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**Evaluating Large Projects when there are
Substitutes:
Looking for Possible Shortcuts**

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Abstract

This paper discusses how to evaluate a large project when there is a substitute. The new large project causes discrete price adjustments in the substitute market. For example, a new high-speed rail may shift the demand curve for flight tickets to the left and reduce their price, in turn shifting the demand curve for train tickets to the left. There are several different ways to handle this complication, and we hopefully provide some guidance how to proceed. In particular, we point at an approach that captures the general equilibrium effects of a considered project in its output market. In theory at least, this approach provides a simple shortcut in cost–benefit analysis of (infrastructure and other) projects that are so large that they have a noticeable impact on equilibrium prices in other markets. A similar shortcut for transport projects that affect time costs is also supplied.

Keywords: Cost–benefit analysis; large projects; substitutes; time costs.
JEL classification: D61; H43; R40

1 Introduction

Most manuals on cost–benefit analysis provides only evaluation rules for small or marginal policy changes. The theoretical underpinnings often seem unclear. Do they consider marginal projects in the mathematical sense or are the rules rough approximations, perhaps based on textbook treatments? Apparently, the scale of a project may have a tremendous impact on how to evaluate it. The study by Dietz and Hepburn (2013) makes this obvious. For discussion how to assess infrastructure projects, including high speed rail, the reader is referred to de Rus (2010, Ch. 10.2-10.3). The current paper adds to the literature by deriving cost–benefit rules for projects that are so large that they cause significant price changes in ”secondary” markets.

The paper is structured as follows. In Section 2 the model is introduced. As a point of departure for the analysis, a ”small” or marginal project is introduced. Section 3 evaluates a large project that impacts on the equilibrium price in a secondary market. A simple cost–benefit rule incorporating all direct and induced effects in a single market is derived. A simple data-saving shortcut or approximation is added. In Section 4 we turn to a project involving time costs, i.e., a transport sector investment. Once again, we derive a compact measure that captures all direct and induced effects in a single market and suggest a simple shortcut that can be used to approximate the measure. Section 5 adds a few concluding remarks. Numerical illustrations are collected in an Appendix. Section A.1 provides a numerical illustration based on a CES indirect utility function. Section A.2 uses a simple general equilibrium model to illustrate different ways of evaluating a large project involving travel time savings.

2 The Model and a Rule for a Marginal Project

In deriving our cost–benefit rules, we set aside distributional issues. Therefore, there is just a single, representative household in the economy. The indirect utility function of this household serves as the social welfare function:

$$V = V(p^z, p^x, p^c, w, y + \pi^z(\cdot) + \pi^x(\cdot) + \pi^c(\cdot) + T) = V(p^z, p^x, p^c, w, m) = V(\mathbf{p}, m) \quad (1)$$

where p^z denotes the consumer price of z , the commodity of interest here, p^x denotes the consumer price of a substitute, denoted x , of z , p^c denotes the consumer price of an aggregate composite commodity consisting of all commodities except z and x (and the numéraire whose price is normalized to unity and suppressed), w denotes the producer price of a homogeneous input, $y \geq 0$ denotes a lump-sum income (not specified here), π^z denotes profit income earned by the representative firm producing z , π^x denotes profit income earned by the representative firm producing x , π^c denotes profits earned by the representative firm supplying the composite commodity x^c , and T denotes a lump-sum (tax) revenue. For the moment, all taxes are set equal to zero.

Throughout, z is considered to be an exogenous policy variable. The profit of the (private sector or public sector) firm supplying z is assumed to be:

$$\pi^z(\cdot) = p^z \cdot z - w \cdot l^z = p^z \cdot z - C(z), \quad (2)$$

where l^z denotes demand for the single input, and $C(\cdot)$ is a short-cut for the cost function. Producers of other commodities are price-takers in all markets and maximize profits using l as the single input. Their profit functions are $\pi(p^i, w)$ with $i = x, c$.

In order to provide a simple point of departure for the evaluation of discrete projects, let us consider a marginal shift in z evaluated at z^0 :

$$\begin{aligned} dV &= \frac{\partial V}{\partial p^z} \frac{\partial p^z}{\partial z} dz + \frac{\partial V}{\partial p^x} \frac{\partial p^x}{\partial z} dz + \frac{\partial V}{\partial p^c} \frac{\partial p^c}{\partial z} dz + \frac{\partial V}{\partial w} \frac{\partial w}{\partial z} dz = \\ &V_m \cdot [(-z^{d0} + z^0)dp^z + p^z dz - w dl^z] = V_m \cdot [p^z - C_z(\cdot)]dz, \end{aligned} \quad (3)$$

where V_m denotes the marginal utility of lump-sum income, and $C_z(z) = w dl^z$. If the initial price p^{z0} of z clears the z -market, the supply z^0 equals demand z^{d0} , explaining why the first term within brackets in the left-hand expression of the second line of (3) vanishes. The remaining markets are also in equilibrium. Therefore, the terms $(\partial V / \partial p^x)(\partial p^x / \partial z)dz$ and so on all equal zero; the same holds true for the input l because supply of the factor equals aggregate demand. If the firm providing z maximizes profits, the entire expression would equal zero because price would equal marginal cost in the right-hand side expression of (3).

A conventional, i.e., a monetary, cost-benefit rule is obtained by multiplying through equation (3) by $1/V_m$, the exchange rate between units of utility and monetary units. Thus, the cost-benefit rule signals that provision of the commodity should be increased as long as the price of the commodity, i.e., the marginal willingness-to-pay (WTP), exceeds the marginal cost of providing the commodity.¹ This cost-benefit rule is easily extended so as to account for more general and complex evaluation situations. However, the aim of this note is to focus on some other evaluation issues. In any case, and setting distributional matters aside, it is straightforward to show that dV/V_m reflects the net WTP or the compensating variation (or the equivalent variation) for the considered project. Figure 1 provides an illustration where the firm supplies z^0 units. At this quantity the equilibrium price p^{z0} exceeds the marginal cost $C_z(z^0)$. However, if the firm supplies z^* units, price p^{z*} equals marginal cost $C_z(z^*)$. For further illustration, the reader is referred to Johansson and Kriström (2016, 2018).

¹If z was a pure public good, p^z could be interpreted as the marginal WTP for the good. To obtain the aggregate marginal WTP, one would have to sum across households.

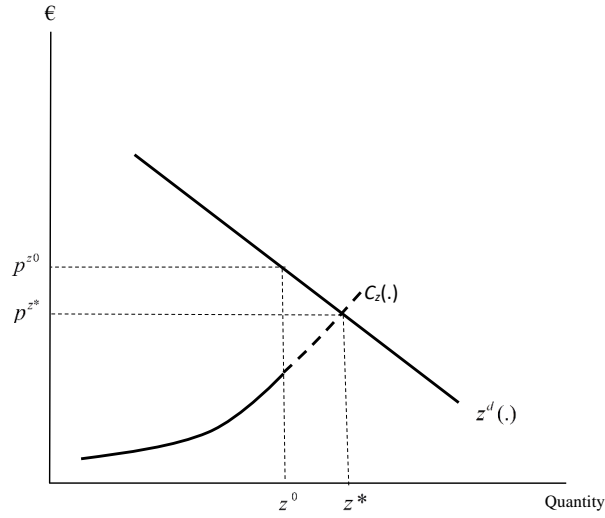


Figure 1: A marginal increase in the provision of z .

3 A Discrete Project

We now leave the marginal project and turn attention to a large or discrete project. The project is interpreted as a discrete change of z from z^0 to z^1 . This causes a change in the general equilibrium price/income vector from $[\mathbf{p}^0, m^0]$ to $[\mathbf{p}^1, m^1]$. The resulting compensating variation is implicitly defined by the following equation:

$$V(p^{z^1}, p^{x^1}, p^{c^1}, w^1, m^1 - CV) = V(\mathbf{p}^0, m^0). \quad (4)$$

CV denotes the maximal (positive or negative) WTP for the project. Thus, the household pays or receives compensation for the whole "package" of changes.

In order to focus on the project and its substitute, it is assumed that it only marginally affects the price of the composite commodity and the price of the input in the way elaborated upon in Section 2.

Obviously, one could alternatively use the expenditure function to define CV :

$$CV = m^1 - m^0 + e(\mathbf{p}^0, V^0) - e(\mathbf{p}^1, V^0), \quad (5)$$

where $e(\cdot)$ denotes an expenditure function, and V^0 denotes the initial level of welfare. A particular problem not further addressed in this paper is the fact that ordinary demand functions, say, $z^d(\mathbf{p}, m)$, typically differ from their compensated counterparts, say, $z^{dH}(\mathbf{p}, V^0)$; a quasi-linear utility function provides the exception. Hence, in general, the compensated equilibrium with the project will differ from the market equilibrium; the general equilibrium prices will differ. Here, we simply assume that income effects are so small that the two equilibria are very close to each other.

It is extremely difficult to estimate a social welfare function or even an expenditure function. Therefore, we will take a look at some different ways of estimating CV .

1. A first variation solves equation (4) for CV and provide both an exact measure and a useful approximation that is easier to estimate in empirical applications:

$$\begin{aligned} CV = & - \int_{p^{z0}}^{p^{z1}} z^{dH}(p^z, p^{x0}, \dots) dp^z - \int_{p^{x0}}^{p^{x1}} x^{dH}(p^x, p^{z1}, \dots) dp^x + \Delta\pi^z + \Delta\pi^x \approx \\ & - [2 \cdot z^{dH0} + \Delta z^{dH}] \frac{\Delta p^z}{2} - [2 \cdot x^{dH1} + \Delta x^{dH}] \frac{\Delta p^x}{2} + \Delta m, \end{aligned} \quad (6)$$

where a superscript H refers to a Hicksian demand function, $\Delta p^i = p^{i1} - p^{i0}$ for $i = z, x$, the WTP/WTa for a change in lump-sum income equals the change itself, and in the right-hand side expression linear (Hicksian) consumer surplus measures are used to approximate WTP/WTa. The compensated consumer surplus triangle plus the change in consumer surplus on the initial units reduce to the rule of half. The integrals in (6) are line integrals. Therefore, we choose to integrate $z^{dH}(\cdot)$ holding all relative prices but p^z at their initial levels. Then, we integrate $x^{dH}(\cdot)$ holding p^z at its final level (and p^c and w at their initial levels).

A graphical illustration is found in Figures 2 and 3. Here we consider the entry of the z -commodity in the market. This assumes that the commodity is non-essential so that it is possible to derive positive overall utility even when $z = 0$. The equilibrium price is reduced from the choke price p^{z0} along the demand curve to the right in the figure. The dotted upward sloping line indicates a supply curve (just indicated because it is not needed as is explained below), and the provision of z is exogenous and need not, but could be, compatible with profit maximization. The price reduction causes the demand curve for the substitute depicted in Figure 3 to shift to the left, reducing the equilibrium price

in that market. In turn, this shifts the demand curve for z to the left in Figure 2. This illustrates that the two markets are interrelated. The final equilibrium is given by $[p^{z^1}, z^1, p^{x^1}, x^1]$. A simple example is provided by a new high-speed rail causing the demand curve for flights to shift to the left and reducing the price of flight tickets, in turn causing the demand curve for train tickets to shift to the left.

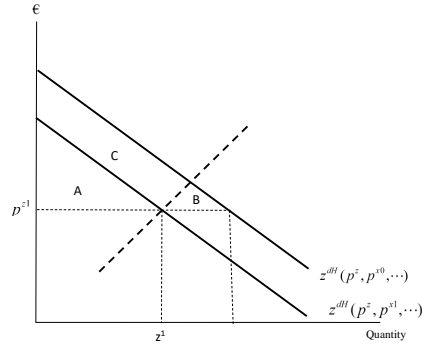


Figure 2: Calculating the consumer surplus change in the z -market.

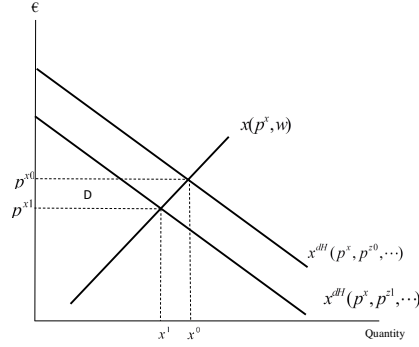


Figure 3: Calculating the consumer surplus change in the x -market.

The compensated consumer surplus in Figure 2 equals the area to the left of the outer demand curve between the choke price and the new equilibrium price p^{z^1} , i.e., Area A+B+C. Turning to the market for the other commodity, the change in compensated consumer surplus is evaluated conditional on $p^z = p^{z^1}$, i.e., to the left of the inner demand curve in Figure 3. This area is denoted D in the figure. Summing the areas, we obtain an estimate of the overall WTP for the project.

The path of integration can be reversed without impact on the total compensated consumer surplus. Then, we evaluate the consumer surplus in the x -market holding all other prices at their initial levels, i.e., to the left of the outer demand curve in Figure 3. Next, the surplus in the z -market is evaluated conditional on p^x held at its final level, i.e., to the left of the inner demand curve in Figure 2 (to obtain Area A). Although the individual areas differ in magnitude if we take this path rather than the initial one, the sum will equal A+B+C+D (although this need not be the case in the hand-drawn figures). The reason is the fact that indirect utility functions and expenditure functions typically are assumed to have symmetric cross derivatives on their domains. This is a necessary and almost sufficient condition for path independency of a line integral, i.e., that the value of the evaluated function (integral) only depends on the end-points and is independent of the particular path between them. Adding

some mild restrictions on the open set of paths' allowed results in sufficiency.²

A particular approximation of the considered consumer surplus measure is provided by the rule of half. The concept, as applied to transport interventions, seems to date back to the late 1960s, but the first mathematical treatment of the concept is due to Williams (1976); refer to, for example, Jara-Diaz and Friesz (1982) and Winkler (2015). Consider the choke price of z (here denoted p^{ch}) where the outer demand curve in Figure 2 intersects the € -axis. Use a straight line to connect this point and the point (p^{z^1}, z^1) . Next, calculate the consumer surplus area to the left of this line segment. This is equivalent to estimating $(1/2) \cdot (z^1 + 0) \cdot (p^{ch} - p^{z^1})$ and add the corresponding measure for the other commodity in Figure 3. Thus, the consumer surplus change is underestimated (overestimated) in the z -market (x -market), while the opposite holds if the path of integration is reversed. Ultimately, it is an empirical question whether the approximation suggested by Williams (1976) is a reasonable one (but note that Williams' approximation was based on Marshallian demand concepts).

What about changes in profit incomes? The integral-loving person would integrate to the left of supply curves between initial and final prices. The lazy of us prefer to simply plug in prices in the profit expressions and calculate changes in producer surpluses to obtain $\Delta m = m^1 - m^0$. Then, we have $A+B+C+D+\Delta m = CV$, at least as long as p^c and w remain approximately unchanged in the sense explained in Section 2.

It may seem a bit surprising that price changes in the "secondary" market are accounted for. If the supply curve for the x -commodity was completely inelastic, i.e., vertical, surpluses would only be reshuffled and sum to zero. However, if capacity/production is added, we must somehow account for such changes. Nevertheless, the next approach considered in this paper seemingly neatly gets off this complication.

2. The second variation draws on the marginal project. Let us begin by stating something similar to equation (3) with p^c , w and T constant:

$$\frac{dV}{V_m} = [-z^d(\cdot) + z(\cdot)]dp^z + [-x^d(\cdot) + x(\cdot)]dp^x + [p^z - C_z(\cdot)]dz. \quad (7)$$

For the truly marginal project, supplies equal demands in equation (7), implying that only the two final terms on the right-hand side of the equation remains; compare the Envelope Theorem which stipulates that the total effect of a small parameter change can be obtained by simply taking the partial derivative of a value function (e.g. an indirect utility function) with respect to the parameter. Refer to a textbook on microeconomics, for example, Jehle and Reny (2011, pp.

²To illustrate, let $F(p^z, p^x) = (z^d(p^z, p^x), x^d(p^z, p^x))$. If $z^d(\cdot)$ and $x^d(\cdot)$ have continuous first order derivatives on a simply connected region $D \in \mathbb{R}_{>0}^2$, then the line integral $\int_C z^d(\cdot)dp^z + x^d(\cdot)dp^x$, where C is a (piecewise) smooth curve between the end-points, is path independent iff $\partial z^d(\cdot)/\partial p^x = \partial x^d(\cdot)/\partial p^z$. The term 'simply connected' can be interpreted as: for any two given points in D , there is one and "essentially" only one path connecting them. See, for example, Courant and John (1974, pp. 95-106) and Johansson (1987, p. 26 and pp.38-40). For a historical account of the concept of a line integral, see Katz (1999).

604-6071) for details. What about a non-marginal project? For such a project we must define equilibrium paths' for both prices and solve them *simultaneously* as functions of z , here considered to be exogenous; compare the equation system (A.2) in Section A.1 of the Appendix. Thus we get something like:

$$\begin{aligned} p^z &= f(z), \\ p^x &= g(z). \end{aligned} \tag{8}$$

Now, if we integrate equation (7) along the paths' defined by equation (8), market equilibrium is maintained throughout from z^0 to z^1 . Therefore the two first terms within brackets on the right-hand side of (7) are throughout equal to zero. In other words, evaluated along the optimal price paths', the sum of consumer surplus plus producer surplus changes in a market sum to zero:

$$\int_{z^0}^{z^1} [x(z) - x^d(z)] dz = \int_{z^0}^{z^1} [z - z^d(z)] dz = 0. \tag{9}$$

What remains to be evaluated in equation (7) is then the final term. Using (8) one obtains:

$$\int_{z^0}^{z^1} [f(z) - C_z(\cdot)] dz = CV, \tag{10}$$

where CV coincides with CV in equation (6). According to the approach stated in equation (10), it is not necessary to account for price changes in "secondary" markets.³ Instead one focuses on the market under scrutinization. The area under the curve traced out by the equilibrium price path in equation (10) reflects the *total* WTP for the considered project, i.e., the measure accounts for non-marginal changes in prices in secondary and other markets as well as income changes. Thus, an area of the kind illustrated in Figure 4 in Section A.1 of the Appendix provides the total WTP of the considered large project. Deducting the project's costs, as in equation (10), one arrives at a simple CBA. An early application, relating to the Swedish forest sector (pulp and paper, sawmills, and forestry) is provided by Brännlund and Kriström (1996).

A straightforward approximation of the area below the $f(z)$ -curve in equation (10) is provided by a variation of the rule of a half, based on a straight line connecting the initial and final (z, p^z) configurations, to obtain:

$$CV \approx ((1/2)(p^{z^0} + p^{z^1})(z^1 - z^0) - \Delta C). \tag{11}$$

It can be shown that one arrives at the same approximation if one calculates the area to the left of the line connecting initial and final configurations and add the change in the firm's revenue.⁴ An exact measure of the area below

³At an abstract level, the result dates back to at least Johansson (1993, Ch. 5.3), but we are unaware of any explicit treatment of the kind provided by the current paper.

⁴Use $(1/2)(p^{z^0} - p^{z^1})(z^1 + z^0)$ plus the change in revenue, $p^{z^1} \cdot z^1 - p^{z^0} \cdot z^0$, to show this. This confirms that it is correct to ignore other markets when using the rule of a half in this way.

the curve is obtained if one is able to determine and replace the prices in (11) by $p^{z^M} \in (p^{z^0}, p^{z^1})$ such that the rule of a half is turned into an equality (a simple application of the Integral Mean Value Theorem). If p^x remains more or less constant, $f(z)$ reduces to the inverse demand curve for z . In such cases, it is legitimate to assess the change in (compensated) consumer surplus in the z -market (and add the change in the operator's revenue to obtain the total WTP). When other prices change, the above discussion shows that there is in principle an infinite number of consumer surplus measures, depending on the order of integration. Hence, in such more general cases it seems meaningless to interpret (10) in terms of compensated consumer surplus changes.

Figure 4 in Section A.1 of the Appendix provides a graphical illustration of how a large project might be assessed in a single market. The section also provides a few suggestions how the measure could be approximated.

3. A straightforward variation of the approach presented above is to take the derivative of the indirect utility function with respect to z and integrate along the path's solved for in equation (8). This yields the same cost-benefit rule as the left-hand side expression in equation (10).

The cost-benefit rules derived in this section can easily be generalized so as to account for non-marginal changes in p^c and w . Changes in p^c are handled in the same way as changes in p^x . Changes in the supply of the input (labor) is evaluated to the left of a compensated supply curve between initial and final input prices. However, as above, one must follow a particular path of integration. Changes in profits are most easily evaluated by plugging in final and initial prices in the profit expressions, but one could alternatively integrate the relevant supply and demand functions. The approach chosen in 2) and 3) above would require an extension of equation (8) from two to four equations. Finally, we ignore discussing the treatment of taxes because there are different approaches, for example the one used by the European Commission, see European Commission (2014), and the EIB, see European Investment Bank (2013), and the one used in Johansson and Kriström (2016).

4 Time Costs

An important class of infrastructure projects cause changes in travel costs. These costs could be the direct monetary travel (ticket) cost as well as time costs. Adding these one arrives at the generalized travel cost. In this section we take a brief look at large changes in generalized travel costs. Drawing on de Rus and Johansson (2018) we consider a simple indirect utility function, basically replicating much of the previous discussion:

$$V(p^z + v \cdot t, p^x, p^c, w, m) = V(\mathbf{p}, m), \quad (12)$$

where p^z now denotes the ticket, v denotes the valuation of time, and t travel time. The generalized travel cost is defined as $g = p^z + v \cdot t = p^z + tc$, where tc denotes the time cost. Taking the partial derivative of (12) with respect to tc

and multiplying by dvc , one obtains:

$$\frac{\partial V(\cdot)}{\partial vc} dvc = -[V_m \cdot z^d(\cdot)] dvc. \quad (13)$$

An increase in travel time (t) and/or the cost (v) per time unit causes a decrease in welfare. The typical transport project changes the travel time leaving v unchanged, i.e., corresponds to the case for which $dvc = vdt$ in equation (13).

Consider now a project changing the generalized travel cost from g^0 to g^1 and the price of a substitute from p^{x0} to p^{x1} , for notational simplicity suppressing any time cost for that transport mode. Just as in the previous section, there are different ways or paths of evaluation to choose among. For example, we could proceed as follows:

$$CV = - \int_{g^0}^{g^1} z^{dH}(g, p^{x0}, \dots) dg - \int_{p^{x0}}^{p^{x1}} x^{dH}(g^1, p^x, \dots) dp^x + \Delta m. \quad (14)$$

In this case, the generalized travel cost is changed. Given the change in g , the impact on the demand for the substitute is assessed, and the change in income is added. However, we could reverse the path and obtain the same overall CV . This is similar to the illustrations in the previous section; compare Figures 2 and 3.

Sometimes, one is interested in distinguishing between the impacts of the different parts of the generalized travel cost. Let us provide an illustration:

$$CV = - \int_{tc^0}^{tc^1} z^{dH}(p^{z0} + tc, p^{x0}, \dots) dvc - \int_{p^{z0}}^{p^{z1}} z^{dH}(p^z + tc^1, p^{x0}, \dots) dp^z - \int_{p^{x0}}^{p^{x1}} x^{dH}(g^1, p^x, \dots) dp^x + \Delta m. \quad (15)$$

The first integral yields the change in (compensated) consumer surplus due to any change in the time cost of trips.⁵ Given this "payment", the consumer surplus change due to the change in the ticket is captured by the second integral in the right-hand side equality. The remaining terms are the same as in equation (14). Once again, there are other paths of integration providing the same overall answer with respect to the project's social profitability. However, changing the order of integration obviously affect the magnitude of individual integrals and hence the valuation of changes in p^z and tc , respectively.

Just as in Section 3, it is possible to evaluate a small project in the following way:

$$dV/V_m = dCV = -[z^d(\cdot) - z(\cdot)] dp^z - [x^d(\cdot) - x(\cdot)] dp^x - z^d(\cdot) dvc = -z^d(\cdot) dvc, \quad (16)$$

⁵In order to evaluate a ceteris paribus change in t , multiply the integral by v , and integrate from t^0 to t^1 .

where market-clearing prices ensure that supply equals demand in both markets so that surplus changes associated with induced price changes sum to zero; compare the Envelope Theorem according to which $\partial V(\cdot)/\partial tc = -V_m \cdot z^d$ captures the total effect of a change in the time cost. Next, solving for the equilibrium price paths in the way suggested by equation (8), one can change tc in a discrete way while preserving market equilibria throughout the change. Thus, one obtains:

$$CV = - \int_{tc^0}^{tc^1} z^{dH}(f(tc) + tc, \dots) dtc \approx \frac{1}{2}(tc^0 - tc^1)(z^0 + z^1). \quad (17)$$

As mentioned, the surplus changes due to induced changes in p^z and p^x sum to zero when we evaluate the project along the equilibrium paths for prices. The integral evaluates the change in demand along the equilibrium path for trips as a function of the exogenous time cost. In this case, the project is evaluated as an area to the left of the equilibrium demand path between initial and final time costs. Note that CV 'incorporates' any change in costs. Hence, with the same assumptions with respect to the supply function of the representative firm supplying z , CV in equation (17) coincides with CV in equations (14) and (15); the equations only take different but permitted paths between initial and final generalized travel costs. The final line in equation (17) provides a suggestion how to easily approximate CV . According to the Integral Mean Value Theorem there is an intermediate z , $z^M \in (z^0, z^1)$, such that the approximation is turned into an equality, but z^M might be difficult to locate in an empirical CBA.

Equations (11) and (14) suggest that another variation of the rule of a half could be used to value the considered project. In this case it is based on the generalized travel cost:

$$CV \approx \frac{1}{2}(g^0 - g^1)(z^0 + z^1) + \Delta\pi^z. \quad (18)$$

The first term in the right-hand side expression approximates the change in consumer surplus and the second term captures the change in producer surplus. However, if markets are competitive, as assumed here, the measure does not reduce to the variation in the second line of equation (17), in general. The assumption of perfect competition in all markets implies that resources that produce EUR 1 in the z -market are able to produce the same value elsewhere in the economy. Hence there is no surplus to capture beyond the direct one in the "time market". Another way to arrive at this result is by noting that in equation (17) we follow a path such that the price of trips equals the marginal cost throughout from z^0 to z^1 . Applying the same assumption to the approximation formula (18), it reduces to the approximation in (17).⁶

⁶Suppose p^x remains constant (while p^z clears the z -market). Then applying (18) to (14) one arrives at the approximation in equation (17); recall that CV is the same in (14) and (17). However, if p^x adjusts, one must add (18) for the x -market (with obvious changes in notation) in order to approximate equation (17). See also the discussion following equation (A.10) in Section A.2 of the Appendix.

If the price of z differs from the marginal cost, equation (10) suggests that one should add the approximation in (11) to the one in the second line of equation (17). It is easily verified that this approximation reduces to the one in (18).

A caveat is justified. Simply adding equations (10) and (17) seems reasonable in an analysis of a project jointly affecting capacity and travel times. However, the location of the locus in equation (10) is affected by the time cost, and the location of the locus in (17) is affected by capacity. Hence, the analysis of such a project might become more complicated than just adding areas. Moreover, the relationship between capacity and travel times could be quite involved (*à la* the chicken or the egg causality dilemma). Therefore, this intriguing issue must be left for future research.

5 Conclusions

This paper is devoted to a discussion how to evaluate projects that are so large that they have a significant impact on prices in other markets.

Equation (10) suggests that cost-benefit analysis of a project could be confined to evaluating an area under a curve in its output market and deducting costs evaluated at initial prices (if factor prices are roughly constant). As is further demonstrated in Section A.1 of the Appendix, bounds for the likely magnitude of the outcome are easily established. However, the real theoretical challenge is to provide even closer approximations, ideally a shortcut just like (but hardly similar to) the Willig (1973) formula. Similarly, we have provided a discussion of transport projects involving time costs. Equation (17) in Section 4 provides a rule incorporating all direct and induced effects of a project causing a change in the time cost of a project. According to a simple general equilibrium analysis in Section A.2 of the Appendix, the shortcut provided by the second line of the equation performs extremely well.

Obviously, there might be effects, for example, externalities, market power in the secondary market, and distorting taxation which complicate an application of the measure. In effect, when market power and/or taxes are present, it might be easier to apply the rule based on, for example, equations (6) or (14), because they allow the investigator to insert initial and final prices in profit expressions and similarly for taxes rather than evaluating distorting components along (hard to estimate) equilibrium paths for prices. But this is a claim which is left for future research. The analysis has also been restricted to an economy with a single, representative household. In a multi-household society, where preferences are heterogeneous, choices are discrete, i.e. you make 0, 1, 2, ..., trips, and possibly new capacity is added, price changes could induce particular groups of travelers to either enter or leave the market. Almost trivially, the results presented in the current paper need not generalize to such more complex general equilibrium settings. Nevertheless, we believe that our results provide some guidance and ideas how to evaluate large projects.

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A Appendix

A.1 Numerical Illustration I

In order to provide an illustration of the results provided in Section 3, this section develops a simple numerical model which is used to arrive at some evaluations when demand functions are interdependent. In contrast to Section 3, this section focuses on an increase of the provision of z from a strictly positive level. The model is partial equilibrium in the sense that factor markets (and hence incomes from such factors) are ignored. This is done in an attempt to keep the analysis as simple and transparent as possible. Section A.2 introduces a simple numerical general equilibrium model which easily can be modified so as to fit the analysis in the current section.

The indirect utility function of the representative household is CES and defined as follows:

$$V(p^z, p^x, m) = m[(p^z)^r + (p^x)^r + 1]^{-1/r} \quad (\text{A.1})$$

where the price of the third good, acting as numéraire, is normalized to unity, $r \equiv \rho/(\rho - 1)$, where the parameter $0 \neq \rho < 1$ (and the two other commodities appearing in Section 3 are ignored). The income argument is $m = \pi^z + \pi^x(\cdot) = p^z \cdot z + (p^x)^2/2$. This formulation means that we initially suppress any factors of production. Refer to Jehle and Reny (2011, Ch. 1) for details on the indirect utility function in equation (A.1).

In order to determine equilibrium prices one has to solve the following system of equations:⁷

$$\begin{aligned} z^{dH} &= V^0[(p^z)^r + (p^x)^r + 1]^{-1/r-1}(p^z)^{r-1} = z, \\ x^{dH} &= V^0[(p^z)^r + (p^x)^r + 1]^{-1/r-1}(p^x)^{r-1} = p^x, \end{aligned} \quad (\text{A.2})$$

where V^0 refers to the initial level of utility, the left-hand expressions are the Hicksian demand functions for z and x , respectively, and the right-hand side expressions are the exogenous supply of z and the supply function for x ($= \partial \pi^x(\cdot)/\partial p^x = p^x$). The paths' are functions of the exogenous z and the constants. If $r = -1$ so that the commodities are substitutes, $y = 10$, $z^0 = 1$, and $z^1 = 5$, the initial and final equilibrium prices are: $p^{z0} = 2\sqrt{2}$, $p^{x0} = 2$, $p^{z1} \approx 0.80138$, and $p^{x1} \approx 1.47531$.⁸ Thus, the increase in z causes both equilibrium prices to drop. Unfortunately, Mathematica supplies an extended and very

⁷We base (A.2) on Hicksian demands because the cross-price derivatives of the Hicksian demand functions are symmetric: $\partial z^{dH}/\partial p^x = \partial x^{dH}/\partial p^z$; see, for example, Jehle and Reny (2011, Ch. 1). As noted, this is a necessary condition for path-independence. The Marshallian (CES) demand functions too have symmetric cross-price effects. However, there is an income effect because $\Delta m \neq 0$. Hence, there is an infinite number of aggregate Marshallian consumer surpluses; each corresponding to a particular income in the interval $[m^0, m^1]$.

⁸ $V^0 (\approx 27.485)$ was estimated by using the Marshallian demand functions to solve for the initial equilibrium prices, and then plugging these prices and initial income into equation (A.1).

complicated solution to (A.2) and only for particular r -values. The conditional path for p^z , given $r = -1$ and strictly positive prices, is found in (A.11) (and the solution for p^x is available upon request; p^x is monotonically decreasing in z over the considered region).

The compensating variation associated with the increase in z is implicitly determined by the following equation:

$$(m^1 - CV)[(p^{z1})^r + (p^{x1})^r + 1]^{-1/r} = m^0[(p^{z0})^r + (p^{x0})^r + 1]^{-1/r}, \quad (\text{A.3})$$

and $CV \approx 5.70065$.

Equation (A.3) includes the change in profit income $\Delta m = m^1 - m^0$. The change in profit income in the x -sector can be calculated using the profit function as $\Delta \pi^x = (p^{x1})^2/2 - (p^{x0})^2/2$ or one integrates the supply function $x(\cdot) = p^x$ between p^{x0} and p^{x1} (to obtain around -0.91173). Hardly surprising, this sector faces a loss in profit income. The change in profits of the exogenous z -sector is calculated as $\Delta \pi^z = p^{z1} \cdot z^1 - p^{z0} \cdot z^0$ (and is around 1.17847). Thus, $\Delta m \approx 0.266743$.

Another way to arrive at CV is to integrate the negative of the demand functions and add the change in income to obtain:

$$CV = \int_{p^{z0}}^{p^{z1}} V^0[(p^z)^r + (p^{x0})^r + 1]^{-1/r-1} (p^z)^{r-1} dp^z + \int_{p^{x0}}^{p^{x1}} V^0[(p^{z1})^r + (p^x)^r + 1]^{-1/r-1} (p^x)^{r-1} dp^x + \Delta m, \quad (\text{A.4})$$

where we integrate the z^{dH} -function holding p^x at its initial level p^{x0} , and then integrate the x^{dH} -function holding p^z at its final level p^{z1} . Once again, CV is around 5.70065 .

Next, let us reverse the order of integration:

$$CV = \int_{p^{z0}}^{p^{z1}} V^0[(p^z)^r + (p^{x1})^r + 1]^{-1/r-1} (p^z)^{r-1} dp^z + \int_{p^{x0}}^{p^{x1}} V^0[(p^{z0})^r + (p^x)^r + 1]^{-1/r-1} (p^x)^{r-1} dp^x + \Delta m, \quad (\text{A.5})$$

In this case too, CV is around 5.70065 .

The third approach involves integrating p^z along the path determined by equation system (A.2):

$$CV = \int_{p^{z0}}^{p^{z1}} p^z(\cdot) dz = \int_{p^{z0}}^{p^{z1}} f(z) dz, \quad (\text{A.6})$$

where $f(z)$ refers to the equilibrium path of p^z as z is changed in the way defined by the equation system (A.2). This approach replicates the one suggested by equation (8) in Section 3. If $f(z)$ as stated in equation (A.11) is integrated from $z = 1$ to $z = 5$, once again, CV is around 5.70065 .

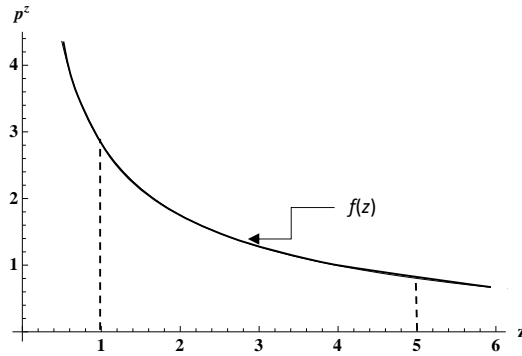


Figure 4: The equilibrium path for z when $r = -1$.

Figure 4 provides an illustration, where the equilibrium price path for z is shown, and the dotted boundary lines identify the outer edge of the region of interest. Note that we integrate under the curve from initial to final z -values, not to the left of it. A number of shortcuts or approximations of the measure are suggested.

1. An *upper bound* (a *lower bound*) is obtained by calculating the area of the rectangle given by the change in z times the initial (final) price of z .
2. The rule-of-half-approximation, $(1/2)(p^{z^0} + p^{z^1})(z^1 - z^0)$, equals the average of the aforementioned bounds (around 7.26). The curve in Figure 4 is convex to the origin. In other words, the (general equilibrium) WTP for further increases in the provision of z diminishes as z increases. This suggests that the rule of half approximation, equal to the area *under* a straight line connecting initial and final prices, *overestimates* areas of the kind depicted in Figure 4.
3. Still another approximation is obtained by drawing on the inverse demand function for z as stated in equation (A.12). Then, the suggested approximation is the average of the areas under the initial and final inverse demand curves between z^0 and z^1 , recognizing that, just as above, a *double counting* results if changes in the secondary market (however calculated) or income are added. In the considered example this measure performs extremely well underestimating CV by just 0.04 units, as is illustrated in equation (A.12).

A nice feature of the first two approximations is that they only require information of initial and/or final price-quantity configurations in the primary market. No further information is needed. In any case, deducting the project's costs from whatever (approximate) measure of CV is available, one arrives at a simple CBA of the considered project.

In this analysis input markets have been ignored. Such a partial analysis overlooks the surpluses added or deducted by price adjustments in these markets. The question is whether the shortcuts suggested above, e.g., the different rules of a half, will perform even better if general equilibrium adjustments are accounted for. Indeed, the numerical example in Section A.2 suggests this is the case.

A.2 Numerical Illustration II

In order to illustrate how to handle time costs, a simple numerical general equilibrium illustration is added. Consider the following quasi-linear indirect utility function with two commodities and labor:

$$V(\cdot) = \ln\left(\frac{1}{p^z + t \cdot w}\right) + \ln\left(\frac{1}{p^x}\right) + \pi + w \cdot \left(T - t \cdot \frac{1}{p^z + t \cdot w}\right) - p^z \cdot \frac{1}{p^z + t \cdot w} - 1, \quad (\text{A.7})$$

where the demand functions are found in the logarithms, with the first one referring to the number of trips as a function of the travel cost and the time cost, π denotes aggregate profit income, T is the time endowment, and time is split between trips and work. In this case, the time cost is related to the wage rate, i.e., the time cost of a trip is sensitive to income (displaced). A firm's profit function is assumed to be $\pi^i(\cdot) = (p^i)^2/(4 \cdot w)$ for $i=z, x$. Therefore, supply equals $p^i/(2 \cdot w)$ and demand for labor $(p^i)^2/(4 \cdot w^2)$.

A general equilibrium require that supply equals demand in all three markets. The equilibrium price paths can be obtained as functions of the exogenous time per trip variable t : Assuming that the time endowment T is equal to 20, the general equilibrium price paths are:

$$\begin{aligned} p^z(t) &= \frac{-t + \sqrt{640 + 9t^2}}{80 + t^2} \\ p^x(t) &= 0.158113883 \sqrt{\frac{320 + 3t^2 + t\sqrt{640 + 9t^2}}{80 + t^2}} \\ w(t) &= \frac{4 + 0.0375t^2 + 0.0125t\sqrt{640 + 9t^2}}{80 + t^2}. \end{aligned} \quad (\text{A.8})$$

Consider a project that reduces t from 2 to 1. The initial equilibrium prices are $p^{z0} \approx 0.285714$, $p^{x0} \approx 0.338062$, $w^0 \approx 0.0571429$, and the final ones are $p^{z1} \approx 0.302166$, $p^{x1} \approx 0.327955$, $w^1 \approx 0.0537771$. The reduction in travel time causes the number of trips to increase from 2.5 to 2.80943. The travel cost p^z increases because the supply curve is upward sloping. More time is available for

work, causing the equilibrium wage to fall, and the x-market is affected through the change in the wage rate which reduces the cost of providing the commodity.

One straightforward way to assess the project is by evaluating "sequentially" market after market to obtain:

$$CV = - \int_{p^{z0}}^{p^{z1}} \frac{1}{p^z + t^0 \cdot w^0} dp^z - \int_{t^0}^{t^1} \frac{1}{p^{z1} + t \cdot w^0} dt - \int_{p^{x0}}^{p^{x1}} \frac{1}{p^x} dp^x + \int_{w^0}^{w^1} (T - t^1 \cdot \frac{1}{p^{z1} + t^1 \cdot w}) dw + \Delta\pi \approx 0.147046 \quad (\text{A.9})$$

where $\Delta\pi \approx 0.0673155$. Note that we have chosen one out of possibly an infinite paths of integration. It is left to the reader to show that any other path produces the same outcome. One might fear that adding the value of additional trips and the value of additional working time results in a double counting of benefits. However, here trips provide utility (for some unspecified reason). Therefore, reducing time costs allows for more valued trips as well as additional production of valued commodities. If trips provided zero benefits per se, shortening travelling times would "only" add to production.

Finally, applying equation (17), but now also allowing the wage to adjust so as to preserve equilibrium in the labor market throughout, one obtains:

$$CV = - \int_{t^0}^{t^1} w(t) \frac{1}{p^z(t) + t \cdot w(t)} dt \approx 0.147046$$

$$(\frac{1}{2})(t^0 \cdot \bar{w} - t^1 \cdot \bar{w})(z^0 + z^1) \approx 0.14723, \quad (\text{A.10})$$

where \bar{w} denotes the average or mean wage rate. Thus, the shortcut performs extremely well.

Applying the rule of a half as stated in the approximation formula (18) estimates CV to around 0.1843. Adding the corresponding approximation for the x -market and the change in producer surplus in the labor market⁹ improves the approximation to end up around 0.161.¹⁰ An even more precise result is obtained by using (18), with obvious adaptations for other sectors, to approximate a particular path, for example, the one in equation (A.9) but with the first two terms "merged". Then, the change in g is assessed for the change in z that results if all other prices are held at their initial levels, while the change in p^x is assessed for the actual change in x (because demand is independent of other prices than p^x). Adding (a rule of a half) estimate of the producer surplus change in the labor market results in a total surplus around 0.151. However, this is a more data-demanding approach than one based on initial and final demands. This suggests that if perfect competition prevails, one either ignore induced surplus changes or sum across all of them (perhaps to highlight how different types

⁹ $(1/2)(w^1 - w^0)(l^0 + l^1)$.

¹⁰ If only p^z is endogenous the price path reduces to $p^* = (1/2)(\sqrt{w^0} \sqrt{8 + w^0 \cdot t^2} - w^0 \cdot t)$. The reader is invited to show that the approximation provided by (18) is close (≈ 0.1491) to CV (≈ 0.149) and equal to the approximation in the second line of equation (17).

of agents are affected) based on initial and final equilibrium demands, just as in the approximation formula (18).

A.3 Some Equations

Solving the equation system (A.2), given $r = -1$, $p^z, p^x > 0$ and $0 < z < 6$, for z yields:

$$\begin{aligned} f(z) = & \text{Root}[-10963 - 6930\sqrt{2} + 1299z + 684\sqrt{2}z - 57z^2 - 18\sqrt{2}z^2 + z^3 + \\ & (2712z + 1404\sqrt{2}z - 226z^2 - 72\sqrt{2}z^2 + 6z^3)\#1 + \\ & (1414z + 720\sqrt{2}z - 336z^2 - 108\sqrt{2}z^2 + 15z^3)\#1^2 + \\ & (-222z^2 - 72\sqrt{2}z^2 + 20z^3)\#1^3 + (-55z^2 - 18\sqrt{2}z^2 + 15z^3)\#1^4 + \\ & 6z^3\#1^5 + z^3\#1^6 \&, 2] \end{aligned} \quad (\text{A.11})$$

where $f(z)$ is the equilibrium price path for p^z depicted in Figure 4, $\#1$ represents the first argument supplied to a pure function.¹¹ In Mathematica, simply insert a z -value to obtain an equilibrium price, as is done beneath equation (A.6), or insert the right-hand side expression of (A.11) in Plot to draw a curve, as is done in Figure 4. The expression for the equilibrium price path for p^x is available from the authors on request. Also note that one could arrive at seemingly very different $f(z)$ -expressions depending on how the equation system (A.2) is set up and solved, and different editions of Mathematica might generate different expressions, but they all provide one and the same solution.

For convenient reference, we provide the equilibrium prices generated by a number of different z -values:

$$[z, \mathbf{p}^z] = [(1.5, 2.1526), (2.5, 1.4767), (3.5, 1.1186), (4.5, 0.8890)]$$

These prices can be obtained by using Mathematica's Solve, NSolve or FindRoot to evaluate the equation system (A.2) for the considered z -values.

A simple approximation (around 5.64) of the integral in equation (A.6) is obtained by using this vector to evaluate $f(z)$, multiplying each value by $\Delta z = 1$ and summing.¹² Adding the endpoints to the above vector and using ListLinePlot, generates a good approximation of the curve in Figure 4.

The inverse Hicksian demand function for z (when $r = -1$) is stated as follows:

$$p^z = p^x \cdot (\sqrt{z \cdot V^0} - z) / (z \cdot (1 + p^x)). \quad (\text{A.12})$$

The initial (final) inverse demand function is obtained by setting p^x equal to 2 (0.801384). Integrating over $[z^0, z^1]$, summing and taking the average, one obtains $CV^E \approx (1/2)(5.97368 + 5.34056) = 5.65712$.

¹¹<https://reference.wolfram.com/language/ref/Slot.html>. For further documentation and interpretation, see <https://reference.wolfram.com/language/>.

¹² $\int_{p^z}^{p^z} p^z(\cdot) dp^z \approx \Delta z \sum_i f(z^i)$ for $z^i = 1.5, \dots, 4.5$.

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