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Access pricing and investment in vertical structures with complementary or rival facilities

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ABSTRACT

This paper considers the existence of a given facility and analyzes the optimal conditions for investing in a complementary or rival new facility. The facilities are publicly owned and are operated by private firms. First, we analyze the optimal access prices to be charged to private operators. We find that the optimal access price to be charged for the use of a particular facility depends on the degree of complementarity and substituibility between facilities. Second, we analyze under which circumstances the investment in a new facility is socially desirable both in a context with and without budget constraints. The alternative to be considered if the investment is not carried out necessarily requires optimal pricing. Thus, the investment is socially desirable if there is a positive difference in social welfare for the cases in which the new facility is and is not constructed and optimal pricing is applied.

KEYWORDS: investment, access pricing, vertical structure, product differentiation.

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1. Introduction

This paper considers the existence of a given facility and analyzes the optimal conditions for investing in a complementary or substitute new facility. The industry is characterized by a vertical structure, that is, each facility is owned by a firm that provides the infrastructure to downstream firms which in turn produce output for final consumers. This is the case, for example, of the transport industry where airports or rail infrastructures are used by operators to provide transport services to final consumers.

Private economic goods like those provided by transport operators require specific infrastructure investment. The ownership of key infrastructure facilities is commonly public. The standard model for the provision of these goods consists in the separation between ownership (usually public) and operation (usually provided by private firms). Sometimes the infrastructure is subject to intermodal competition, like airports facing competition from high speed rail infrastructure in medium distance corridors. In other cases, the facilities are considered as complementary, like the use of the rail infrastructure to reach airports. In all cases access pricing for the use of the infrastructure is critical.[1]

There are some papers in the literature analyzing pricing and capacity investment in facilities with vertical structure. In particular, some recent papers have applied this setting in the analysis of airports, including the analysis of a non-competing airport (Brueckner, 2002; Zhang and Zhang, 2006), complementary airports (Basso, 2005; Brueckner, 2005; Pels and Verhoef, 2004), or an airport competing with other airports or transport facilities (Basso and Zhang, 2007). However, none of these papers study how access pricing is affected by the degree of complementarity or substituibility between facilities, or even more important, the conditions under which a new complementary or rival facility should be constructed.

[1] Some contracts leave pricing decisions to the concessionaire but we are interested here in the case where, even with infrastructure private operators, infrastructure pricing is in the hand of the regulator. This is the case of variable term concessions where efficient pricing and profit maximization with demand uncertainty is possible through the variability of the concession life (see Engel *et al*., 2001)

Public investments in dedicated high speed rail (HSR) infrastructure that compete directly with air transport serves as an excellent case for the analysis of access pricing, investment and intermodal competition. The base case is characterized by the following: an existing infrastructure (airport), public or regulated, is demanded by privately owned commercial airlines operating in open competition (intramodal competition) and another existing infrastructure (rail tracks) is provided by a monopoly (the rail operator). In this context, we analyze the optimal access prices to be charged to private operators for the use of public infrastructures in order to maximize the social welfare of the overall economy in the case in which there is and there is not a budget constraint.

Real world access pricing for the use of a particular public infrastructure is usually performed by independent agencies that analyze the specific characteristics of such an infrastructure and take pricing decisions in an independent manner. The Office of Rail Regulation, for example, is the independent safety and economic regulator for Britain's railways and the Civil Aviation Authority has as its prime focus to ensure that the airports at Heathrow, Gatwick and Stansted do not exploit their position as a monopoly service. There are no interconnections between the decisions of both agencies.

In this paper, we show that if consumers do not consider goods as independent the socially optimal access price for the use of each public infrastructure cannot be set in an independent manner. Moreover, we show that the optimal access pricing strongly depends on the budget constraints faced by the regulator.

Public infrastructures involve significant amounts of public funds and are, essentially, irreversible so the optimal timing of investment is critical as it is not a “now or never” decision (Dixit and Pindyck, 1994). To evaluate whether an infrastructure should or should not be constructed today we do not only need the project to have a positive net present value compared with an alternative in which optimal pricing is not applied, but also to increase the social welfare compared to the situation in which the infrastructure is not constructed and optimal pricing is used. In this sense, if the regulator is not subject to any budget constraint we show that the new infrastructure is more likely to be constructed the higher the private revenues, the lower the demand elasticity and the lower the social cost of the investment. Given these values, a key parameter is the population. High individual willingness to pay, low opportunity cost of capital and low construction costs are not enough unless we have a reasonable level of demand, which is proportional to the number of passengers.

Governments and supranational agencies argue that one of the main benefits of rail infrastructure investment is the reduction of environmental externalities. In general, the plane is considered as a mode of transport more harmful to the environment than the HSR, especially in regard to its impact on climate change (Eurocontrol, 2004; Givoni, 2007; Schreyer *et al*., 2004). However, we would like to highlight that the environmental impact of investment in HSR points in two directions: one of them is the reduction in air and road traffic. In such cases its contribution to reducing the negative externalities of these modes could be positive, though it requires a significant deviation of passengers from these modes. Moreover, the use of capacity must be high enough to offset the pollution associated with the production of electric power consumed by high speed trains (and in the construction period), as well as noise pollution (Kageson, 2009). Rail infrastructure also has a negative environmental impact such as the barrier effect as well as the land taken for the access roads needed for construction and the subsequent maintenance and operation. The net balance of these effects depends on the value of the affected areas, the number of people affected, the benefit from diverted traffic, etc.

Given the intense debate in the literature about the environmental costs of rail and air transport, we have included such impacts in our model. However, we have not included other indirect effects. The reason is that in undistorted competitive markets the net benefit of marginal change in secondary market is zero: no welfare changes as long as the price changes are small. As Greenwald and Stiglitz (1986) points out: "… if firms are maximizing profits and individuals are maximizing utility, both facing prices that correctly reflect opportunity costs, then standard envelope theorem arguments imply that changes in profits or utility induced by changes in allocations (resulting from any small change in prices) are negligible" .

The rest of the paper is organized as follows. Section 2 is dedicated to present the main features of the model. The model is solved backwards. Thus, section 3 analyzes the operators' maximization programs while in section 4 we discuss the optimal access prices. Section 5 is devoted to analyze whether or not a new facility should be constructed by a welfare-maximizing regulator and a budget-constrained welfare-maximizing regulator. Finally, section 6 concludes.

2. The Model

Following Dixit (1979) and Singh and Vives (1984), we consider an economy composed of an oligopolistic transport sector and a competitive (*numeraire*) sector summarizing the rest of the economy. The transport sector contains two public transport infrastructures (a rail infrastructure and an airport) that are used by private operators.[2] In particular, the rail infrastructure is used by a private rail operator while two private airlines operate in the airport. In this context, a benevolent regulator must decide the access prices to be charged to private operators for the use of public infrastructures in order to maximize the social welfare of the overall economy.

Denote by q_1, q_2, q_t the quantity offered by airline 1, airline 2 and the rail operator, respectively. Consumers are all identical with a utility function separable and linear in the *numeraire* good, $m : U(q_1, q_2, q_t) + m$. Therefore, there are no income effects on the transport sector, and we can perform partial equilibrium analysis.

$U(q_1, q_2, q_t)$ is assumed to be quadratic and strictly concave:

$$U(q_1, q_2, q_t) = u_a q_1 + u_a q_2 + u_t q_t - \frac{1}{2}(q_1^{\,2} + q_2^{\,2} + q_t^{\,2} + 2\gamma q_1 q_2 + 2\delta q_1 q_t + 2\delta q_2 q_t), \quad (1)$$

where u_a and u_t are positive parameters, γ represents the degree of product differentiation between airlines, and δ represents the degree of product differentiation between airlines and the railway. We assume that passengers consider that airlines are substitutes but exhibit brand loyalty to particular carriers, that is, airlines sell differentiated products.[3] Therefore, γ ranges from zero when airlines are independent to one when airlines are perfect substitutes (homogenous market). On the contrary, passengers may consider the railway and airlines either as substitutes or complements.

[2] Other transport modes can be included in the *numeraire* sector.

[3] Product differentiation between airlines may be due to different reasons such as brand loyalty, the existence of frequent flier programs, etc. (see, for example, Brueckner and Whalen, 2000, or Flores-Fillol and Moner-Colonques, 2007).

Therefore, we assume that δ ranges from minus one when rail and airlines are considered perfect complements to one when rail and airlines are considered perfect substitutes. The parameter δ is equal to zero when passengers consider rail and airlines as independent goods. However, we assume that $\gamma > \delta$, that is, passengers consider that one airline can better substitute the other airline than the train. In order to have the model well defined, we also assume that $\gamma > \delta^2$.

Passengers' generalized cost is defined as the sum of the ticket price, p_i with $i = 1, 2, t$, and the monetary value of time and/or any disutility component associated with the specific transport mode, t_a and t_t, which includes access, egress, waiting and in-vehicle time, discomfort, etc. Thus, the representative consumer solves:

$$\underset{q_1, q_2, q_t}{Max}\ U(q_1, q_2, q_t) - (p_1 + t_a)q_1 - (p_2 + t_a)q_2 - (p_t + t_t)q_t, \tag{2}$$

where t_a and t_t denote all costs associated with the specific transport mode except the ticket price.

The above maximization program can be rewritten as:

$$\underset{q_1, q_2, q_t}{Max}\ \alpha q_1 + \alpha q_2 + \beta q_t - \frac{1}{2}(q_1{}^2 + q_2{}^2 + q_t{}^2 + 2\gamma q_1 q_2 + 2\delta q_1 q_t + 2\delta q_2 q_t) - p_1 q_1 - p_2 q_2 - p_t q_t, \tag{3}$$

where α and β denote the maximum (net of all except price) willingness to pay for travelling by air or by rail, respectively.

The utility function described above gives rise to a linear demand structure for the representative consumer, and direct demands can be written as:

$$\begin{aligned} q_1 &= a_a - b_a p_1 + d_a p_2 + d_t p_t, \\ q_2 &= a_a - b_a p_2 + d_a p_1 + d_t p_t, \\ q_t &= a_t - b_t p_t + d_t p_1 + d_t p_2, \end{aligned} \tag{4}$$

where:

$$a_a = \frac{(\alpha - \beta\delta)}{1 + \gamma - 2\delta^2},\ b_a = \frac{(1 - \delta^2)}{(1 - \gamma)(1 + \gamma - 2\delta^2)},\ d_a = \frac{(\gamma - \delta^2)}{(1 - \gamma)(1 + \gamma - 2\delta^2)}$$

$$a_t = \frac{\beta(1 + \gamma) - 2\alpha\delta}{1 + \gamma - 2\delta^2},\ b_t = \frac{1 + \gamma}{1 + \gamma - 2\delta^2},\ d_t = \frac{\delta}{1 + \gamma - 2\delta^2}.$$

We assume that $\alpha - \beta\delta > 0$, and $\beta(1+\gamma) - 2\alpha\delta > 0$. Notice that, given our assumptions, a_a, b_a, d_a, a_t, and b_t are strictly positive, while d_t may be positive or negative depending on whether airlines and the railway are substitutes or complements, respectively.

We assume that when operating airlines and the rail operator produce an environmental damage. Let us denote by A the environmental damage that airlines produce per passenger and by T the environmental damage per passenger produced by the rail operator.[4]

The timing of the game is as follows: In the first period, a benevolent regulator must decide whether or not to construct a new rail infrastructure, taking into account that there already exists an airport and that consumers perceive the airlines services either as substitutes or complements to the railway services. In the second period, the regulator must decide the access price to be charged to private operators for the use of public infrastructures. We distinguish between a welfare-maximizing regulator and a budget-constrained welfare-maximizing regulator, that is, a regulator that maximizes social welfare but must achieve financial breakeven. In the last period and given the access prices, private operators compete in prices with differentiated products. The game is solved by backward induction.

3. Third period: Private operators' maximization programs

In the third period, private operators consider access prices as given. Let us denote by μ_a the access price charged to private airlines for the use of the public airport, and by μ_t the access price charged to the rail operator for the use of the public rail infrastructure. Let us denote by c_a and c_t the constant marginal operating cost for airlines and rail, respectively.

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[4] For the sake of simplicity, we just consider operating environmental costs, ignoring all fixed environmental costs that might arise during the construction of the public infrastructures.

Each airline i maximizes its own profits and, thus, airline i solves the following maximization program:

$$\underset{p_i}{Max}\ \pi_i = (p_i - c_a - \mu_a) q_i, \tag{5}$$

where q_i is given by expression (4), with $i = 1,2$.

The rail operator solves the following maximization program:

$$\underset{p_t}{Max}\ \pi_t = (p_t - c_t - \mu_t) q_t, \tag{6}$$

where q_t is given by expression (4).

First order conditions for airline 1, airline 2 and the rail operator's maximization program are given by:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial p_1} &= a_a + \mu_a b_a - 2 p_1 b_a + p_2 d_a + b_a c_a + d_t p_t = 0, \\
\frac{\partial \pi_2}{\partial p_2} &= a_a + \mu_a b_a - 2 p_2 b_a + p_1 d_a + b_a c_a + d_t p_t = 0, \\
\frac{\partial \pi_t}{\partial p_t} &= a_t + \mu_t b_t + p_1 d_t + p_2 d_t + b_t c_t - 2 b_t p_t = 0.
\end{aligned} \tag{7}$$

Privately optimal ticket prices are then given by:

$$\begin{aligned}
p_1^* = p_2^* &= \frac{1}{4 b_a b_t - 2 d_t^2 - 2 d_a b_t}(2 a_a b_t + a_t d_t + 2 \mu_a b_a b_t + \mu_t b_t d_t + 2 b_a c_a b_t + b_t c_t d_t) \\
p_t^* &= \frac{1}{4 b_a b_t - 2 d_t^2 - 2 d_a b_t}(2 b_a a_t + 2 a_a d_t - d_a a_t + 2 \mu_a b_a d_t + 2 \mu_t b_a b_t - \mu_t d_a b_t + 2 b_a c_a d_t + 2 b_a b_t c_t - d_a b_t c_t).
\end{aligned}$$

(8)

Let us now analyze how optimal ticket prices change when both the access price for the use of the own public infrastructure and the access price for the use of the other public infrastructure are increased. Both results are summarized in the following lemmas.

Lemma 1: *The higher the access price for the use of the public airport μ_a, the higher the ticket price to be charged to consumers by airlines. The higher the access price for the use of the rail infrastructure μ_t, the higher the ticket price to be charged to consumers by the rail operator.*

Proof: See the Appendix. ■

Lemma 2: *If airlines and the railway are substitutes (complements), the higher the access price for the use of the public airport* μ_a*, the higher (the lower) the ticket price to be charged to consumers by the rail operator. If airlines and the railway are substitutes (complements), the higher the access price for the use of the rail infrastructure* μ_t*, the higher (the lower) the ticket price to be charged to consumers by airlines.*

Proof: See the Appendix. ■

4. Second period: optimal access prices

In the second period, anticipating how private operators react to access prices, a benevolent regulator must decide the socially optimal access prices to be charged for the use of public infrastructures. Let us denote by C_a and C_t the marginal operating and maintenance cost of each public infrastructure.

Willingness to pay for capacity ($WTP^{capacity}$) per individual is defined as the sum of the individual consumer surplus (CS), private operators surplus per individual and the infrastructure operator surplus per individual, minus environmental costs per individual. Formally:

$$WTP^{capacity} = CS + \pi_1 + \pi_2 + \pi_t + (\mu_a - C_a)(q_1 + q_2) + (\mu_t - C_t)q_t - A(q_1 + q_2) - Tq_t, \quad (9)$$

where $CS = U(q_1, q_2, q_t) - (p_1 + t_a)q_1 - (p_2 + t_a)q_2 - (p_t + t_t)q_t$.

Thus, willingness to pay for capacity per individual is given by:

$$\begin{aligned} WTP^{capacity} &= \alpha q_1 + \alpha q_2 + \beta q_t - \frac{1}{2}(q_1{}^2 + q_2{}^2 + q_t{}^2 + 2\gamma q_1 q_2 + 2\delta q_1 q_t + 2\delta q_2 q_t) \\ &\quad - (A - C_a - c_a)(q_1 + q_2) - (T - C_t - c_t)q_t. \end{aligned} \quad (10)$$

Suppose that there are N identical consumers in the society and that K_t denotes the investment required to construct the rail infrastructure, K_a the investment in airport capacity and r is the opportunity cost of capital. Then, social welfare (SW) is defined as the difference between the social willingness to pay for capacity and the social cost of capacity:

$$SW = N\ WTP^{capacity} - rK_a - rK_t, \tag{11}$$

that is:

$$\begin{aligned} SW = N[&\alpha q_1 + \alpha q_2 + \beta q_t - \frac{1}{2}(q_1{}^2 + q_2{}^2 + q_t{}^2 + 2\gamma q_1 q_2 + 2\delta q_1 q_t + 2\delta q_2 q_t) \\ &- (A - C_a - c_a)(q_1 + q_2) - (T - C_t - c_t)q_t] - rK_a - rK_t. \end{aligned} \tag{12}$$

Notice that short-run social marginal costs for the use of public infrastructures are given by the sum of the marginal environmental cost, the marginal operating and maintenance cost of the public infrastructure and the marginal operating costs for private operators, that is:

$$\begin{aligned} SMC_a &= A + C_a + c_a, \\ SMC_t &= T + C_t + c_t, \end{aligned} \tag{13}$$

where SMC_a and SMC_t denotes the short-run social marginal cost for the use of the airport and the short-run social marginal cost for the use of the rail infrastructure, respectively.

We assume that the maximum (net of all except price) willingness to pay for travelling by air or by rail is higher than the short-run social marginal cost for the use of the airport or the rail infrastructure, respectively, that is, $\alpha > SMC_a$ and $\beta > SMC_t$. Moreover, in order to have the model well defined (positive quantities) we need to assume that $\beta - SMC_t > \dfrac{2\delta}{1+\gamma}(\alpha - SMC_a) > \dfrac{2\delta^2}{1+\gamma}(\beta - SMC_t)$.

We will consider two possibilities: a regulator that maximizes social welfare and a regulator that maximizes social welfare but must achieve financial breakeven.

4.1. Optimal access prices for a welfare-maximizing regulator

A welfare-maximizing regulator chooses the access prices for the use of the airport and the rail infrastructure, μ_a and μ_t, in order to maximize the social welfare given by expression (12). Thus, socially optimal access prices are given by:

$$\mu_a^* = A + C_a + \frac{1-\gamma}{1-\delta^2}[\delta(\beta - SMC_t) - (\alpha - SMC_a)]$$
$$\mu_t^* = T + C_t + \frac{1}{1+\gamma}[2\delta(\alpha - SMC_a) - (1+\gamma)(\beta - SMC_t)]. \tag{14}$$

Proposition 1*: If consumers do not consider airlines and rail as independent goods ($\delta \neq 0$) the socially optimal access price for the use of each public infrastructure cannot be set in an independent manner.*

The socially optimal access price for the use of a particular infrastructure is decreasing in the maximum willingness to pay for travelling in that transport mode and increasing in the social marginal costs of that infrastructure. However, Proposition 1 implies that if airlines and rail are not considered as independent goods the decisions on how much to charge for the use of a particular infrastructure must be taken by considering the willingness to pay and social costs not only of that particular good but also those of the other good. In particular, if goods are substitutes, the socially optimal access price for the use of a particular infrastructure is increasing in the maximum willingness to pay for travelling in the other transport mode and decreasing in the social marginal costs of the other infrastructure. On the contrary, if goods are complements, the socially optimal access price for the use of a particular infrastructure is decreasing in the maximum willingness to pay for travelling in the other transport mode and increasing in the social marginal costs of the other infrastructure. All these results are summarized in the following proposition.

Proposition 2: *The socially optimal access price for the use of a particular public infrastructure is higher:*

- *the lower is the maximum willingness to pay for travelling in that transport mode:* $\partial\mu_a / \partial\alpha < 0,\ \partial\mu_t / \partial\beta < 0$;
- *the higher are the social marginal costs of that particular infrastructure:* $\partial\mu_a / \partial A > 0,\ \partial\mu_a / \partial C_a > 0, \partial\mu_a / \partial c_a > 0, \partial\mu_t / \partial T > 0,\ \partial\mu_t / \partial C_t > 0, \partial\mu_t / \partial c_t > 0$;

Moreover, if consumers consider airlines and rail as substitutes (complements) the socially optimal access price for the use of a particular public infrastructure is higher:

- *the higher (lower) is the maximum willingness to pay for travelling in the other transport mode:* $\partial\mu_a / \partial\beta > (<) 0,\ \partial\mu_t / \partial\alpha > (<) 0$;

- *the lower (higher) are the social marginal costs of the other infrastructure:* $\partial\mu_a / \partial T < (>)0,\ \partial\mu_a / \partial C_t < (>)0, \partial\mu_a / \partial c_t < (>)0, \partial\mu_t / \partial A < (>)0,\ \partial\mu_t / \partial C_a < (>)0,$ $\partial\mu_t / \partial c_a < (>)0.$

Proof: See the Appendix. ■

Let us look at the socially optimal ticket prices and quantities per individual that are induced by the regulator. Socially optimal ticket prices are obtained by substituting the socially optimal access prices given by expression (14) into the privately optimal tickets prices given by expression (8).

Proposition 3: *Independently of the degree of product substitution of airlines (γ) and the degree of product substitution between airlines and the railway (δ), socially optimal access prices for a welfare-maximizing regulator always induce private operators to charge ticket prices equal to short-run social marginal costs, that is,* $p_1^* = p_2^* = A + C_a + c_a$ *and* $p_t^* = T + C_t + c_t$.

Proof: See the Appendix. ■

The result in Proposition 3 generalizes what Zhang and Zhang (2006) obtained in a different model setting with homogeneous goods (perfect substitution) and just one facility.

Socially optimal quantities are obtained by substituting the socially optimal ticket prices given by Proposition 3 into the demand functions given by expression (4). Thus, socially optimal quantities for airlines and the rail operator are given by:

$$
\begin{aligned}
q_1^* = q_2^* &= \frac{1}{1+\gamma-2\delta^2}\left[\alpha - SMC_a - \delta(\beta - SMC_t)\right] \\
q_t^* &= \frac{1}{1+\gamma-2\delta^2}\left[(1+\gamma)(\beta - SMC_t) - 2\delta(\alpha - SMC_a)\right]
\end{aligned}
\tag{15}
$$

The maximum willingness to pay for capacity per individual $WTP^{capacity*}$ is obtained by substituting the socially optimal quantities given by expression (15) into the function given by expression (10). The maximum social welfare SW^* is obtained by substituting the socially optimal quantities given by expression (15) into the function given by expression (12).

Taking into account expressions (14) and (15), optimal access prices can be rewritten as:

$$\mu_a^* = A + C_a - \frac{1}{b_a} q_1^*,$$
$$\mu_t^* = T + C_t - \frac{1}{b_t} q_t^*. \tag{16}$$

Proposition 4: *In absence of environmental costs, socially optimal access prices for a welfare-maximizing regulator are always set below the infrastructure's marginal operating and maintenance costs.*

Let us define by ε_1^* the demand elasticity of airline 1 or 2 with respect to his own price evaluated in the social optimum, that is, $\varepsilon_1^* = -b_a \frac{p_1^*}{q_1^*}$. Let us also denote by ε_t^* the demand elasticity of the rail operator with respect to his own price evaluated in the social optimum, that is, $\varepsilon_t^* = -b_t \frac{p_t^*}{q_t^*}$. Then, optimal access prices can be also written as:

$$\mu_a^* = A + C_a - \frac{p_1^*}{\left|\varepsilon_1^*\right|},$$
$$\mu_t^* = T + C_t - \frac{p_t^*}{\left|\varepsilon_t^*\right|}. \tag{17}$$

Proposition 5: *The lower the demand elasticity is, the lower is the socially optimal access price for the use of public infrastructures.*

The intuition behind Proposition 5 is that the higher the monopoly power of private operators, the lower are access prices in order to induce them to charge a ticket price equal to short-run social marginal costs. Moreover, if the demand elasticity (in absolute value) is low enough socially optimal access prices may be even negative, that is, the regulator gives subsidies to private operators in order to induce them to charge a ticket price equal to short-run social marginal costs. The demand elasticity (in absolute value) for airlines (railway) is higher the higher is b_a (b_t).

4.2. Access prices for a budget-constrained welfare-maximizing regulator

In this subsection we consider a benevolent regulator that maximizes social welfare but must guarantee that for each infrastructure revenues cover all costs, including both operating and maintenance, and investment costs.

A regulator that maximizes social welfare but must achieve financial breakeven solves for each representative consumer the following maximization program:

$$\begin{aligned}
\underset{\mu_a,\mu_t}{Max} \quad & SW = \alpha q_1 + \alpha q_2 + \beta q_t - \frac{1}{2}(q_1^{\,2} + q_2^{\,2} + q_t^{\,2} + 2\gamma q_1 q_2 + 2\delta q_1 q_t + 2\delta q_2 q_t) \\
& \qquad -(A - C_a - c_a)(q_1 + q_2) - (T - C_t - c_t)q_t - rK_a - rK_t \\
s.t. \quad & (\mu_a - C_a)(q_1 + q_2) - (rK_a)/N \geq 0, \\
& (\mu_t - C_t)q_t - (rK_t)/N \geq 0.
\end{aligned} \tag{18}$$

If the optimal solutions for a welfare-maximizing regulator satisfy the restrictions given by expression (18), that is:

$$(\mu_a^* - C_a)(q_1^* + q_2^*) - (rK_a)/N \geq 0, \tag{19}$$

$$(\mu_t^* - C_t)q_t^* - (rK_t)/N \geq 0, \tag{20}$$

then, the social optimum for a budget-constrained welfare-maximizing regulator is given by expression (17). Therefore, the maximum social welfare SW^* obtained by a welfare-maximizing regulator coincides with the maximum social welfare obtained by a budget-constrained welfare-maximizing regulator.

Let us denote by sw^* the maximum social welfare achieved by a budget-constrained welfare-maximizing regulator when any of the conditions given by expression (19) and (20) is not satisfied. Then, the social optimum for a budget-constrained welfare-maximizing regulator is obtained by solving the maximization program given by (18) and in the optimum at least one of its restrictions is binding. Therefore, the maximum social welfare SW^* obtained by a welfare-maximizing regulator is higher than the maximum social welfare obtained by a budget-constrained welfare-maximizing regulator, that is, $sw^* < SW^*$.

5. First period: the rail infrastructure investment

5.1. Rail infrastructure investment for a welfare-maximizing regulator

Suppose as a benchmark the case in which there is no rail infrastructure. Then, the representative consumer's utility function is given by:

$$U(q_1,q_2)=u_a q_1+u_a q_2-\frac{1}{2}(q_1^{\;2}+q_2^{\;2}+2\gamma q_1 q_2). \tag{21}$$

The representative consumer maximizes his utility minus his generalized cost:

$$\underset{q_1,q_2}{Max}\; U(q_1,q_2)-(p_1+t_a)q_1-(p_2+t_a)q_2, \tag{22}$$

where t_a denote the total time required to make a trip by air and/or any disutility component associated with the air transport.

The above maximization program can be rewritten as:

$$\underset{q_1,q_2}{Max}\; \alpha q_1+\alpha q_2-\frac{1}{2}(q_1^{\;2}+q_2^{\;2}+2\gamma q_1 q_2)-p_1 q_1-p_2 q_2, \tag{23}$$

where α denotes the maximum (net of all except price) willingness to pay for travelling by air.

The utility function described above gives rise to a linear demand structure, and direct demands can be written as:

$$\begin{aligned} q_1 &= a-bp_1+dp_2,\\ q_2 &= a-bp_2+dp_1, \end{aligned} \tag{24}$$

where $a=\dfrac{\alpha(1-\gamma)}{1-\gamma^2}$, $b=\dfrac{1}{1-\gamma^2}$ and $d=\dfrac{\gamma}{1-\gamma^2}$. Notice that given our assumptions, the parameters a, b, and d are strictly positive.

In this context, a benevolent regulator must decide the access price $\overline{\mu}_a$ to be charged to private airlines for the use of the public airport. Given this access price, airlines compete in prices with differentiated products. In particular, each airline i maximizes its own profits and, thus, airline i solves the following maximization program:

$$\underset{p_i}{Max}\; \pi_i=(p_i-c_a-\overline{\mu}_a)q_i, \tag{25}$$

where q_i is given by expression (24), with $i = 1, 2$.

Privately optimal ticket prices are then given by:

$$\overline{p}_1^* = \overline{p}_2^* = \frac{1}{2b-d}(a + b\mu_a + bc_a). \tag{26}$$

If there is no rail infrastructure, a benevolent regulator chooses the access price for the use of the public airport in order to maximize social welfare. In this case, willingness to pay for capacity per individual is defined as the sum of individual consumer surplus, airlines' surplus per individual and public airport's surplus per individual minus individual environmental costs. In order words:

$$\overline{WTP}^{capacity} = \alpha q_1 + \alpha q_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) - (A - C_a - c_a)(q_1 + q_2). \tag{27}$$

Social welfare is then defined by:

$$\overline{SW} = N\,\overline{WTP}^{capacity} - rK_a. \tag{28}$$

If there is no rail infrastructure, a welfare-maximizing regulator chooses the access price $\overline{\mu}_a$ in order to maximize the social welfare given by expression (28). Thus, the optimal access price is given by:

$$\overline{\mu}_a^* = A + C_a - (1-\gamma)(\alpha - SMC_a). \tag{29}$$

In this social optimum, tickets prices and quantities are given by:

$$\begin{aligned} \overline{p}_1^* &= \overline{p}_2^* = SMC_a, \\ \overline{q}_1^* &= \overline{q}_2^* = \frac{1}{1+\gamma}(\alpha - SMC_a). \end{aligned} \tag{30}$$

Thus, if there is no rail infrastructure the maximum social welfare that can be achieved is given by:

$$\overline{SW}^* = \frac{N}{1+\gamma}(\alpha - SMC_a)^2 - rK_a, \tag{31}$$

where $\overline{WTP}^{capacity*} = \frac{1}{1+\gamma}(\alpha - SMC_a)^2$ is the maximum willingness to pay for capacity per individual.

Let us now compare the maximum social welfare that can be achieved if there is no rail infrastructure, $\overline{SW}^*$, with the maximum social welfare when there is a rail operator,

SW^*. Let $SW^* - \overline{SW}^*$ represent the gain in social welfare due to the existence of a rail infrastructure and let $WTP^{capacity*} - \overline{WTP}^{capacity*}$ denote the difference in the maximum individual willingness to pay for capacity due to the existence of a rail infrastructure. Then, the gain in social welfare due to the existence of a rail infrastructure is given by:

$$SW^* - \overline{SW}^* = N(WTP^{capacity*} - \overline{WTP}^{capacity*}) - rK_t, \tag{32}$$

where $WTP^{capacity*} - \overline{WTP}^{capacity*} = \frac{1}{2b_t}(q_t^*)^2$.

Denoting by ε_t^* the demand elasticity of the rail operator with respect to his own price evaluated in the social optimum, the difference in the maximum individual willingness to pay for capacity due to the existence of a rail infrastructure can be rewritten as:

$$WTP^{capacity*} - \overline{WTP}^{capacity*} = \frac{1}{2}\frac{p_t^* q_t^*}{|\varepsilon_t^*|}. \tag{33}$$

Proposition 6: *If the regulator is a welfare-maximizer, in the social optimum the society is always willing to pay more for the existence of a rail infrastructure.*

Proposition 6 states that, if the regulator is a welfare-maximizer, the difference in the maximum individual willingness to pay for capacity due to the existence of a rail infrastructure is always positive. However, this might not be enough. In order to construct today the rail infrastructure we need the society to be willing to pay for the extra costs, that is, we need a gain in social welfare due to the existence of the rail infrastructure.

Proposition 7: *If the regulator is a welfare-maximizer the rail infrastructure must be constructed today if there is a gain in social welfare for the cases in which the new facility is and is not constructed and optimal pricing is applied, that is,* $SW^* - \overline{SW}^* > 0$. *In other words, the rail infrastructure must be constructed today iif and only if:*

$$\frac{1}{2}\frac{p_t^* q_t^*}{|\varepsilon_t^*|} > \frac{rK_t}{N}. \tag{34}$$

The social profitability of investing in the new infrastructure depends on the difference between the individual willingness to pay for capacity in the case in which there is a rail infrastructure and the case where the rail infrastructure is not constructed, summarized by the left hand side fraction of condition (34), the cost of the infrastructure and the opportunity cost of capital. The higher the private revenues generated by the rail infrastructure and the lower the demand elasticity of the train, the more likely is the investment to be welfare enhancing. Given these values, a key parameter is the population (*N*) served by the new infrastructure. High individual willingness to pay, low opportunity cost of capital and low construction costs are not enough unless we have a reasonable level of demand,[5] and this critically depends on geographic and demographic conditions.

Notice that, in order to be optimal to construct the rail infrastructure today, we not only need to have a positive social welfare. What we need is the investment to be welfare enhancing, that is, induce a higher social welfare than in the case in which there is no rail infrastructure. This is summarized in the following corollary.

Corollary 1: *A positive social welfare for the case in which a rail infrastructure is constructed,* $SW^* > 0$ *is a necessary but not sufficient condition. The necessary and sufficient condition implies a positive difference in social welfare for the cases in which the rail is and is not constructed and optimal pricing is applied, that is,* $SW^* - \overline{SW}^* > 0$.

All the conditions found in this section imply the availability of public funds without restrictions. In particular, we have shown that for a welfare-maximizing regulator socially optimal access prices may be set below marginal operating and maintenance costs. However, the idea of a regulator subsidizing private operators for the use of public infrastructures may be inacceptable, unfeasible or even inefficient. Thus, in the next subsection we consider a benevolent regulator that maximizes social welfare but is subject to cost recovery.

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5 From actual construction, rolling stock, maintenance and operating costs of European HSR lines, average values of time, a reasonable range of potential travel time savings, and a five per cent discount rate, de Rus and Nombela (2007) find that HSR investment is difficult to justify when the expected first year demand is below 8–10 million passengers for a line of 500 km, an optimal length for HSR to compete with road and air transport.

5.2. Rail infrastructure investment for a budget-constrained welfare-maximizing regulator

Consider as a benchmark the case in which there is no rail infrastructure. If there is no rail infrastructure, a regulator that maximizes social welfare but must achieve financial breakeven solves the following maximization program:

$$\begin{aligned} &\underset{\mu_a,\mu_t}{Max} \quad \overline{SW} = N\ \overline{WTP}^{capacity} - rK_a \\ &s.t. \qquad N[(\mu_a - C_a)(q_1+q_2)] - rK_a \geq 0, \end{aligned}$$

that is:

$$\begin{aligned} &\underset{\mu_a,\mu_t}{Max} \quad \overline{SW} = N[\alpha q_1 + \alpha q_2 - \frac{1}{2}(q_1{}^2 + q_2{}^2 + 2\gamma q_1 q_2) - (A - C_a - c_a)(q_1+q_2)] - rK_a \\ &s.t. \qquad N[(\mu_a - C_a)(q_1+q_2)] - rK_a \geq 0. \end{aligned} \tag{35}$$

If the optimal solutions for a welfare-maximizing regulator in the benchmark case in which there is no rail infrastructure satisfy the condition given by expression (35), that is:

$$(\overline{\mu}_a^* - C_a)(\overline{q}_1^* + \overline{q}_2^*) - (rK_a)/N \geq 0, \tag{36}$$

then, the maximum social welfare $\overline{SW}^*$ obtained by a welfare-maximizing regulator if there is no rail infrastructure coincides with the maximum social welfare obtained by a budget-constrained welfare-maximizing regulator if there is no rail infrastructure .

Let us denote by $\overline{wtp}^{capacity*}$ the maximum willingness to pay for capacity per individual and by $\overline{sw}^*$ the maximum social welfare achieved by a budget-constrained welfare-maximizing regulator both if there is no rail infrastructure and the condition given by expression (36) is not satisfied. Then, we can conclude that $\overline{wtp}^{capacity*} < \overline{WTP}^{capacity*}$, and thus, $\overline{sw}^* < \overline{SW}^*$.

Let us now compare the maximum willingness to pay for capacity per individual if there is no rail infrastructure with the maximum willingness to pay for capacity per individual when there is a rail operator and the regulator is subject to cost recovery. We can distinguish the following cases:

Case 1: Conditions (19), (20), and (36) are satisfied. Then, the difference between the maximum willingness to pay for capacity per individual when there is a rail operator and the maximum willingness to pay for capacity per individual if there is no rail infrastructure and the regulator is subject to cost recovery is given by: $WTP^{capacity*} - \overline{WTP}^{capacity*} = \frac{1}{2}\frac{p_t^* q_t^*}{\left|\varepsilon_t^*\right|}$.

Case 2: Condition (36) is satisfied but conditions (19) and/or (20) is not satisfied. Then, the difference between the maximum willingness to pay for capacity per individual when there is a rail operator and the maximum willingness to pay for capacity per individual if there is no rail infrastructure and the regulator is subject to cost recovery is given by: $wtp^{capacity*} - \overline{WTP}^{capacity*} < WTP^{capacity*} - \overline{WTP}^{capacity*}$.

Case 3: Conditions (19) and (20) are satisfied but condition (36) is not satisfied. Then, the difference between the maximum willingness to pay for capacity per individual when there is a rail operator and the maximum willingness to pay for capacity per individual if there is no rail infrastructure and the regulator is subject to cost recovery is given by: $WTP^{capacity*} - \overline{wtp}^{capacity*} > WTP^{capacity*} - \overline{WTP}^{capacity*} > 0$.

Case 4: Condition (36) and at least one of the conditions (19) and (20) are not satisfied. Then, the difference between the maximum willingness to pay for capacity per individual when there is a rail operator and the maximum willingness to pay for capacity per individual if there is no rail infrastructure and the regulator is subject to cost recovery is given by: $wtp^{capacity*} - \overline{wtp}^{capacity*}$.

Proposition 8: *If the regulator is a budget-constrained welfare-maximizer, in the social optimum consumers may be willing to pay for capacity less when there exists a rail infrastructure, that is,* $wtp^{capacity*} - \overline{wtp}^{capacity*} < 0$.

Proof: To prove that this possibility can indeed arise, suppose the following counter example. Counter example: Suppose $A = 0$ and $T = 0$ so conditions (19), (20) and (36) are not satisfied (case 4) and in the optimum all restrictions are binding. Suppose the following values for the parameters: $\gamma = 0.8$, $\delta = 0.5$, $\alpha = 212$, $\beta = 190$, $c_a = 40$, $c_t = 20$, $C_a = 10$, $C_t = 20$, $N = 250000$, $(rK_t)/N = 2000$, and $(rK_a)/N = 200$. The following table summarizes the results:

Table 1: Budget-constrained optimal solutions in Counter example

Budget-constrained optimal solutions	There is a rail infrastructure	There is not a rail infrastructure (benchmark case)
Access prices	$\mu_a = 135.26,\ \mu_t = 48.548$	$\overline{\mu_a} = 11.344$
Quantities	$q_1 = q_2 = 0.79713,\ q_t = 70.058$	$\overline{q}_1 = \overline{q}_2 = 74.378$
Willingness to pay for capacity per individual	$wtp^{capacity*} = 11853$	$\overline{wtp}^{capacity*} = 14141$
Gain in willingness to pay for capacity per individual due to the existence of rail	$wtp^{capacity*} - \overline{wtp}^{capacity*} = -2288 < 2000$	
Social welfare	$sw^{*} = 2413250000$	$\overline{sw}^{*} = 3485250000$
Gain in social welfare due to the existence of rail	$sw^{*} - \overline{sw}^{*} = -1072000000$	

This completes the proof. ■

Let us now compare the decision whether to construct or not the rail infrastructure for a welfare-maximizing regulator and a budget-constrained welfare-maximizing regulator. In general, the rail infrastructure must be constructed today if the gain in willingness to pay for capacity for the society is higher than the cost of the investment, that is, if there is a positive gain in social welfare due to the existence of a rail infrastructure. In order to illustrate the differences between a welfare-maximizing regulator and a regulator subject to cost recovery, let us consider the above counter example. In particular, let us now compare the budget-constrained optimal results in Table 1 with the optimal results for a welfare-maximizing regulator.

Table 2: Optimal solutions for a welfare-maximizing regulator in Counter example

Optimal solutions without budget restrictions	There is a rail infrastructure	There is not a rail infrastructure (benchmark case)
Access prices	$\mu_a = -13.2,\ \mu_t = -40$	$\overline{\mu_a} = -22.04$
Quantities	$q_1 = q_2 = 66.923,\ q_t = 83.077$	$\overline{q}_1 = \overline{q}_2 = 90$
Willingness to pay for capacity per individual	$WTP^{capacity*} = 17072$	$\overline{WTP}^{capacity*} = 14580$
Gain in willingness to pay for capacity per individual due to the existence of rail	$WTP^{capacity*} - \overline{WTP}^{capacity*} = 2492 > 2000$	
Social welfare	$SW^* = 3718000000$	$\overline{SW}^* = 3595000000$
Gain in social welfare due to the existence of rail	$SW^* - \overline{SW}^* = 123000000$	

Comparing the results in Table 1 and Table 2 we can conclude that for a welfare-maximizing regulator the rail infrastructure must be constructed today since the gain in social welfare is higher than the cost of the investment. However, for a budget-constrained welfare-maximizing regulator the existence of a rail infrastructure implies a loss in social welfare and it should never be constructed.[6] In general, we can distinguish the following cases.

Corollary 2: *If conditions* (19), (20)*, and (36) are satisfied (case 1)*, *the decision whether to construct or not the rail infrastructure coincides for a welfare-maximizing regulator and for a welfare-maximizing regulator subject to cost recovery.*

Corollary 3: *If condition (36) is satisfied but conditions* (19) *and/or* (20) *is not satisfied (case 2), it might be the case that a welfare-maximizing regulator decides to construct the rail infrastructure but a budget-constrained welfare-maximizing regulator does not, that is:* $sw^* - \overline{SW}^* < 0 < SW^* - \overline{SW}^*$.

Corollary 4: *If conditions* (19) *and* (20) *are satisfied but condition (36) is not satisfied (case 3), it might be the case that a budget-constrained welfare-maximizing regulator*

[6] Results in Table 1 suggest that under budget constraints, even if we do not take into account the cost of the investment, the existence of a rail infrastructure might be welfare detrimental if fixed and marginal maintenance costs are too high.

decides to construct the rail infrastructure but a welfare-maximizing regulator does not, that is: $SW^* - \overline{sw}^* > 0 > SW^* - \overline{SW}^*$.

Corollary 5: *If condition (36) and at least one of the conditions* (19) *and* (20) *are not satisfied (case 4), it might be the case that a budget-constrained welfare-maximizing regulator decides to construct the rail infrastructure but a welfare-maximizing regulator does not, that is:* $sw^* - \overline{sw}^* > 0 > SW^* - \overline{SW}^*$. *However, it might be also the case that a welfare-maximizing regulator decides to construct the rail infrastructure but a budget-constrained welfare-maximizing regulator does not, that is:* $sw^* - \overline{sw}^* < 0 < SW^* - \overline{SW}^*$.

6. Conclusions

In this paper we analyze the optimal conditions for investing in a complementary or substitute new facility. The industry is characterized by a vertical structure, that is, each facility is owned by a firm that provides the infrastructure to downstream firms which in turn produce output for final consumers. Although we use as an example the case of airports and rail infrastructure, we would like to highlight that the main conclusions of the paper regarding access prices and investment can be also applied to other public infrastructures with differentiated products considered either as substitutes or complements by consumers.

One of the main conclusions of this paper is that the decision on the optimal access price to be charged for the use of a particular infrastructure must be taken by considering the existence of intermodal substitutions or complementarities. This result has important implication in terms of the institutional design of public agencies such as the ministry of transport in many countries, where the division of management units is usually based on technological and engineering characteristics (road, air or rail) with decisions taken in isolation and without considering the overall picture and the important cross-effects between different modes of transport.

Another important result of the paper is related with the decision on whether to construct or not today a particular infrastructure. In this regard, traditional cost-benefit analysis needs to predict ticket prices and quantities. However, access prices strongly affect operators' profits and consumer surplus and hence ticket prices and quantities. Thus, access prices must be considered in any cost-benefit analysis. Moreover, optimal access prices vary depending on the restrictions faced by the regulator and thus, such restrictions must be taken into account in cost-benefit analysis.

In this paper we show that even in the case in which society´s willingness to pay for the construction and operation of rail infrastructure is higher than the investment, investing today is not necessarily the best option. If the base case implies no optimal pricing, the positive net present value of the investment is not a sufficient condition for implementing the project. The necessary and sufficient condition implies a positive difference in social welfare for the cases in which the new infrastructure is and is not constructed, with optimal pricing being applied in both cases. This is not a result derived from the presence of uncertainty and irreversibility but from the interaction of access pricing and investment decisions and the need to consider as a benchmark the case in which social welfare is maximized, that is, the case in which the infrastructure is not constructed and optimal pricing is considered.

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Appendix

Proof of Lemma 1: We need to check the sign of the following partial derivatives:

$$\frac{\partial p_1^*}{\partial \mu_a} = \frac{\partial p_2^*}{\partial \mu_a} = -b_a \frac{b_t}{d_t^2 - 2b_a b_t + d_a b_t}, \quad \frac{\partial p_t^*}{\partial \mu_t} = \frac{1}{2}\frac{d_a b_t - 2b_a b_t}{d_t^2 - 2b_a b_t + d_a b_t}.$$

We know that b_a and b_t are strictly positive. Given the definitions of b_a, d_a, b_t, and d_t, we have that: $d_t^2 - 2b_a b_t + d_a b_t = -\frac{1}{(1-\gamma)(1+\gamma-2\delta^2)^2}(2+\gamma-\gamma^2-2\delta^2)$, which given our assumptions is clearly negative; $d_a b_t - 2b_a b_t = \frac{1+\gamma}{(1-\gamma)(1+\gamma-2\delta^2)^2}(\delta^2+\gamma-2)$, which given our assumptions is clearly negative. Thus, $\frac{\partial p_1^*}{\partial \mu_a} = \frac{\partial p_2^*}{\partial \mu_a} > 0$ and $\frac{\partial p_t^*}{\partial \mu_t} > 0$, as we wanted to prove. ■

Proof of Lemma 2: We need to check the sign of the following partial derivatives:

$$\frac{\partial p_1^*}{\partial \mu_t} = \frac{\partial p_2^*}{\partial \mu_t} = -\frac{1}{2}b_t \frac{d_t}{d_t^2 - 2b_a b_t + d_a b_t}, \quad \frac{\partial p_t^*}{\partial \mu_a} = -b_a \frac{d_t}{d_t^2 - 2b_a b_t + d_a b_t}.$$

We know that b_a and b_t are strictly positive while d_t may be positive or negative depending on whether airlines and the railway are substitutes or complements, respectively. Given the definitions of b_a, d_a, b_t, and d_t, we have that: $d_t^2 - 2b_a b_t + d_a b_t = -\frac{1}{(1-\gamma)(1+\gamma-2\delta^2)^2}(2+\gamma-\gamma^2-2\delta^2)$, which given our assumptions is clearly negative. Thus, $\frac{\partial p_1^*}{\partial \mu_t} = \frac{\partial p_2^*}{\partial \mu_t} > 0$ and $\frac{\partial p_t^*}{\partial \mu_a} > 0$ if airlines and the railway are substitutes, while $\frac{\partial p_1^*}{\partial \mu_t} = \frac{\partial p_2^*}{\partial \mu_t} < 0$ and $\frac{\partial p_t^*}{\partial \mu_a} < 0$ if airlines and the railway are complements. This completes the proof. ■

Proof of Proposition 2: We need to check the sign of the following derivatives:

$$\frac{\partial \mu_a}{\partial \alpha} = -\frac{1-\gamma}{1-\delta^2} < 0; \quad \frac{\partial \mu_t}{\partial \beta} = -1 < 0; \quad \frac{\partial \mu_a}{\partial A} = \frac{\partial \mu_a}{\partial C_a} = \frac{2-\gamma-\delta^2}{1-\delta^2} > 0; \quad \frac{\partial \mu_a}{\partial c_a} = \frac{1-\gamma}{1-\delta^2} > 0;$$

$$\frac{\partial \mu_t}{\partial T} = \frac{\partial \mu_t}{\partial C_t} = 2 > 0; \quad \frac{\partial \mu_t}{\partial c_t} = 1 > 0; \quad \frac{\partial \mu_a}{\partial \beta} = \delta \frac{1-\gamma}{1-\delta^2} > (<)0 \text{ if } \delta > (<)0;$$

$$\frac{\partial \mu_t}{\partial \alpha} = 2\frac{\delta}{1+\gamma} > (<)0 \text{ if } \delta > (<)0; \quad \frac{\partial \mu_a}{\partial T} = \frac{\partial \mu_a}{\partial C_t} = \frac{\partial \mu_a}{\partial c_t} = -\delta \frac{1-\gamma}{1-\delta^2} < (>)0 \text{ if } \delta > (<)0;$$

$$\frac{\partial \mu_t}{\partial A} = \frac{\partial \mu_t}{\partial C_a} = \frac{\partial \mu_t}{\partial c_a} = -2\frac{\delta}{1+\gamma} < (>)0 \text{ if } \delta > (<)0.$$ This completes the proof.■

Proof of Proposition 3: Substituting the socially optimal access prices given by expression (14) into the optimal ticket prices given by expression (8), we get that $p_1^* = p_2^* = A + C_a + c_a$ and $p_t^* = T + C_t + c_t$. This completes the proof. ■

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