others. The measurement problems associated with those costs are big, making the calibration of parameter \( C \) in our model challenging. Our approach is then to limit the direct monetary costs in the model to the (relatively well measured) average price of child care in those ages in which schooling is not covered by the public education system. According to Consumer (2005) the average monthly price of a nursery for children of up to two years of age is around 33% of the monthly earnings of a low educated female. Therefore we set the monetary cost \( C \) in our model economy and assume this cost is constant for the first four years of the children life. This is likely to be a lower bound of the real monetary cost, and we may in consequence be overestimating the utility cost, but the empirical performance of the resulting model (see section 3.2.3 below) is good.

---

17 This is higher than the hourly wage gender gap as measured by Encuesta de Estructura Salarial. There are at least two reasons for this discrepancy: first, females tend to work less hours than males, second, unemployment incidence is higher among females.

18 This means that we do not allow for changes in the relative productivity by age, by education or by gender (i.e. there is no improvement in the estimated gender gap, conditional on age and education). However the average earnings gender gap in our model evolves endogenously according to changes in the composition of the female labour force. In the data there are several forces driven the evolution of the wage gender gap. On the one hand, as women work harder the gender gap goes down due to the returns to education, on the other hand, the selection process that brought more women to the labor market may increase the gender gap (see Attanasio et al. (2008)). We only have the second mechanism operating in our model economy throughout the heterogeneity in education.

19 For example, children-related costs estimates based on expenditures of two-earners households may be affected by selection. In Spain there is a special concern in this respect as the role of grandparents taking care of their grandchildren while their parents work has been documented to be important, see Tobio (2003). Concerning foregone home production, Hong (2008) shows that its estimation can not rely on inputs only. Home production productivity varies across individuals and along the life-cycle. Then observed inputs (hours in home production and capital) are affected by selection as household choose optimal allocation of time to home production and labor market according to their productivity and to the monetary cost of working that may vary across households.

20 As documented in de Evaluación y Calidad del Sistema Educativo (2004) children in Spain start attending the state-provided public school when they are four years old. Before that age, public or private nurseries or kindergarten have to be used in general (the public ones are actually in very low supply).

21 An alternative strategy is to calibrate or estimate these costs within a structural model using as a source of
Averaging Early Retirement Contrib. Legal Survival
period penalties penalties Ages pension

\[ D=15 \]
\[ \alpha_E^0 = \begin{cases} 
0.60 & (h \leq 30) \\
0.65 & \text{(other)} \\
0.7 & (h \geq 40) 
\end{cases} \]
\[ \alpha_H^0 = 0.5 \]
\[ \tau_m = 61 \]
\[ \tau_N = 65 \]
\[ \alpha^V = 0.52 \]
\[ \Delta \alpha_E^{+65} = 0.02 \]
\[ \Delta \alpha_H = \begin{cases} 
3\% & h \in (15, 25) \\
2\% & h \in (25, 35) 
\end{cases} \]

Table 1: Pension System parameters in the benchmark simulation. All notation is as in section 2.2.

**Pension System**

The institutional parameters are set to reproduce the General Regime (RGSS) of the Spanish social security system. Our benchmark simulation corresponds to the structure in place after the latest implemented reforms in 1997 and 2002, in particular, the institutional values observed in 2002 that are shown in table 1.

Additionally, we model the discretionary components of the pension system (the annual values of minimum pensions and maximum contributions) as linear functions of the annual income per worker. The parameters of those functions are calibrated to reproduce the corresponding 2001 ratios:

\[ \frac{b_{m1}}{W^{01}} = \begin{cases} 
0.30 & \text{with Dependant Spouse} \\
0.35 & \text{without Dependant Spouse} 
\end{cases} \]
\[ \frac{C_M^{01}}{W^{01}} = 1.76 \]
\[ \frac{b_{m1}}{W^{01}} = 0.26 \]

Note, finally, that some of the parameters of the system are without direct observable counterparts and have to be calibrated. First the model pay-roll tax, \( \varsigma \), does not fit with the real-world contribution rate. The problem is that our contribution only finances pension obligations, while the real-world counterpart also provides for health, disability and other contingencies. To proceed, we calibrate the value of \( \varsigma \) to reproduce the average balance of the pension system (as a proportion of the annual GDP) in the interval 2001/2007. Panel (c) in table 2 shows both the target value (a positive imbalance slightly above 1% of the GDP) and the parameter value selected for the simulation (a rate equal to 12.1% of the pensionable income). Second the assessment of the wedges separating gross income and covered earnings, \( \chi_j \), are unreliable due to data scarcity. Therefore, we calibrate them through indirect inference by selecting the values that simultaneously reproduce (i) the ratio of the average pension to average productivity (\( \bar{y}/\bar{y} \)) in the interval 2001/2007; and (ii) the proportion of workers receiving the minimum pensions. The former statistic identifies the wedge for workers of medium and above medium education while the latter is associated with the wedge for low income workers. As reproduced in table 2, the calibrated wedges are

---

22The Spanish pension systems features several different regimes that differ to varying degrees. At the moment, however, 76% of the affiliated workers and 57% of the pensioners belong to the General Regime. Besides, there is a general political agreement (Pacto de Toledo) to progressively eliminate all Special Regimes but the one for the self-employed. See Boldrin et al. (2004) for a complete description of the Spanish pension system.
rather substantial. Finally, to add more realism to the model we allow the ratio of pensions per employee, $\phi$, to differ from one. This reflects some relatively minor regulations in the Spanish General Regime (i.e., the possibility of earlier retirement for some specific occupations) and, more importantly, to alleviate our rigid assumptions on the life-cycle participation of individuals - specially women. To minimize the loss of discipline implied by this extra parameter we keep the value of $\phi$ constant by cohort and education. We calibrate to the value that best reproduce the coverage rate of the system ($P/N_{+64}$, the total number of pensions per person age 65 or older) in the interval 2001/2007. The resulting figure is 1.12 (table 2).

3.2.2 Initial conditions

As our simulation path starts in 2001, the values of a number of relevant variables are already predetermined by the decisions previously taken by the households. Using the notation of previous sections and denoting the life-span by $I = \{20, \ldots, 100\}$ and the cohorts whose pension benefit is already fixed in 2001 by $u$, the variables whose starting values are obtained empirically are the following:

- **Asset holdings** by age and education, $\{a_{i,j}^{2001}\}$, $i \in I, j \in J$ for all cohorts alive at the beginning of the simulation. In the model we reproduce the wealth to income ratios observed in the Bank of Spain’s EFF (Encuesta Financiera de las Familias). Proceeding in this way, we get an aggregate capital to GDP ratio of 2.4 in 2001, very close to value measured by BBVA-Ivie (2005) (2.3).

- **Retirement ages** of the more senior Spaniards, by education and gender, $\{\tau_{u,j,g}\}$, $u \in U, j \in J, g \in \{1,2\}$. This information is obtained from the latest released of the MCVL-08 (Muestra Continua de Vidas Laborales). When interacted with our other model assumptions we generate an aggregate average retirement age in 2001 of 62.7, very close (although slightly below) the empirical figure of 62.9 years.

- **Pensions benefits** of the more senior citizens, $\{B_{u,j,g}\}$, $u \in U, j \in J, g \in \{1,2\}$. As with asset holdings, we reproduce in our model the ratio of old-age pensions (by cohort, education and gender) to the average wage in 2001, according to the information in MCVL-08. After aggregation, we generate a pension expenditure to GDP ratio ($PP/Y$) slightly above the data: 6.94% vs 6.75%.

- **Female labor participation** by education for cohorts born before 1970. We use the Spanish Labor Survey (EPA) to calculate the proportion of double-earner households, $\Lambda_{u,j}$, by cohort $u < 1971$ and education type $j$; In our model (recall our assumption about the full life-cycle participation of a working female) this proportion identifies the participation rate of the females.
### (a) Macro-Aggregates

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
<th>Parameter &amp; value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K/Y$</td>
<td>2.40</td>
<td>2.42</td>
<td>$\beta=0.985$</td>
</tr>
<tr>
<td>$I/Y$ %</td>
<td>28.3</td>
<td>28.7</td>
<td>$\delta=6.0%$</td>
</tr>
<tr>
<td>$rk/Y$ %</td>
<td>32.6</td>
<td>32.6</td>
<td>$\theta=0.326$</td>
</tr>
<tr>
<td>$CP/Y$ %</td>
<td>14.8</td>
<td>14.8</td>
<td>$c_p=0.148$</td>
</tr>
<tr>
<td>Productivity growth %</td>
<td>1.7</td>
<td></td>
<td>$\rho$</td>
</tr>
</tbody>
</table>

### (b) Labor Supply

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
<th>Parameter &amp; value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female participation* % (Low educ)</td>
<td>67.8</td>
<td>67.2</td>
<td>(\mu_z=1.07)</td>
</tr>
<tr>
<td>(Medium education)</td>
<td>76.5</td>
<td>75.8</td>
<td>(\sigma_z=0.67)</td>
</tr>
<tr>
<td>(High education)</td>
<td>84.6</td>
<td>82.7</td>
<td>(\alpha=0.75)</td>
</tr>
<tr>
<td>Average retirement age</td>
<td>63.0</td>
<td>62.7</td>
<td>(\zeta=12.1%)</td>
</tr>
</tbody>
</table>

### (c) Pension System

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
<th>Parameter &amp; value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSB/Y %</td>
<td>1.08</td>
<td>1.12</td>
<td>(\varsigma=12.1%)</td>
</tr>
<tr>
<td>Minimum pension coverage (OA)%</td>
<td>28.1</td>
<td>26.6</td>
<td>(\chi_1=0.92)</td>
</tr>
<tr>
<td>$\bar{b}/\bar{y}$ replacement rate</td>
<td>20.86</td>
<td>21.3</td>
<td>$\chi_2 = \chi_3 = 0.65$</td>
</tr>
<tr>
<td>Number of Survival pensions</td>
<td>2.14</td>
<td>2.15</td>
<td>(\tau_m^Y=45)</td>
</tr>
<tr>
<td>Pension coverage rate %</td>
<td>88.5</td>
<td>86.7</td>
<td>(\phi=1.12)</td>
</tr>
</tbody>
</table>

Table 2: Calibrated parameter values and empirical averages in the interval 2001-2007. *Female labor force participation by members of the cohort born in 1970. OA=Old Age pensions.
3.2.3 Preference, technology and distributional parameters

For the rest of the parameters of the model we select values that make the model mimic the performance of some fundamental variables of the Spanish economy in the interval 2001/2007. As reflected in Table 2, the targeted properties capture the aggregate behavior of the pension system, the labor market and some basic macroeconomic ratios. The parameters and the properties are introduced and discussed in the next two sections.

Preferences

As indicated in section 2.3, the period utility function at age $i$, $U_i(c, E, \tau, z)$, depends on consumption $c$ and leisure (ie. retirement age, $\tau$, and female participation decision, $E$). More specifically, we use a separable CES function:

$$U_i = \frac{c^{1-\eta}}{1-\eta} - z E + \alpha I(i > \tau)$$

with intertemporal elasticity of substitution (IES=$1/\eta$), relative value of leisure $\alpha$ and utility cost of female participation $z$. These parameters are calibrated as follows:

The consumption contribution is modeled as a logarithm (IES=1), in accordance with econometric evidence (Hurd (1989) for US or Jiménez-Martín and Sánchez-Martín (2007) for Spain). It has the additional property that it is the only functional form that guarantees the invariance of the retirement decision in presence of exogenous productivity growth. $\alpha$ is selected to reproduce the average retirement age (63.0). The best figure we get simulating the model is 62.7, which is close to the empirical figure, but not entirely successful. This reflects the limitations derived from the existence of a relatively small number of households at each age. Note that, conditioning on female labour market decision, the heterogeneity in the utility cost has no effect on the retirement decision of the head (due to the additive separability assumed in the utility function). The utility cost of a working-female is instrumental for the alignment of the employment rates of the model and of the data. We assume $z$ to be normally distributed across the members of each cohort. Two parameters, then, have to be calibrated: the mean ($\mu_z$) and standard deviation ($\sigma_z$) of the normal. They are chosen to guarantee a seamless transition from the predetermined participation rates of older cohorts (directly obtained from the data), in particular those born before 1970, to the endogenous rates produced by the model for younger generations, those born after 1970. This is done in order to minimize the distance between the female participation rates by education in the model and in the data. The results are reported in panel (b) of table 2. Concerning participation rates of males, they are assumed to be constant throughout the simulated path and equal to their observed value in 2001 (83.3, 89.8 and 92.9 for low, medium and highly educated workers respectively). Finally, to complete the specification of individual

23 Relating to the household income equation during the retirement phase of the life cycle (eq (12) in section 2.3), the coefficients $\xi_{i,g}$ are obtained by multiplying the employment rate by $\phi$.

24 For instance, it is possible for a number of second-earners to remain inactive during large parts of their life cycle and reenter the labor force at very advanced ages precisely to qualify for an old-age pension of their own. This type of behavior is not captured by the model.

25 The original Auerbach and Kotlikoff (1987) approach was to take those values from an initial steady state. In this work, however, we follow more recent practices where, whenever possible, the predetermined variables are taken directly from the data.

26 This is an extraction from the archive of administrative records of the Social Security System.

27 This makes the computation of the equilibrium more tractable, but also results in a relatively inflexible aggregate retirement age.
preferences we choose a value for $\beta$, the pure time impatience parameter, that makes our model reproduce the average 2001/2007 $K/Y$ ratio of the economy (as reported in table 2).

Technology

Aggregate output is generated with a Cobb-Douglas production function, $Y = K^\zeta L^{1-\zeta}$. Therefore, the supply side of the model is completely specified by choosing the capital share in aggregate income $\zeta$, the rate of capital depreciation, $\delta$, and the constant productivity growth rate, $\rho$. Panel (a) in table 2 reproduce the macro-targets and the parameters employed to achieve them. $\zeta$ is set to reproduce the average capital income share in the interval 2001/2007, as measured in Puch and Licandro (1997). The depreciation rate is selected to mimic the average Investment-to-GDP ratio, while the exogenous productivity growth rate, $\rho$, is set to 1.7% to reproduce the long term value employed in the central scenario of EPC (2006). The final macro aggregate to be calibrated is the government expenditure to output ratio. In this case we simply use the average 2001/2007 value as an input in the model (parameter $c_p$) and assume it constant throughout the equilibrium path.

3.3 Testing the model: behavior in 2001-2007

We use the observable performance of the Spanish economy in the 2001-2007 interval to test the calibration of the model. Needless to say, we expect a close performance in the dimensions that have been explicit calibration targets in the previous section. We consequently pay more attention to other indicators.

Graphs in figure 1 illustrate the time-series behaviour of both the real economy and our simulated equilibrium path in 2001/2007. The aggregate capital to GDP ratio, $K/Y$, is a good starting point, conveniently bundling the saving and labor supply decisions of the agents in the model (top-left graph in figure 1). The model reproduces the average level in the data, but fails to capture the intensity of the upward trend shown by the real world. This is not surprising, as the model is not built to reproduce the cyclical patterns of the economy (which have been particularly acute in the Spanish case in the last cycle of expansion and downturn)\(^{28}\). A similar pattern will reveal itself repeatedly in other variables subject to a strong cyclical character.

Given the target of our work, it is specially important to scrutinize the performance of the pension system. The top-right and bottom-left graphs of figure 1 show the aggregate pension expenditure and the pension system balance (difference of annual contributions and pension expenditures) expressed as a ratio to GDP ($PP/Y$ and $PSB/Y$ respectively). The degree of coincidence is, in our view, very satisfactory, both in terms of levels and trends.

In order to understand the driving forces of $PP/Y$ and the ability of the model to track them we decompose it as follows:

$$\frac{PP}{Y} = \left(\frac{E}{\bar{E}}\right) \left(\frac{P}{\bar{P}}\right) = \left(\frac{\bar{P}}{\bar{P}^{+65}}\right) \left(\frac{P^{+65}}{P^{20-64}}\right) \left(\frac{P^{20-64}}{E}\right) = rep\ rate \ast cov\ rate \ast dep\ ratio \ast \frac{1}{emp\ rate}$$

So total pension expenditure is the product of the number of people aged 65 or more, $P^{+65}$, and the average pension per person older than 65, $\bar{p}$. Similarly, we express the GDP as the product of the number of employees, $E'$, times the average productivity per employee, $\bar{y}'$. Trivial algebra make then possible to express the $PP/Y$ ratio as the product of four

\(^{28}\)The expansion stage was mainly driven by housing investment and the low interest rates resulting from the ECB monetary policy and the arrival of large flows of foreign capital.
Figure 1: Model vs reality performance in 2001/2007: Model (blue, continuous line) vs Data (red, dashed line).

(a) Pension System

<table>
<thead>
<tr>
<th></th>
<th>Average 2001/2007</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>Pension Expenditure to GDP (PP/Y) %</td>
<td>6.54</td>
</tr>
<tr>
<td>(PP/Y: Old-age pensions) %</td>
<td>5.05</td>
</tr>
<tr>
<td>(PP/Y: Survival pensions) %</td>
<td>1.49</td>
</tr>
<tr>
<td>Number of Retirement Pensions (millions)</td>
<td>4.67</td>
</tr>
</tbody>
</table>

(b) Labor Supply

<table>
<thead>
<tr>
<th></th>
<th>Average 2001/2007</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>Aggregate participation rate</td>
<td>67.2</td>
</tr>
<tr>
<td>Female participation rate (Low education)</td>
<td>38.1</td>
</tr>
<tr>
<td>(Medium education)</td>
<td>61.6</td>
</tr>
<tr>
<td>(High education)</td>
<td>76.5</td>
</tr>
</tbody>
</table>

Table 3: Model performance in non-calibrated dimensions
factors: the replacement rate of the average pension in terms of average productivity, \( \frac{\bar{b}}{\bar{y}} \), the coverage rate or number of pensions per person aged 65 or more, \( \frac{P_{t}}{P_{t+65}} \), a dependency ratio, \( \frac{P_{t+65}}{P_{20-64}} \) capturing demographics and the inverse of the employment rate \( \frac{E_{t}}{P_{20-64}} \), reflecting the performance of the labor market. Figure 2 displays the evolution of the four factors in the interval 2001/2007. We see some differences, but the overall performance of our simulation is remarkable. Our stylized model of the pension system slightly underestimate the number of pensions and overestimate the replacement rate, specially at the beginning of the simulation. But the differences are small. Panel (a) in table 3 provides a few more details on these differences: the total number of old-age pensions is around 3% smaller than in the data and we generate a slightly inflated total expenditure in survival pensions.

The adjustment in terms of labor market performance is also good. The time-series of our simulated participation rates fails to reproduce the overheated labor market observed in the second half of the decade, but that is expectable. The differences by education are a bit bigger (panel (b) in table 3) but clearly reflect the strong negative correlation between participation and education that we observe in the data. Finally, the right-bottom panel of figure 1 shows that we systematically underestimate the average retirement age. Given the relatively small number of discrete households at each possible age, the fine-tuning this variable is not possible.

Overall, we find the discrepancies of the simulated model and the data small enough that we are reinforced on the adequacy of our environment for policy analysis.
4 Simulation Results

In this section we present the results obtained after simulating the model of the previous sections in three different institutional environments. In section 4.1 we discuss the main properties of our benchmark model, representing a projection of the Spanish economy with the current pension system. Section 4.2 discuss the effectiveness of the implemented reforms by exploring counterfactual simulations aimed to replicate the pension system in place before the 1997/2001 reforms. Finally, in section 4.3 we compare our benchmark results with the findings in an environment lacking survival pensions by construction. This is to highlight the potential bias incurred when evaluating the financial future of the pension system with a single-earner household models that only reflect old-age pensions.

4.1 Projection in the benchmark

Demographic projections: an ageing population

Figure 3 summarizes our projection of the main demographic variables for the first half of the 21st century that, unsurprisingly, do not differ appreciably from the projections made by other institutions\(^{29}\). Two phenomena stand out as particularly remarkable. First, the

\(^{29}\)Like the aforementioned INE projections or those in the latest EU Ageing report by the European-Comission (2009)
intensity of the immigration flows is large enough to fuel a rather dramatic increase in the 
absolute size of the Spanish population (top-left panel of figure 3). In the process, the 
share of first-generation immigrants grows to an astonishing 25% of the total population 
(bottom-right panel of figure 3). Secondly, it is clear that immigration flows alleviate the 
aging process of the population at large (as the distribution by age of the immigration flows 
is younger than that of the general population). However, the bottom-left panel of figure 3 
makes clear that they fall short of stopping it altogether. The dependency ratio (defined as 
the ratio of the number of people older than 65 to those between 20 and 64) almost doubles 
in the interval 2000/2050 (from 26.9 to 51.2%). The intensity of the aging process can also 
be appreciated in the change of the shape of the population pyramid in the top-right panel 
of figure 3. These demographic patterns underlie all the simulations that follows.

The Economy Equilibrium path

<table>
<thead>
<tr>
<th>Year</th>
<th>K/Y</th>
<th>Income tax rate (\varphi)</th>
<th>PP/Y</th>
<th>Retirement age</th>
<th>Participation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>2.379</td>
<td>12.74</td>
<td>6.94</td>
<td>62.67</td>
<td>64.00</td>
</tr>
<tr>
<td>2005</td>
<td>2.428</td>
<td>16.89</td>
<td>6.47</td>
<td>62.70</td>
<td>67.10</td>
</tr>
<tr>
<td>2010</td>
<td>2.523</td>
<td>15.31</td>
<td>7.03</td>
<td>62.13</td>
<td>69.30</td>
</tr>
<tr>
<td>2015</td>
<td>2.699</td>
<td>15.82</td>
<td>7.41</td>
<td>62.10</td>
<td>72.00</td>
</tr>
<tr>
<td>2020</td>
<td>2.686</td>
<td>16.03</td>
<td>8.27</td>
<td>62.08</td>
<td>73.80</td>
</tr>
<tr>
<td>2025</td>
<td>2.754</td>
<td>16.36</td>
<td>9.68</td>
<td>62.12</td>
<td>75.00</td>
</tr>
<tr>
<td>2030</td>
<td>2.800</td>
<td>17.36</td>
<td>11.43</td>
<td>62.19</td>
<td>75.90</td>
</tr>
<tr>
<td>2035</td>
<td>2.815</td>
<td>18.89</td>
<td>13.33</td>
<td>62.28</td>
<td>78.00</td>
</tr>
<tr>
<td>2040</td>
<td>2.806</td>
<td>20.89</td>
<td>15.82</td>
<td>62.54</td>
<td>79.10</td>
</tr>
<tr>
<td>2045</td>
<td>2.745</td>
<td>21.96</td>
<td>17.18</td>
<td>62.75</td>
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<td>2050</td>
<td>2.669</td>
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<td>17.82</td>
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<td>79.90</td>
</tr>
<tr>
<td>2055</td>
<td>2.642</td>
<td>21.49</td>
<td>17.81</td>
<td>62.61</td>
<td>79.00</td>
</tr>
</tbody>
</table>

Table 4: Macroeconomic aggregates in the benchmark simulation path

The projected evolution of the Spanish economy is represented by the blue continuous 
line in figures 4 to 7 and some key indicators in table 4. The most important feature of the 
projection is the large increase in the pension expenditure, which is predicted to represent 
almost 18% of the annual GDP by 2050 (fourth column of table 4). This amounts to a 125% 
increase from the starting value, and is the result of expansions in both the old-age pensions 
and survival pensions (bottom graphs of figure 4). At the same time population aging has an 
impact of factor prices via changes in the capital to labor and capital to GDP ratios (K/Y is 
plotted on the top-right panel of figure 4). Labor becomes relatively more scarce that capital 
until around 2040, leading to mild improvements in wages versus interest rates.

As we keep the contribution rate constant the increase in pension expenditure demands 
an adjustment of the income tax rate from 12.7 to 21.7% (see third column in table 4) 
to keep the overall public budget balance period by period (the balanced of the pension 
system alone turns deeply into deficit, as the top-right graph of figure 4 makes clear)\(^{30}\).

Both labor, pension and saving income is taxed, implying that the extra fiscal burden is 

\(^{30}\)Other fiscal paths compatible with the fulfilment of a long-term budget constraint by the public sector are 

clearly possible. We opt for a period by period adjustment for two reasons. First, for simplicity: we avoid the 
complex intergenerational redistributional issues that would appear otherwise. Second, for realism. Spain has
Figure 4: Simulated capital to output ratio and pension system’s indicators in three institutional settings: (i) Current (i.e. benchmark simulation; blue continuous line, −)); Pre-1997/2001-reforms with early retirement (black, dashed line −−) ) and Pre-1997/2001-reforms without early retirement (red dotted line ·). PSB/Y= pension system balance as a proportion of GDP; PP/Y=total pension expenditure to GDP by pension type (Old-age and survival).

Figure 5: Pension Expenditure Decomposition (equation (13)) in three institutional settings: (i) Current (i.e. benchmark simulation; blue continuous line, −)); Pre-1997/2001-reforms with early retirement (black, dashed line −−) ) and Pre-1997/2001-reforms without early retirement (red dotted line ·).
spread widely across the population. Household reaction is complex, due to opposite income and substitution effects. On the one hand, the net return on savings and working (even despite the gains in gross wages) is smaller, pushing towards more current consumption, earlier retirement and less female labor participation. On the other hand, (relatively) lower life-cycle wealth pushes in the opposite direction. With the current model calibration both effects are essentially balanced: there are small changes in the savings and participation rates of cohorts. Only retirement shows a clear (although mild) tendency towards longer working careers (figure 6), specially among workers with higher education.\[31\] Note that the average participation rates by education go up for most of the simulation (see figure 7), but this is entirely due to a composition effect (the replacement of older cohorts of workers with very low participation rates). Similarly, the increase in the aggregate average participation rate (top-left graph of figure 7) reflects both the generational replacement and the change in the distribution by education. Overall, our results confirms the qualitative findings in previous analysis of the Spanish economy (eg. Díaz-Giménez and Díaz-Saavedra (2009) o Sánchez (2010)). The quantitative findings, notwithstanding, differ as we shown in section 4.3.

The traditional explanation for the difficulties of PAYG pension system emphasizes the role of demographic aging. But a decomposition of the pension expenditure to GDP ratio along the lines of eq (13) reveals a very different picture (that can be clearly appreciated in figure 5). It is true that the dependency ratio contributes to higher expenses, but the coverage rate and the replacement rate also play very substantial roles. Again, these phenomena result from the long-term changes in the average participation rate by cohort and in the composition by education. More younger cohorts, with stronger labor market attachment, will result in a larger number of old-age pensions and, with some delay, of survival pensions. The latter reinforces the natural increase in survival pensions derived from an ageing population. The average pension will also grow faster than productivity. This reflects that historical pensions (formed in some cases according to different rules) are, on average, less generous than the newest ones. It also reflects the impact of longer working careers (later retirement) in the individual pensions (ie. lower penalties). Note, finally, that the positive evolution of the labor market contributes to alleviate the increase in PP/Y, but is far from preventing it.

<table>
<thead>
<tr>
<th>Number of Early retirement</th>
<th>Contribution record</th>
<th>Legal</th>
<th>Survival</th>
</tr>
</thead>
<tbody>
<tr>
<td>back-lags penalties</td>
<td>penalties</td>
<td>Ages</td>
<td>System</td>
</tr>
<tr>
<td>( D=8 )</td>
<td>( \alpha_E^E = 0.6 )</td>
<td>( \alpha_H^H = 0.6 )</td>
<td>( \tau_m = 60 )</td>
</tr>
<tr>
<td>( \Delta \alpha_{+65}^E = 0 )</td>
<td>( \Delta \alpha_H^H = 2% )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Pension system parameters before the 1997-2002 reforms
Figure 6: Simulated average retirement ages in three institutional settings: (i) **Current** (ie. benchmark simulation; blue continuous line,−)); **Pre-1997/2001-reforms with early retirement** (black, dashed line −−) ) and **Pre-1997/2001-reforms without early retirement** (red dotted line ·)) .

Figure 7: Simulated female participation rates in three institutional settings: (i) **Current** (ie. benchmark simulation; blue continuous line,−)); **Pre-1997/2001-reforms with early retirement** (black, dashed line −−) ) and **Pre-1997/2001-reforms without early retirement** (red dotted line ·)) .
Table 6: Pension expenditure 2010-2050 in three institutional environments: summary statistics. ER=Early Retirement IUL=Implicit Unfunded Liabilities of the pension system (present discounted value of the difference between present and future pensions and contributions to all cohorts alive in 2001, expressed as a proportion of the 2001 tax base). The discount factor is 5%.

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>Pre 1997, ER</th>
<th>Pre 1997</th>
</tr>
</thead>
<tbody>
<tr>
<td>IUL/Tax-Base %</td>
<td>105.4</td>
<td>114.3</td>
<td>94.2</td>
</tr>
<tr>
<td>Average PP/Y</td>
<td>11.93</td>
<td>12.60</td>
<td>11.35</td>
</tr>
<tr>
<td>Maximum PP/Y</td>
<td>17.82</td>
<td>18.97</td>
<td>17.95</td>
</tr>
</tbody>
</table>

4.2 Impact of the 1997/2002 Reforms

The current institutional environment is the result of several changes introduced in the pension rules during the interval 1997/2002. As discussed in section 1, the reforms pursued two largely antagonist goals:

(i) Fiscal consolidation in the medium-run.

The government introduced changes in the pension formula aimed at reducing pension generosity. The most significant changes were (1) an increase in the length of the averaging period in the pension formula from $D=8$ to $D=15$ and (2) the imposition of stiffer penalties for retirement with a contribution record shorter than 25 years (to 3% from and initial 2%)

(ii) Achieving a more egalitarian distribution of income.

This was pursued through the following three changes. First, the generosity of the survival pension was enhanced by an increase in the associated replacement rate from 45% to 52%. Second, the right to claim the old-age pension before the age of 65 was extended to all cohorts, independently on when they become affiliated for the first time with the Social Security. In compensation, the early retirement age was delayed to 61. Finally, the penalties for early retirement were relaxed for workers with very long contributive records.

Table 5 reflects the pension parameter values in effect before the 1997 changes (compare with table 1). In order to disentangle the impact of all these changes we consider two alternative counterfactual institutional environments, both characterized by the parameters in table 5 but differing on the legal possibility of Early Retirement (ER) and compare them with the benchmark economy.

31 The discontinuity that figure 6 exhibits in year 2040 is created when households made of highly educated spouses decide to delayed retirement for 65 to 66.

32 There has been some other smaller changes that we abstract from on simplicity grounds.

33 In Spain, early retirement was initially introduced as a temporary measure aimed at easing the painful process of industrial restructuring of the 80’s. Then, workers who became affiliated with the social security later than 1967 were originally not allowed to early retirement.
Economic performance under the Pre-1997 reform, with early retirement

The first alternative pension system that we simulate reflects the parameters in place before 1997 except for the prohibition of early retirement at the age of 60 (Pre 1997, ER). The results are represented by a dashed black line in figures 4 to 7. In addition, table 6 reports some summary statistics of the level of pension expenditure reached in each of our simulations and tables 7 and 8 (in appendix D) report the change in individual pensions by cohort, education and labor status of the female.

The extension of the averaging period reduced the level of the individual old-age pension, but with a varying degree of success depending on the educational level. Highly educated workers suffered the largest reductions, with drops of around 8% inflicted on the cohorts of older active workers at the time the reform was implemented. The pensions of low educated workers, in contrast, were little affected and, besides, their effective income was protected by the minimum pension guarantee (whose value was left untouched by the reform). A side effect of the reform was that, by reducing the individual benefit base, it was also undermining the effort to increase the survival pension by increasing (from 45% to 52%) its replacement rate over the very same benefit base. As can be appreciated in the bottom-right of figure 4 the expenditure to GDP ratio in this type of pensions is actually smaller in a very short period of time after the reform.

All in all, the reform reduced the total pension expenditure to GDP in, roughly, 3/4 of a percentage point, throughout a reduction in the average coverage rate (as intended) but also by reducing the replacement rate (figure 5). This latter effect results from a small delay in the average retirement (figure 6), essentially achieved by elevating the early retirement age form 60 to 61. The changes induced in the participation rates of the females were very small. Using the change in the Implicit Unfunded Liabilities (IUL) as a catchall measure of the improvement in the financial prospects of the pension system we find a 9 percentage points relieve resulting from the reforms other than the early retirement (see table 6). This triggered some small macroeconomic gains in the form of a mild process of capital deepening.

To complete the analysis, we measure the change in welfare generated by the reform with an Equivalent Variation (EV) in life-cycle consumption. Figure 8 shows the calculated EV by cohort and educational level conditioning on the female’s labour market status in the original (PRE-Reformed) economy (i.e. we distinguish between single-earner and double-earner households in the original case). The general patterns are clear: welfare losses are concentrated on the cohorts of older active workers, specially those with higher education, and largely with independence of the employment status of the female. Younger cohorts fare better because they profit from the tax cuts and macroeconomic gains generated for

---

34 The differences by education are generated by the different properties of their life-cycle profile of labor productivity. Recall figure B.3.
35 The Implicit Unfunded Liabilities are defined as the present discounted value of the difference between current and future pensions and contributions of all cohorts alive in 2001, expressed as a proportion of the 2001 tax base.
36 The EV assess how much should the entire initial life-cycle profile of household consumption change to generate the same life-cycle utility as that observed after the reform. A positive value, then, implies a welfare gain.
37 Our welfare figures include the endogenous adjustment in retirement, but fail to capture the welfare improvement derived from altering, in response to the reforms, the labor status of the female. This is actually the case in a small number of cohorts born immediately after the reform (peaking around 2.2% for low educated households and 0.5% for medium educated workers). These are all marginal households, in the sense that their utility cost of female participation is very close to the threshold. For those households, our EV values overestimate the losses and underestimate the gains derived from the reform.
longer period. Within cohorts, low income workers benefit most, thanks to the protection provided by minimum pensions. The welfare losses among top educated cohorts can be as large as almost 2% of life-cycle consumption.

**Economic performance under the Pre-1997 reform**

To complete the evaluation of the implemented reforms we simulate the economy under the pension system in place before 1997. That include the prohibition to early retire for workers who became affiliated with the social security later than 1967. We refer to this environment as simply the Pre 1997 framework. The simulation results are represented by a dotted red line in figures 4 to 7.

Unsurprisingly, the biggest difference observed with the previous simulations occurs in retirement behavior, as the cohorts of workers without the right to retire early choose to wait till the normal retirement age to leave the labor force. This automatically implies that the Pre 1997 environment exhibits lower coverage rates and higher replacement rates than the benchmark (with the latter point reinforced by the higher benefit base associated with the other Pre 1997 institutional parameters). In the aggregate, these two opposite forces tend to cancel out, with the effect that annual old-age pension expenditure after the reform is larger until around 2040 and slightly smaller thereafter. Survival pensions, are by the end of the simulation period a bit more expensive to the increase in the replacement rate of the benefit base. Overall, the reform has increased the average annual total pension expenditure in 2010/2050 in around 1/2 of a percentage point. The total IUL is 94.2% of the tax base in the Pre 1997 scenario, in contrast with the 105.4% in the benchmark (see table 6). The macroeconomic side-effects of the increase in public pensions are some crowding out of private capital (leading to a higher K/Y ratio) and a different time-path of tax hikes featuring a higher fiscal burden at the beginning of the path and slightly lower after 2050. Furthermore, on the labour market side, female participation of low and medium educational groups is about 5 percentage points lower as a result of the extension of early retirement. The lower coverage rate that the Pre 1997 renders should lay behind this change in behavior.

Figure 9 displays a more marked pattern of welfare redistribution than that found with constant legal ages. Being the group that benefit most from the right to early retire, low income workers stand out as the big winners (specially in households with a double earner, which tend to anticipate retirement). Remarkably, all the rest of households alive at the time the reform is implemented experience welfare losses (this time including already-retired workers). Future cohorts suffer progressively less severe losses and eventually benefit from the reform, again in inverse order to their educational attainment.

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38 Younger cohort fare better with the exception of the cohorts of already retired households, that benefit from the tax relief generated by the reform without suffering any pension cut.
Figure 8: Equivalent Variation associated with reforms 1997-2001 (allowing early retirement) by education level: low (blue continuous line (−)), medium (black dashed line (−−)) and high (red dotted line (·)).

Figure 9: Equivalent Variation associated with reforms 1997-2001 without extending early retirement, by education level: low (blue continuous line (−)), medium (black dashed line (−−)) and high (red dotted line (·)).
4.3 The importance of modeling survivors pensions

As we emphasize in section 1, up to now, the standard modeling framework employed in the pension literature has featured individuals rather than households. In a typical case, the individuals in the model are given a pension benefit whose value is set according to the regulations for old-age pensions in the country analyzed. Survival pensions and other transfers based on family relations are customary ignored (or, at best, added in present value terms to the old-age benefit). This modeling strategy makes perfect sense for a large number of research questions. However, we doubt that this is a satisfactory framework when projecting the aggregate pension system of a country. The issue is probably trivial on qualitative grounds: survival pensions are granted according to their own specific rules which, in general, are very different to the old age rules. As a consequence, the level and dynamics of aggregate pension expenditure has to be different. It may well be the case, however, that the quantitative difference is small and the error incurred when using the standard model is acceptable. So the aim of this section is to assess this issue for the Spanish case.

The experiment is simple: to mimic standard models we assume that the old-age pension is the only available and disregard any other transfer based on family links. Survival pension are, therefore, ignored while keeping all the other ingredients of the model unaltered. To make the new simulation quantitatively meaningful we recalibrate the model aiming to reproduce the pension to GDP expenditure in the interval 2001/2007. This effort involves three changes: (1) inflate the initial condition on old-age pension to represent both retirement and survival benefits; (2) change the number of retirement pensions until the model reproduces the initial total coverage rate (which we do by increasing the number of pensions per employee, φ); and (3) reduce the size of the average old-age pension until we reproduce the initial replacement rate of the data. This final target is accomplished by increasing the “wedges” separating gross income and pensionable income, χ. The simulated behavior of the resulting economy is presented in figures 10 and 11. Note that the new calibration slightly overestimates the initial coverage and replacement rate. Still, its long term predictions in term of the aggregate pension to GDP expenditure fall wide short of the predictions of the more complete model of the previous sections. The long term divergence in the predicted PP/Y peaks at a value of more than 1.5 percentage points (or a 10% error in the estimated value for 2050). The discrepancy is the result of significantly different simulated paths for the total replacement rate (which is underpredicted in a largely systematic way) and the coverage rate (which is overpredicted after 2020). Overall, the omission of survival pensions leads to an appreciable underestimation of the size of the implicit liabilities of the pension system (98.3% vs 105.4% with survival pensions). In the light of these results, we think researchers should proceed with caution when using single-agent models for the prediction of long-term pension expenditures.

5 Conclusions

We use an dynamic General Equilibrium model calibrated to the Spanish economy over the period 2001-2007 to assess the impact of the most recent reforms of the pension system, those undertaken in 1997/2002. The reforms aimed at improving the financial prospects of the system as well as enhancing solidarity. They combined a set of institutional parameter changes with contradicting financial and welfare consequences. Our target in this paper is to assess those financial and welfare implications. We introduce a novelty in the type of macroeconomic setup used to evaluate these issues in an ageing population economy.

The parameters values employed in the re-calibrated model are as follows: $\alpha^V = 0$, $bm^V = 0$, $\phi = 1.6$, $\chi_1 = 0.2$ and $\chi_2 = \chi_3 = 0.6$. 

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39 The parameters values employed in the re-calibrated model are as follows: $\alpha^V = 0$, $bm^V = 0$, $\phi = 1.6$, $\chi_1 = 0.2$ and $\chi_2 = \chi_3 = 0.6$. 

28
Figure 10: Pension expenditure to GDP ratio in the benchmark economy (blue continuous line) and in an economy without survival pensions (red dotted line (·)).

Figure 11: Pension Expenditure Decomposition (equation (13)) in the benchmark economy (blue continuous line) and in an economy without survival pensions (red dotted line (·)).
Instead of the standard single earner household model, we base on a two potential earners household model that allows to explicitly account for some essential elements of the pension system (such as survivor pensions and other rules contingent on household composition, like eg. minimum pensions). Furthermore, we allow for the endogenous reaction of both female labour participation and of the retirement decisions of households.

Our main findings are as follows. First, as other studies in the literature, we conclude that the Spanish pension system is clearly unsustainable and pension expenditure will become about 18% of the GDP in 2050 (compared to the current 7%). Survival pensions will also roughly double and so contribute 15% of the total increase. Second, our assessment of the reforms is that they were clearly ineffective in tackling the financial imbalances of the system, in spite of their financial impact: excluding the extension of the early retirement entitlement, the implemented reforms have reduced the implicit unfunded liabilities (as a proportion of the tax base) by around 9 percentage points, however, including the extension of early retirement, we find an increase of around 11 pp. Third, the reforms had significant welfare consequences by redistributing the cost of ageing against more educated individuals and in favor of less educated individuals. Also, they are detrimental for current cohorts of workers. Forth, concerning labour supply reactions, the reforms have a small impact on female labour supply, but a significant impact on the retirement decision of low educated group. Finally, we show that the omission from the analysis of the survival pensions turns the implicit unfunded liability value to be around 7 percentage points smaller than the one predicted by our benchmark economy. In this sense, we think that our analysis makes a contribution of broader interest than the assessment of the financial and welfare implications of the reforms implemented in Spain: the projections of pension expenditures with standard, single-earner households models that abstract from survival pensions in an explicit way may deliver biased estimates of the future pension expenditures to GDP ratio.

We think that models with two-potential-earner households provide a fruitful framework to explore questions relating pension and population aging. For example, they may be specially well suited to study the impact of pension reforms on poverty at advance ages, on the labor supply behaviour (specially retirement and female participation decisions) and on the economic effects of increasing longevity. Actually, we think that it will be hard to address these matters without an explicit modeling of the economic links within the household. We expect sustain progress along these lines in the immediate future.
References


**APPENDIX**

**A Formal definition of equilibrium**

An *equilibrium path* over the time interval $T$ consists of the following objects:

- Time series of the aggregate number of households $\{P_t\}$ and their distribution by age, education and labor status of the female, $P_{tij}^e$ for all $i \in I$, $j \in J$, $e \in \{0, 1\}$, $t \in T$. Note that, for each cohort and education, the distribution according to the labor status of the female is determined by a threshold on the utility cost, $\bar{z}_{ij}$, that is determined according to eq (14). The utility cost itself is assumed to be normally distributed.

- Assignments of consumption and asset holdings $\{c_t^e, a_t^e\}$ for all cohorts alive in $t \in T$ and for all educational levels $j \in J$, and female participation situation $e \in \{0, 1\}$.

- Time series of the inputs employed by the competitive firms of the economy $(K_t, L_t)$ $t \in T$.

- A Public Policy consisting of the time series of taxes, public expenditure, minimum pensions (old-age and survivors), maximum old-age pensions and maximum contributions: $\{\varphi^t, CP^t, bm^1, bmV^t, bM^t, C_M^t\}$ $t \in T$

- A price system: $\{r^t, w^t\}$ $t \in T$

such that the following properties apply:

1. **Endogenous population dynamics**
   Population aggregates and distributions are coherent with our demographic model, given the exogenous patterns specified for fertility, mortality, immigration flows and education distribution.

2. **Rational behaviour by the households.**
   Household decisions are optimal (ie. solve problem (9)) given the price system and the public policy.

3. **Clearance of the markets for capital and labor.**
   The capital and labor effectively employed by firms come from the aggregation of the household savings and labor supply:

   $L^t = A^t H^t$
   $H^t = \sum_{j=1}^{J} \sum_{e=0}^{1} P_{tij}^e l_{ij}^e$
   $K^t = \sum_{j=1}^{J} \sum_{i=20}^{I-1} \sum_{e=0}^{1} P_{tij}^e a_{ij}^t$

   Where $l$ is the household supply of efficient labor units: $l_{ij}^e = \pi_{i,1} \varepsilon_{ij} + \pi_{i,2} \varepsilon_{ij} I(e = 1)$ (with $\pi$ and $\varepsilon$ are defined as in section 2.3 and $I(.)$ an indicator function).

4. **Competitive prices.**
   
   $r + \delta = \frac{\partial F}{\partial K}(K^t, L^t)$
   $w^t = \frac{\partial F}{\partial H}(K^t, L^t)$

5. **Balanced Public budget.**

   $FI^t(\varphi^t) + PSB^t = CP^t$
where the public expenditure is a fixed proportion of aggregate output \( CP^t = c_p Y^t \); the fiscal income, \( FL^t \), and the income from bequest, \( BI^t \), takes the form:

\[
FL^t = \varphi^t \sum_{j=1}^J \sum_{i} \sum_{e=0}^{1} p_{ij,e} Inc_{ij,e} + BI^t
\]

\[
BI^t = \sum_{j=1}^J \sum_{i} \sum_{e=0}^{1} (1 - hs_{i,j}^{t-i}) p_{ij,e}^{t-1} a_{t+1}^{t-1}
\]

where \( Inc \) stands for the total income of the household: \( Inc_{ij,e} = H_I j_{i,e} + r^t a_{i,j,e} \), and \( hs_{i,j} \) is the household survival probability (ie, the probability of survival of at least one of its members).

The pension system balance is given by

\[
PSB^t = COT^t - PP^t
\]

where \( PP^t \) and \( COT^t \) stand for the aggregate pension expenditures and the aggregate social contributions:

\[
PP^t = \sum_{j=1}^J \sum_{i} \sum_{e=0}^{1} p_{ij,e} H_I j_{i,e} \quad COT^t = \sum_{j=1}^J \sum_{i} \sum_{e=0}^{1} p_{ij,e} cot_{ij,e}
\]

With \( cot \) standing for the social contributions paid by the household,

\[
cot_{ij,e} = \varsigma (\pi_{i,1} cov_{i,j,1} + \pi_{i,2} cov_{i,j,2} I(e = 1))
\]

6. Aggregate feasibility

\[
Y^t + (1 - \delta) K^t + BI^t = K^{t+1} + BI^{t+1} + \sum_{i=20}^{1} \sum_{j=1}^J \sum_{e=0}^{1} p_{ij,e} e_{ij,e} + CP^t + CC^t
\]

Where the Child-Care Cost depends on the number of children of cohort \( u \) and education \( j \) \( n_{j}^{u} \), that are in the age range 20/24 in \( t \): \( CC^t = C \sum_{i=20}^{24} \sum_{j=1}^J p_{ij} e_{ij}^{t-i+1} \)

\[\text{B Economic and demographic calibration}\]

\[\text{B.1 Demographic Transition}\]

Figure 12 illustrates the changing patterns of fertility, mortality and immigration flows assumed in our simulation.

\[\text{B.3 Life cycle labor-income profiles}\]

The data source to estimate individual life-cycle profiles of earnings is the European Community Household Panel (ECHP) 1994-2004. In particular, we use annual gross labour earnings (variable PI110: total income from work).
B.2 Education

The education classification explained in section 3.2.1 is chosen to mimic as closely as possible the distribution implemented in the European Community Household Panel (ECHP), from where we estimate the household earnings by education and gender. However, the historical evolution of labour market participation was taken from the Labor Force Survey whose large sample size allows a disaggregate analysis by cohort, gender, and education. There is a more detailed description of the individual educational level in this survey, with more categories, but we reclassified individuals according to the categories in the ECHP.

Figure 13: Simulated distribution by education. Time series of the proportion of households in each education level: low (blue continuous line (−)), medium (black dashed line (−−)) and high (red dotted line (·))
C  The female participation decision

Under the assumptions in section 2.3 and 3.2.3, it is not difficult to express the life-cycle utility of single and two-earner households as, respectively:

\[ V(\tau) = u(Y) \Psi_1 + \alpha \Psi_2(\tau) \]
\[ V_2(\tau_2, z) = (u(Y_2) + z) \Psi_1 + \alpha \Psi_2(\tau_2) \]

where \( u \) is the period utility function, \( \tau \) and \( \tau_2 \) are the optimal retirement decisions for single and two-earner households; \( Y \) and \( Y_2 \) are the life-cycle wealth in both cases and \( \Psi_i, i = \{1, 2\} \) are constants that depends on the discount factor, survival probabilities and risk aversion. So for the female to participate in the labour market (i.e. \( E = 1 \)) when \( V_2(\tau_2, z) > V(\tau) \).

There is a threshold level \( \bar{z} \) such that those for which \( z < \bar{z} \) participate, whereas the rest stay out of the labor market. Formally: \( V(\tau) = V_2(\tau_2, \bar{z}) \).

Linking the life-cycle wealth obtained in both cases as follows:

\[ Y_2 = Y + \Delta = (1 + \gamma)Y \]

with \( \gamma = \) growth rate of LC wealth after joining the labor force

we can derive the formal expression of the threshold:

\[ \bar{z} = \frac{Y^{1-\eta}}{1-\eta} \left[ (1 + \gamma)^{1-\eta} - 1 \right] + \alpha \Psi \]

where \( \Psi = \Delta \Psi_2/\Psi_1 \) with \( \Delta \Psi_2 = \Psi_2(\tau_2) - \Psi_2(\tau) \).

This simple theory of female labor participation combines substitution effects (driven by the additional life-cycle income generated by female labor participation, \( \gamma \), and the elasticity of substitution) and wealth effects (\( Y \)). With log-utility the threshold turns a simple function of \( \gamma \) and the changed induced in retirement ages. Leaving aside the latter element (just
for illustration purposes), we can informally describe how the model can reproduce the main stylized facts found in the data. The higher participation rates of highly educated females observed in the crossed section would, therefore, be the result of a larger $\gamma$ for those women (their added life-cycle labor income, net of participation costs, represents a larger proportion of the single-household $Y$ than for lower educated females). The trend towards higher participation rates by females of lower education implies that this difference is being progressively cut down for younger cohorts.

The foundations of optimal retirement behavior can be worked out in a similar way: see, for example, Jiménez-Martín and Sánchez-Martín (2007) where a continuous time version of the same model is explained in detail.

### D Pension changes in the simulated reforms

<table>
<thead>
<tr>
<th>Cohort</th>
<th>Low Education</th>
<th>Med Education</th>
<th>High Education</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With ER</td>
<td>Without ER</td>
<td>With ER</td>
</tr>
<tr>
<td>1920.</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1940.</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1960.</td>
<td>0.982</td>
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<td>0.956</td>
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<tr>
<td>1980.</td>
<td>0.989</td>
<td>0.833</td>
<td>0.965</td>
</tr>
<tr>
<td>2000.</td>
<td>0.995</td>
<td>0.835</td>
<td>0.969</td>
</tr>
<tr>
<td>2020.</td>
<td>0.980</td>
<td>0.901</td>
<td>0.959</td>
</tr>
</tbody>
</table>

Table 7: Initial pension ratios for **males**: ratio of the average initial pension (by education and cohort) in the institutional setting before the 1997-2002-reforms to the average initial pension in the benchmark simulation (post reforms).

<table>
<thead>
<tr>
<th>Cohort</th>
<th>Low Education</th>
<th>Med Education</th>
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<td>1940.</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1960.</td>
<td>1.082</td>
<td>0.707</td>
<td>0.952</td>
</tr>
<tr>
<td>1980.</td>
<td>1.090</td>
<td>0.722</td>
<td>0.962</td>
</tr>
<tr>
<td>2000.</td>
<td>1.098</td>
<td>0.726</td>
<td>0.970</td>
</tr>
<tr>
<td>2020.</td>
<td>1.083</td>
<td>0.710</td>
<td>1.001</td>
</tr>
</tbody>
</table>

Table 8: Initial pension ratios for **females**: ratio of the average initial pension (by education and cohort) in the institutional setting before the 1997-2002-reforms to the average initial pension in the benchmark simulation (post reforms).
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