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NOTIONAL DEFINED CONTRIBUTION ACCOUNTS (NDCs): SOLVENCY AND RISK; APPLICATION TO THE CASE OF SPAIN [*]

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ABSTRACT

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The aim of this work is twofold, on the one hand, to demonstrate the actuarial imbalance of the Spanish pension system in its current configuration, and on the other, to measure the aggregate economic risk to which the pensioner would be exposed if it were decided to apply ten formulas for the calculation of the retirement pension based on notional accounts. Given the uncertainty involved in working with a long term horizon, a model of generation of multi-periodic scenarios is used, based on the predictions of mean values of Alonso and Herce (2003) for the period 2006-2050. This provides up to ten thousand trajectories of the macroeconomic indices needed to calculate such parameters as the initial pension, the replacement rate (RR) or the internal rate of return (IRR), and the value-at-risk (VaR) of the pensioner. The results obtained are analyzed in both objective and subjective terms. The main conclusions are that, applying the notional philosophy, the expected average RR and IRR would be much lower than those obtained under the current rules of the pay-as-you-go system. If the projections used were slightly probable, the pension system would build up such a large additional financial imbalance in the future that it would require either a considerable reduction in the initial pension or a severe combination of parameter adjustments. From the risk perspective, the preferred formulas for a beneficiary most averse would be those based on future variations in salaries with a pension constant in real terms, whereas those beneficiaries less averse to risk would prefer formulas supplying a lower initial pension which grows in real terms in line with future variations in salaries. (JEL: H55, J26).

Keywords: Risk aversion, Spain, scenario generation, retirement, pensions.

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NOTIONAL DEFINED CONTRIBUTION ACCOUNTS (NDCs): SOLVENCY AND RISK, APPLICATION TO THE CASE OF SPAIN

1. INTRODUCTION

In Spain numerous works have been carried out that study the supposed inviability, unsustainability, insolvency, and financial insufficiency of the public pension system, and that propose measures aimed at avoiding the long-term financial insolvency of the system. According to Alonso and Herce (2003), the Spanish contributory pension system is doomed to face serious financial inadequacy from 2020 onwards, with the acceleration of the ageing of the Spanish population and the shrinkage of the margins of its activity. Not even the foreseeable increase in fertility and in migratory flows would add enough human resources to avoid that deterioration.

In a similar line for Ahn *et al.* (2005), the financial situation of the pension system in Spain is perceivably affected by an adverse future demographic situation. During the first decades of this century it will have a small surplus. However, the deficit during the following decades will be high and will grow with time. In 2050, the deficit will be greater than 6% of the GDP with a probability of 90%, and greater than 15% with a probability of 10%. In the same year, the accumulated deficit[1] would be between 77% and 260% of the GDP, with a confidence interval of 80%.

The study conducted by the EU (2005) concludes that the percentage expenditure in contributory pensions –retirement, orphan, widow(er), relative, etc.– on GDP will pass from 8.8% in 2005 to 15.7% in 2050. It is necessary to emphasize, however, that this projection is more optimistic than the previous projection made in 2001 which situated pension spending in 2050 at 17.3%.

On the other hand, not all researchers agree with the previous diagnosis. Thus, for example, Del Brio and González (2004) find that almost all the predictions on the solvency of the pension system have had numerous errors in the past, and that they have turned out to be excessively pessimistic. In their work, they present projections based on the observed flow of new immigrants, concluding that the financial problems of the pension system will be deferred beyond the year 2045. However, even the official sources, MTAS (2005), admit that in the year 2015 the first deficit of the system will appear for an amount equivalent to 0.04% of the GDP. In the same year the Reserve Fund will begin to be used in order to balance the results of the period 2015-2020, and from 2021 effective deficits would begin to be produced. What is more, they agree with the need to set a new reform process in motion, with *a priori*ty character in the framework of the last recommendations of the "Pacto de Toledo" [Toledo Pact], although they consider that there is a long enough time period in which to carry out the reforms.

Vidal-Meliá and Domínguez-Fabián (2006) have strongly recommended the necessity of establishing a profound overhaul of the Spanish public pension system. In the present work, an alternative reform of the system is proposed based on notional defined-

[1]Accumulated deficit must not be confused with implicit debt. According to the last estimates carried out by Devesa and Devesa (2005), the implicit debt at 31-12-2003 of the overall public pension system (general regime, special, and passive classes), only for the contingency of retirement, varies between 152% and 214% of GDP, according to the method of estimation.

contribution schemes (NDCs) [2]. This is the most important innovation that has been introduced into the field of public pensions in recent years, and these schemes are already in operation in different countries[3]. This model has been proposed, Holzmann (2006), as a fundamental referent for the future unified pension system of the European Union.

Devesa-Carpio and Vidal-Meliá (2004) study retrospectively how the introduction of pension formulas based on the notional philosophy would have affected the initial amount of the retirement pension and the internal rate of return of the pension system in Spain. Their conclusion is that the effect of their implantation would have notably reduced both the amount of the pensions and the replacement rate (RR) currently produced by the pension formula based on the traditional defined benefits. Also, the expected theoretical real internal rate of return (IRR) of the contributions, assuming survival to retirement age, would have passed from around 5.07% (5.64%, in the case of women) to less than 2.21% (2.91%, for women) for whichever of the applied formulas.

Vidal-Meliá *et al.* (2006) carried out a prospective analysis of the pension system, in which they quantify the aggregate "economic" risk to which the beneficiary in Spain would be exposed if a system of retirement pensions based on notional accounts were introduced. To this end they used the scenario generation technique to make projections of the factors that determine the real expected IRR for the beneficiary as a function of sixteen retirement formulas obtained from the most widely accepted notional indices or rates. It was concluded that the IRR, under whichever of the applied retirement formulas, would also be less than the corresponding value under current Spanish legislation, while the most favourable replacement rate would only just reach 50.5% for a person of 65 years and with 40 years of contribution.

The present work, which is directly related to that of Vidal-Meliá *et al.* (2006), is aimed at perfecting the scenario generation technique used for the projections of the macroeconomic indices that determine the values of such variables as the initial pension or IRR, amongst others. The number of possible scenarios that could arise for each formula of the calculation of the pension is increased to ten thousand, in order for the results to gain in robustness. We use as mean values to obtain the trajectories of the macroeconomic indices three new, more up-to-date, basic macroeconomic projections —Alonso and Herce (2003), MTAS (2005) and EU (2005). We then analyze the utility of the pension from both an objective and a subjective perspective, taking the individual's risk aversion into account.

Another contribution with respect to the referred work is that we perform a sensitivity analysis of the anticipated and unanticipated changes in the survival rate, of the expected average growth, and of the change in the base macroeconomic projection.

The work is organized as follows. Following this Introduction, Sec. 2 describes the model, differentiating two parts — first, a brief overview of the actuarial concept of the notional account, and second, details of the model used for the projections of the macroeconomic variables that intervene in the notional formulas. Section 3 presents the principal assumptions, starting data, and notional account formulas used. Section 4 presents and analyzes the principal results: RR, average IRR, value-at-risk (VAR) of the IRR, Markowitz

[2]As Gronchi and Nisticò (2006) note, the original idea of the NDC was present in two works published in the 1960s by Buchanan (1968) and Castellino (1969) which were rediscovered in the late 1990s.

[3]Italy (1995), Latvia (1996), Khirghitzia (1997), Poland (1999), Sweden (1999), and Mongolia (2000). According to Williamson (2004), other countries such as China and Russia, are seriously considering the introduction of this system of pensions.

ranking of the IRR and the utility of the pension. Section 5 describes the sensitivity analysis of the anticipated and unanticipated changes of the survival rates, the expected average growth, and the change in the base macroeconomic projection. The work ends with the main conclusions, future lines of research, the literature references, and a technical appendix.

2. THE MODEL

A notional account is a virtual account containing the individual contributions of each participant and the fictitious returns that these contributions generate over the course of the working life. The returns are calculated in accordance with a notional rate that may be the growth rate of the GDP, of average wages, of aggregate wages, of contribution payments, etc. When the individual retires, he or she (henceforth, he) receives a benefit that is derived from the accumulated notional fund, the specific mortality of the cohort retiring in that year, and the applied notional rate.

According to Valdés-Prieto (2000, 2005), the system of notional accounts is a very useful way in which to minimize the political risk associated with pay as you go systems, and increases the financial solvency of the system in the long term, although it also increases the explicit economic risk affecting the contributors. As Diamond (2006) correctly points out, all the advantages attributed to NDCs[4] could be obtained with a well designed system of defined benefits, although of course this is precisely the difficulty inherent in such systems — the ease with which *erroneous political decisions* convert them into badly designed systems.

Following the development of Vidal-Meliá et Alt. (2006), in order to calculate the initial pension of an individual at the age of retirement, in the models of notional accounts, the contributions made and valued at the date of retirement are made equal to the pension that he is going to receive until his death, also valued at the age of retirement, *i.e.*:

$$\overbrace{\sum_{x=x_e}^{x_r-1} CR_x CB_x \prod_{i=x}^{x_r-1}(1+r_i)}^{K=\text{Notional Accumulated Fund}} = P_r \underbrace{\ddot{a}_{x_r}^{\beta}}_{G=\text{Conversion Factor}} \quad \textbf{[1]}$$

where:

CR_x: Contribution rate at age "x", defined as the percentage to apply on the base contribution.

CB_x : Contribution base for the contingency of retirement at age "x".

$CR_x CB_x$: Effective contribution for an age "x".

x_e : Age of entry to the labour market.

x_r : Age of retirement.

[4] See in this regard the work of Barr and Diamond (2006), Börsch-Supan (2006), Lindbeck and Persson (2003), Palmer (2006), Vidal *et al.* (2004), and Williamson (2004), *inter al.*

r_i : Notional rate that is applied to capitalize the contribution.

P_r : Initial pension at the age of retirement.

K: Notional accumulated fund when the individual reaches the age of retirement.

$\ddot{a}_{x_r}^{\beta}$: Present value, at the age x_r, of a life pre-payable actuarial annuity due of 1 per year, while "x_r" survives, increasing at the accumulative annual rate of β, which is also the so-called conversion factor (G).

The retirement pension will be:

$$P_r = \frac{K}{\ddot{a}_{x_r}^{\beta}} = \frac{K}{G} = Kg \quad [2]$$

if $(1+\beta) = (1+i)$, can be expressed as:

$$P_r = \frac{K}{1+e_{x_r}} \quad [3]$$

where

e_{x_r} : Life expectancy at retirement age.

With the assumption of Equation 3, since the initial pension is assumed to be lower, it is considered to be increased in real terms. If a higher initial pension is desired, to maintain actuarial equilibrium between the notional capital and the expected pensions the adjustment would obviously have to be less than in the supposition shown.

Assuming that $(1+\beta) = (1+\alpha\,\rho); \alpha > 1$, and therefore:

$$P_r < P_r^{*} = \frac{K}{\ddot{a}_{x_r}^{\beta=\alpha\,\rho}} \quad [4]$$

As was noted above, the equivalence between a system of notional accounts and a system of defined benefits under certain conditions was demonstrated in the work of Cichon (1999) and Devolder (2005).

Since the principal objective of the present work was to measure what is the aggregate economic risk to which the pensioner would be exposed, projections were made of different macroeconomic variables for a standard individual who enters the labour market in 2005 at x_e years of age. It is not easy to establish a long-term macroeconomic scenario in view of how the Spanish economy has changed over the last fifty years. The aim therefore is not to estimate the future value of the variables, but the consequences for the beneficiary of the introduction of a system of notional accounts in a context of uncertainty.

Following the approach of Devolder (1993), the model used to get the trajectories for the different macroeconomic indices is:

$$I_t^s = \mu_t + \lambda_t^s \sigma_I \qquad [5]$$

where:

I_t^s: Macroeconomic index "I" in period "t" and under scenario "s". Two indices are considered — variation of the gross domestic product (VGDP) and variation of the average earnings index (VAEI).

μ_t : Real mean value of the macroeconomic index in the period "t". Information based on the projections of Alonso and Herce (2003), MTAS (2005) and EU (2005).

λ_t^s : Residue N(0,1) is different for each period "t" and scenario "s".

σ_I Real standard deviation of each index, that incorporates past information of the variables into the model. In order to calculate this value the real historical values of each index for the period 1961-2005 will be taken into account.

This scenario generation model assumes perfect correlation between the indices in the long term, since the analysis of the series of real historical data of the indices in Spain for 1961–2005 shows that the correlation between GDP and VAEI for quinquennial data is 0.65, and increases to 0.92 if decennial data are used.

3. NOTIONAL FORMULAS, DATA, AND ASSUMPTIONS

3.1. NOTIONAL FORMULAS

The formulae that will be explored in order to determine the initial pension and its subsequent variation are ten and are given in Table 1.

In the ten models analyzed, the notional rate of the contributions is either the variation in GDP (VGDP) or wage variation (VAEI). The basis used to calculate the initial pension is Formula 4 for the first six models. In theory, these models will provide higher initial pensions, since later the pension will be constant in real terms. The basis for the calculation of the pension in the remaining models, 7–10, is Formula 3. In this case, the initial pension (replacement rate) will be smaller, although the pension will be increased (or decreased) in real terms.

If the notional rate of the pensions includes ± differential (Formulae 3-6), this rate will be adjusted through a positive or negative difference. For example, for Model 3, the pensions will take the real behaviour of the GDP with respect to the expected behaviour into account (average value of the used macroeconomic projection) in such a way that, if the real value is greater or less than the expected value, a positive or negative difference will be obtained, respectively.

3.2. DATA

The projections of the macroeconomic indices given in Table 1 are based on the scenario generation model described above. According to Alonso and Herce (2003), MTAS (2005) and EU (2005), the mean values of these projections have in common that they predict a marked future fall in the rate of GDP growth with respect to the average of the Spanish economy of the period 1961-2005.

We will take as the base macroeconomic scenario that of Alonso and Herce (2003). The mean scenario depicted by the MTAS (2005) is more pessimistic than that of Alonso and Herce (2003), and it also bases the future GDP growth on the basic contribution of employment rather than on productivity growth (wages). The scenario of Alonso and Herce (2003) holds that the future basis of GDP growth, given the expected restriction of labour, will be the growth in productivity, which will in turn be transmitted entirely to wage growth. The projection of the MTAS (2005) is far from complete, and therefore a series of additional assumptions has to be adopted in order to be able to disaggregate the GDP. It is assumed that the percentage represented by the productivity in the increase of the GDP in the projection of Alonso and Herce (2003) is also maintained in that of the MTAS (2005).

The EU (2005) projection is more optimistic than that of Alonso and Herce (2003) for the first ten years. Subsequently, however, as can be seen in Figure 1, the forecast growth rate for the GDP and productivity is lower, although the profiles are very similar.

Figure 1 shows the real historical variations of the last eleven years and the evolution of the macroeconomic variables (VGDP and VAEI) in real terms, using the average value projections Alonso and Herce (2003), MTAS (2005), and EU (2005). The mean projections of the studies considered reach up to the year 2050. The calculations that are to be carried out involve working as a minimum with a horizon of 75 years, since the maximum age of the PEMF-98-99 tables reaches 100 years. It was assumed that the mean value for the year 2050 will remain constant in subsequent periods.

Figure 2 represents the 10.000 scenarios generated for each of the indices, VGDP and VAEI, taking as the mean scenario that of Alonso and Herce (2003). This plot gives an idea of the volatility and the extreme values that the indices can reach over the course of the projection period. To show this graphically, Figure 3 shows the VGDP and VAEI density functions for three years chosen purely for illustrative purposes –2010, 2030, and 2050. The mean scenario was taken to be that of Alonso and Herce (2003). Although one can perceive extreme values in the figure, the probability that they will occur is very low. Thus the VGDP will take values greater than 10% in some 0.11% of cases and less than -10% in 0.001%. The probabilities that VAEI will be outside the interval [-10%, 10%] will be 0.07% and 1.55%, respectively.

3.3. ASSUMPTIONS

The working assumptions were the following:

CR_x: Contribution rate at age x. This will be assumed constant and equal to 15% throughout the period. This is an approximation, and it is considered that, according to the data of the Social Security budget, approximately 50% of the common contingencies are assigned to the retirement contingency.

The profile of the representative individual that is considered evolves according to the variation of wages in the projections used, *i.e.*, it will always be increasing in real terms. Neither maximum nor minimum pensions are taken into account. It is considered that wages and contribution bases coincide at all times.

x_e: Age of entering the labour market. It is considered equal to 25 years.

δ: Pure preference rate in time, which reflects the impatience of the pensioner to consume. It is considered constant throughout the period and equal to 2%.

Initially, we use the mortality tables of the Spanish population 1998-1999 (PEMF-98-99)[5], although subsequently we perform various sensitivity analyses taking into account the GRMF-95 tables[6] that some insurance companies apply, and that might better represent the demographic tendencies of the future Spanish population.

For the determination of the initial pension of the first six, it is considered that the notional capital is actuarially equal to a constant pre-payable annuity at a real interest rate of 3%. For the ages of 60, 65, and 70, this implies a value of G=16.14, 13.94, and 11.65, respectively, considering the PEMF-98-99 tables. For the determination of the initial pension in the remaining models, the real interest rate is taken to be 1.25%, which implies values of G=19.52, 16.38, and 13.30 for the ages of 60, 65, and 70, respectively.

The value is obtained by averaging the life expectancy of men and women. This is of great importance in the determination of the amount of the initial pension, and is the denominator in the formula that gives the initial pension in each case, the numerator being the notional capital (K). It is logical that one interest rate is used in some of the formulas, and another lower one in others, since the initial amount of the pension is subsequently increased differently.

4. RESULTS

The results that are presented below were calculated using the program Mat-Lab®[7], version 6.0 for Windows.

The beneficiary is subject to risk in as much as he does not know with certainty what will be the IRR of his contributions, or the replacement rate reached. This risk could be qualified as non-diversifiable or systematic, as it is directly associated with the overall risk of the economy. The beneficiary's aggregate "economic" risk with respect to the IRR is defined as the possibility that the effective rate of return obtained by the contributions does not coincide with the expected rate due to the uncertain return on some economic asset (behaviour of wages, of the GDP, of the contributing population, etc.) that is providing support to the notional retirement accounts, and may be seen as an indicator of the system's financial health. With respect to the RR, it will be the possibility that the RR differs from the expected value due to the uncertain return on the economic asset that is used to capitalize the contributions.

The risk that is evaluated is the economic risk (basically from reduction of the GDP or wage growth rate), which is also dependent on demographic risks (increase in the longevity of the population, fall in the fertility rate or in the activity rate) that affect economic activity and the health of the pension system.

[5]http://www.ine.es/inebase/cgi/um?M=%2Ft20%2Fp319&O=inebase&N=&L=
[6]GRMF-95 (1996): "Mortality probabilities of the GRMF-95, GKMF-95, and EVK-90 tables". Actuarios 13, 29-33.
[7]See in this regard the Web site of the supplier: http://www.mathworks.com/.

4.1. REPLACEMENT RATE (RR)

Tables 2 give the values of the mean expected replacement rate (RR) for the different formulas as a function of the individual's retirement age, and the replacement rate adjustment coefficient (AC) with respect to the retirement age of 65 years.

One observes that RR increases with increasing retirement age. This is because the fund accumulated until that age is larger, while there are fewer years in which to receive the retirement pension. Early retirement would apparently be discouraged, not so much by the discount with respect to the age of 65 years (adjustment coefficient of 0.723, for Formulae 2-5-6 of Table 2), but because the RR, already very small at 65 years, would be even smaller at 60 years.

The formulas are grouped into four significant sets as a function of whether the notional rate used is VGDP or VAEI, or whether the adjustment of the pension is done in real or remains constant. The formulas that present the highest RR values are those which are based on the variation of wages and the amount of the pension remains constant in real terms.

The RR on the average wage for the age of 65 years, after 40 years of contributions, is 62%, when with the current rules of the Spanish system the rate would be over 90%. The message that is given by this result is clear. If the projections used were even minimally close to the truth, the pension system would show a very important future imbalance that, to be resolved, would need a considerable reduction of the initial pension or a combination of severe parameter adjustments. Only for retirement at 70, after 45 years of contributions, does the RR approach 87% in the best of cases.

An increase of the RR would only be possible with a major extra effort in contributions. If, with the perspectives of growth derived from the projection of AH (2003), one wanted to attain replacement rates of 80% of the average wage of all the working life, the contribution rates for Formulae 2 and 10 would be 19.36% and 22.75%, respectively, in any case far from easily supported in the Spanish labour market[8].

The results presented in this section bring out a manifest problem in the current pension system. Nevertheless, in order to avoid a slanted analysis that only considers the first pension, it is necessary to include an indicator —the IRR— that also takes how the amount of the pension evolves over time into account.

4.2. INTERNAL RATE OF RETURN (IRR)

The results of the expected IRR for each formula will be presented separately for each retirement age considered (Tables 3, 4, and 5), as well as its deviation and the percentage that this represents with respect to the expected IRR.

One notes immediately the significant differences, at the age of 65, which appear between the IRR of men and that of women. The IRR of women is approximately 66% to 106% higher than that of men. This is because when the retirement pension is being calculated the mean life expectancy between men and women is used.

[8]It is not difficult to show that the fraction represented by any replacement rate RR_i at a given contribution rate TC_i relative to a given replacement rate RR at contribution rate TC is equal to the ratio of the contribution rates (TC_i/TC).

Comparison of these results with those obtained relative to the replacement rate shows that there is no clear relationship between the two concepts. Furthermore, the model with the greatest IRR for all ages, Model 10, had a low replacement rate. Also, one observes that, in terms of IRR, delaying retirement age is not rewarded. This is logical since, as the amount of the pension depends upon the future growth and the evolution of the demographic parameters, and the future prognosis is unfavourable due to the slowing of the growth of the macroeconomic variables that intervene in the formula, the return on the investment, IRR, worsens with the passage of time.

There are marked differences in the expected value of the IRR, on the contrary, the percentage standard deviation does not vary notably among the different formulae. It seems clear that formulae 10 and 2, both wage-based, are the most suitable since they have a high IRR and a low volatility.

The formulas that have higher IRR practically coincide for the case of men and women (there are differences for 70 years of age), but women are subject to a smaller variability of the IRR, so that, besides obtaining a greater average IRR, they are subject to a far lower risk.

The values obtained for the IRR are surprisingly low, especially compared to the expected return under unchanging legislation in the current system which approaches 4% for the age of 65 years. But it has to be taken into account that this reference 4% is not a net return of an increase in future contributions. Following the arguments of Geanakoplos et al. (1998), this return should decrease considerably since; to maintain the promised benefits and the financial equilibrium of the pay-as-you-go system, there would have to be a very notable increase in the contributions relative to wages. The IRR obtained from the system of notional accounts is actuarially fair.

Besides, these figures are not so alarming if they are compared with the average growth of the macroeconomic indices considered, which are about 1.66% for the increase in GDP or 1.84% for the variation in wages according to the projections of Alonso and Herce (2003).

Another important question is the fact that the IRR decreases with increasing retirement age, and also the risk to bear is far greater. Therefore, if the analysis made by the contributor-beneficiary is from this perspective[9], he would not likely be willing to prolong his stay in the labour market, even though it is considered, Palmer (1999), that NDCs mitigate the effect of disincentive to work that appears with a pay as you go system of defined benefits, especially if that system is badly designed.

4.3. VALUE AT RISK (VAR)

Besides the calculation of the deviation of the IRR, another important tool for the measurement of risk is the value-at-risk (VaR). This parameter has become one of the most widely used tools with which to measure risk by regulators, economic agents and academics. One of the reasons for its popularity is the simplicity of the concept and, above

[9]In making a decision of whether to take early retirement, the beneficiary will, or should, at least consider the following questions: the amount of the pension he will receive relative to his needs, his valuation, Valdés-Prieto (2002), of his possibilities of leisure, his state of health, his interest in his work, the possible access to a better job position, etc. Palmer (2001) indicates that there is empirical evidence that the individual tends to go into retirement as soon as he is allowed to, so that one must be very cautious when it comes to establishing the minimum retirement age.

all, the intuitiveness of its interpretation, it being an estimate of the maximum possible loss for a given time horizon and significance level under circumstances considered as "normal in the market".

In the analysis to follow, VaR_{ε} is understood to be the minimum value of IRR at a determined confidence level ε. Thus, a $\text{VaR}_{0.95}$ of 0.61% for Formula 2 implies that only 5% of the times (assuming a 95% confidence level) in normal conditions would the IRR of a man be less than 0.61%.

Assuming a confidence level of 95%, the following results are obtained (Table 6). For the men at any age, the model that would provide the best value of $\text{VaR}_{0.95}$ is Model 2, which uses the VAEI to capitalize the contributions, and the pension remains constant in real terms. Women would only choose this model at the age of 60. They would opt for Model 10 at the ages of 65 and 70. It has to be emphasized that Model 10 would give a notably lower initial pension–replacement rate than Model 2 at any age –for instance 37% as against 44% at the age of 65– but the pension will be increased at the same rate as wages in Model 10 as against constancy in real terms in Model 2. The result seems logical given women's longer life expectancy.

The variation of the retirement age with respect to that considered ordinary (65 years) also alters notably the value of $\text{VaR}_{0.95}$. Anticipating of the retirement age increases $\text{VaR}_{0.95}$, and its deferment diminishes it notably. The conclusion therefore is that neither in terms of $\text{VaR}_{0.95}$ would delaying the retirement age be rewarded, since the VaR for an age of retirement of 60 years would be greater than that for 65 years, and this in turn greater than that obtained for 70 years.

In general, the worst model in terms of $\text{VaR}_{0.95}$ is Formula 4, which uses VGDP to capitalize the contributions, and VAEI for the pensions. This is because the corresponding dispersion of IRR around its mean value –already small in itself, as one can see in the tables of the previous subsection– is one of the greatest.

4.4. MARKOWITZ FUNCTION.

In order to carry out an overall analysis of the risk, it is necessary to introduce the subjectivity of its valuation by the beneficiary through his risk aversion. Analyzing the return in terms of Markowitz is equivalent to doing so in terms of a utility function. In the present case, the results are presented after applying the theory of Markowitz.[10]

Table 7 presents the ranking of the formulae according to the Markowitz function criterion for men and women of 65 years, with different values of risk aversion (γ). Following the method used by Feldstein and Ranguelova (2001), values of risk aversion are used in relation to both the mean determinant variable (1+IRR) of each formula and to the differences between the formulas. The values of γ that are used depend to a great degree on the relationship between the values of the formulas that one wishes to rank. If the differences are small, as is the case with the IRR, then the risk aversion γ will take much higher representative values. Individuals (men and women) neutral to risk or not averse to risk would prefer Formula 10 in which return is rewarded. Those individuals (men and women) with a more marked degree of risk aversion, however, would prefer Formula 2,

[10] Identical results were obtained in a calculation of the utility of (1+IRR) through a CRRA utility function.

which is preferred when the $VaR_{0.95}$ criterion is applied for men and for women with lower retirement age.

Table 8 gives the preferred formula for each age according to the Markowitz function for men and women, with different values of risk aversion (γ) and projections. The formulas that are found to be preferable for the beneficiary are 10 and 2 — the former for those individuals more averse to risk, and the latter for the neutral or less averse to risk individuals.

4.5. UTILITY OF THE PENSIÓN.

To complete the risk study, the evolution of the pension is analyzed throughout the passive lifetime of the individual in terms of utility. Table 9 presents the ranking of the formulae according to the utility of the pension for men and women, with different values of risk aversion (β) and projections. The values of risk aversion (β) to be used vary from 1 to 5. Feldstein and Ranguelova (2001) provide qualitative arguments that very high values of risk aversion should not be used in the application of CRRA (constant relative risk aversion) type utility functions to the analysis of pensions.

The results are practically identical to those of the previous subsection. The formulas chosen are 10 and 2 according to the individual's degree of risk aversion. The least averse individuals will choose Formula 10, and as the individuals' degree of aversion increases, they would tend towards a preference for Formula 2 (higher initial pension, but constant in real terms).

Table 10 lists the formula chosen for each age according to the utility of the pension, for men and women, with different values of risk aversion (β) and the AH (2003) base projection. One observes that the decision of individuals who are neutral or not very averse to risk does not vary even though the retirement age is modified. The critical value of aversion is 2 or 3, the values where the change from Formula 10 to 2 occurs. The decision of very averse individuals is unchanged, they all choose Formula 2.

5. SENSITIVITY ANALYSIS

This section describes a sensitivity analysis with respect to changes in the survival rates, the expected average GDP growth derived from the mean macroeconomic projections used and the change in the base macroeconomic projection.

5.1. SENSITIVITY OF THE RESULTS WITH RESPECT TO INCREASES IN THE SURVIVAL RATE

This first sub-section describes the sensitivity analysis of the results with respect to changes in the survival rate. It is a fact that in the last 50 years there has been a notable increase in longevity in most developed countries, including Spain, Goerlich and Pinilla (2005). According to the mortality tables of the INE [Spain's National Institute of Statistics] of 1950, the life expectancy for men and women at the age of 65 was 11.83 and 13.48 years for men and women, respectively. The INE tables of 1998-1999, which are those that have been used in all the foregoing calculations, indicate that, in the intervening approximate 50 years, these figures have risen by 36% for men and 49% for women. The person who enters the labour market now, at the age of 25 years, assuming retirement at 65, would begin to collect his pension in the year 2045. It is reasonable to think, Diamond (2005), that life expectancy will have further increased, and mortality tables that better reflect this increase would have to be used. Also, one of the fundamental virtues that is claimed for the NDCs is that they can deal with demographic and economic changes by means of the design of the formula they use to calculate the initial pension.

5.1.1. ANTICIPATED CHANGES IN THE SURVIVAL RATE

It is assumed that the authority governing the pension system is capable of anticipating this increase in longevity[11], and that this is reflected in a change of the mortality tables valid for the calculation of the amount of the initial pension. The mortality tables used are the Swiss GRMF-95, which insurance companies in Spain have already been applying for several years for the commercialization of life annuities. According to these tables, the life expectancies of men and women at the age of 65 are 20.47 and 27.15 years, respectively. If, for the determination of the initial pension at 65, the notional capital is considered to be actuarially equal to a constant pre-payable annuity at a real interest rate of 3%, a value of G=16.65 is obtained as against the 13.94 that was obtained with the PEMF-98-99 tables; these values are applied to the first six formulae, which correspond to a greater initial pension, but constant in real terms. If a real interest rate of 1.25% is considered, the value of G with the GR-95 tables is 19.52 instead of 16.38. These are the values applied to the last four formulas which correspond to an initial pension that is smaller but that will be increased in real terms.

The results presented in Table 11 are revealing. The notional account system reacts to an anticipated change in the survival rate with an automatic reduction of the replacement rate to try to maintain the actuarial equilibrium between contributions and pensions. As can be observed, the expected average replacement rate would decline in whichever of the cases by around 10%, to, in the best of the cases, some 51% of the average wage. The adjustment, however, would be neither complete nor perfect, since the IRR would increase slightly because the reduction in the initial pension would not sufficiently compensate the foreseen increase in longevity. Hence, some additional fine-tuning would be necessary in order to leave the IRR unchanged. The $VaR_{0.95}$ of the IRR would also increase as a consequence of the foreseen greater longevity.

5.1.2. UNANTICIPATED CHANGES IN THE SURVIVAL RATE

In this sub-section, unlike the previous one, it is assumed that the authority governing the pension system is unable to anticipate the increase in longevity, or that, while recognizing it, it has insufficient political courage to apply it immediately, so that the formula for the calculation of the pension would use the demographic parameters derived from the PEMF-98-99 tables, but the longevity would correspond to the GRMF-95 survival tables. The effect of this atypical situation is given in Table 12, and is compared with the initial situation.

The average RR would remain unchanged, but the expected average IRR would increase notably for both men and women by around an additional 0.7%, because on average men and women would collect 4 and 7 years more of pension, respectively relative to the initial estimate. This would be a particularly critical situation since, as observed by Settergren and Mikula (2005), these unanticipated changes in mortality would lead to the violation of the Samuelson-Aaron rule, since the implicit average IRR of the system, for whichever of the

[11]In Sweden and Brazil, there is a process of automatic annual adjustment of the demographic parameters based on the observed survival rates. In Italy, Brugiavini and Peracchi (2005), the adjustment is made every ten years. To avoid undesired political manipulation, it is considered more appropriate, Diamond (2005), which the adjustments be made annually with real data instead of with projections.

formulae, would surpass the estimated average GDP growth for the period according to the AH (2003) projection, which in all cases is less than 1.8%.

5.2. SENSITIVITY OF THE RESULTS WITH RESPECT TO INCREASES IN THE EXPECTED AVERAGE GROWTH OF THE GDP AND ITS COMPONENTS

A sensitivity analysis was made of the RR, GDP, and resulting $VaR_{0.95}$ with respect to increases of the expected average growth in GDP and its components. Two additional hypotheses were considered: (1) The average growth is 50% greater than the basic assumption, *i.e.*, the values of mean VGDP estimates are multiplied by 1.5. (2) The average growth is 100% greater than the basic assumption, *i.e.*, the values of mean VGDP estimates are multiplied by 2.

Table 13 gives the expected mean RR as a function of the average wage for various projections and their variations. The result was as expected — RR increases until in some cases it reaches almost 85%. The reaction of the notional accounts system to greater economic growth is an increase in the replacement rate. This is because, if the longevity rates remain constant on increasing the notional capital from the effect of a greater capitalization of the contributions, the result is an increase in the initial pension, and consequently the RR is increased.

The effects on the average IRR of men and women (Table 14) and on the $VaR_{0.95}$ of the IRR are even clearer than on the RR. With respect to the IRR, one observes an almost proportional increase in the case of men, and a notable increase, but not so high, in the case of women.

With respect to the $VaR_{0.95}$ of the IRR (Table 14), as could not be otherwise, one also observes an increase in the minimum guaranteed value.

It is thus demonstrated that, if there is no willingness to make a greater effort in contributions, even allowing for working careers that contribute for longer periods than those of today, and given the foreseeable evolution of longevity, the only way that it will be possible to maintain adequate initial pensions that are compatible with the financial viability of the public retirement pension system is for the future average GDP growth to be much greater than that expected by AH (2003).

5.3. SENSITIVITY OF THE RESULTS TO A CHANGE IN THE BASE MACROECONOMIC PROJECTION

In this last subsection, we analyze the sensitivity of the replacement rate as a function of the average wage, GDP, and resulting $VaR_{0.95}$ at the age of 65, to changes in the base macroeconomic projection. As well as the basic macroeconomic scenario of Alonso and Herce (2003), as was noted above, we use the projections of the MTAS (2005) and of the EU (2005).

Table 16 presents the mean expected replacement rate as a function of the average wage at the age of 65. With the new projections, the replacement rates decline as was to be expected, since the average future economic growth is lower relative to the projection of Alonso and Herce (2003).

Table 17 presents the average IRR at the age of 65 for different projections. Two truly significant facts stand out:

1. With the (official) projection of the MTAS (2005), the results for the IRR fall very markedly by more than 40% for men and 20% for women. While the difference is less dramatic with the EU (2005) projection, the results are still lower than those of AH (2003).

2. Also with the (official) projection of the MTAS (2005), there is a marked alteration in the ranking of the formulas in terms of the IRR. There now dominate the formulas which provide a greater initial pension with a stable amount in real terms over time as against those of a smaller initial pension and growth of the amount in real terms.

Finally, Table 18 presents the values of $VaR_{0.95}$ at the age of 65 for the different projections. In this case too, certain results stand out:

1. The ranking resulting from the EU (2005) projection is similar to that of the AH (2003) although the minimum insured values are now lower for all the formulas, and in one case the value is not even positive.

2. With the MTAS (2005) projection, the structure of the ranking clearly changes. According to the VaR criterion, the preferred formula would be number 1, which was sixth in the AH (2003) projection. Similarly, Formula 10, which was ranked second in the AH (2003) projection, is ranked next to last with MTAS (2005), and moreover with a negative value.

These considerations with respect to Tables 17 and 18 are presented graphically in Figure 4. This shows the mean and the 5th and 95th ($VaR_{0.95}$) percentiles of the IRR for each model, for men aged 65, with the different projections of the mean values.

In view of the foregoing results, it is clear that the ranking of the pension formulas is highly sensitive to the structure of the base macroeconomic projection, depending on whether the future growth of the GDP is dominated by the growth of the contributory population or by productivity (wages).

6. CONCLUSIONS AND FUTURE RESEARCH

This work has analyzed the impact on the initial amount of the retirement pension and of the IRR of Spain's system of pensions if it is decided to apply ten formulas for calculating the retirement pension based on notional accounts. The average RR and the expected IRR applying the notional philosophy would be much lower than those obtained under the rules of the system of allotment in force. The average RR as a function of the average wage, for the age of 65 after 40 years of contributions, would be around 62%. This is far from what the value would be if the current system were maintained with its same rules (around 91%). Also, the expected average IRR and the guaranteed minimum IRR in the best of the models would be very significantly distant from what would be the values with the current system, around 4%. This latter aspect is a clear indicator of the actuarial imbalance of the current system since the IRR of the system of notional accounts would be a return free from future increases in contributions and/or reductions in benefits, whereas the 4% would be exposed to the risk of reduction due to the fiscal transfer necessary to ensure the system's financial solvency.

On the other hand, if the increase in longevity observed in the last fifty years in Spain is maintained into the next fifty, even though it were only in part, and it is desired to preserve the financial equilibrium of the system, the RR should be located at 51% of the average wage after 40 years of contribution.

The message that comes from the previous results is clear. If the projections used are even minimally close to the truth, the system of pensions in its current configuration would accumulate a major additional future financial imbalance, which, to be resolved, would require either a considerable reduction of the initial pension or a combination of severe parameter adjustments. If there is no willingness to make a greater effort in contributions, the only way that it will be possible to maintain acceptable initial pensions that are compatible with the financial viability of the public retirement pension system is for the future average GDP growth to be much greater than forecast.

Another important result that coincides with that of Vidal-Meliá *et al.* (2006) is that, under a notional accounts system and a macroeconomic scenario such as that used, it is not at all clear that delaying the retirement age is rewarded. For increasing retirement age, the IRR diminishes and the risk to be supported by the beneficiary increases. This result was to be expected since, as the amount of the pension depends on future growth and on the evolution of the demographic parameters, and as the future prognosis is unfavourable with a slow-down in the growth of the macroeconomic variables that intervene in the formula, the return on investment, IRR, and its volatility, $VaR_{0.95}$ of the IRR, worsen with the passage of time.

Which is the most suitable formula with which to implement the notional philosophy in the case of Spain? According to the past values of the indices for the period 1961–2005 and the macroeconomic scenarios that we used, it seems clear that Formulae 2 and 10 would be the most appropriate since they would give the greatest minimum IRR with a probability of 95% and the greatest expected mean IRR, respectively. If the model is chosen taking the beneficiary's risk aversion into account, the more risk adverse would choose Formula 2 (based on a greater initial pension which is constant in real terms), and the less risk averse Formula 10 (based on an initial pension that is smaller but increases in real terms). Nonetheless, these conclusions, as was seen in the previous section, are highly sensitive to the structure of the macroeconomic scenario used. If this scenario is that of the MTAS (2005), the differences are considerable. It is clear that the predominant pattern of future growth, productivity, or employment has a marked impact on which formulas to choose.

Future research will centre on disaggregating the economic risk that the contributor-beneficiary has to bear. In the present work, we assumed the existence of a single standard contributor with a wage profile (increasing in real terms) determined by the macroeconomic scenario used. In the future, it is intended to develop differentiated wage profiles that model the various socio-economic groups of contributors in order to understand in more detail in the calculation of the economic risk the impact that the introduction of notional formulas could have on the IRR and the initial amount of the retirement pension.

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8. TECHNICAL APPENDIX

REPLACEMENT RATE (RR)

The replacement rate as a function of the average wage, for model "m", is given by the following formulation:

$$\overline{RR(m)}_{x_r,m} = \sum_{s=1}^{10.000} p^s \, RR(m)^s_{x_r,m} \quad [6]$$

$$RR(m)^s_{x_r,m} = \frac{P^s_{x_r,m}}{W(m)^s_{x_r\text{-}1\text{-}x_e,m}} \quad [7]$$

where:

$\overline{RR}(m)_{x_r,m}$: Expected mean replacement rate at the retirement age for model "m" as a function of the average wage of the entire working life.

$RR(m)^s_{x_r,m}$: Expected replacement rate at the retirement age under scenario "s" for model "m" as a function of the average wage of the entire working life.

$P^s_{x_r,m}$: Expected initial pension under scenario "s" for model "m".

$W(m)^s_{x_r\text{-}1\text{-}x_e,m}$: Mean expected wage under scenario "s" for model "m".

INTERNAL RATE OF RETURN (IRR)

According to Devesa-Carpio *et al.* (2002), the *a priori* apparent expectation of the real IRR for a contributor (individual focus) who enters the labour market at the age of x_e years, in a pure pay as you go system with retirement benefits, under the assumption that the norms of the system remain constant, is defined as the value of the parameter (interest rate) of the compound capitalization law that actuarially equals the flow of contributions to that of benefits. Similarly, the *a priori* expected internal rate of return (IRR) for each model and for each scenario "s" can be calculated by actuarially equating the contributions and the benefits. *I.e.*:

$$\sum_{x=x_e}^{x_r-1} RAC_x^s\,(1+IRR^s)^{-(x-x_e)} = \sum_{x=x_r}^{\omega} RAP_x^s\,(1+IRR^s)^{-(x-x_e)} \quad [8]$$

RAC^s_x: Real actuarial contribution paid at age "x" under scenario "s".

IRR^s: Internal rate of return under scenario "s".

RAP^s_x: Real actuarial pension received at age "x" under scenario "s".

The value of the real actuarial contribution for a person aged x:

$$RAC_x^s = CR_x\; W_x^s\; {}_{x-x_e}p_{x_e} \quad [9]$$

CR_x: Contribution rate at age "x".

W^s_x: Salary base at age "x" under scenario "s".

${}_{x-x_e}p_{x_e}$: Probability that an individual of age "x_e" will reach age "x".

The real actuarial pension at age "x" is the real value of the pension affected by the probability of survival from the moment of entry into the labor market:

$$RAP_x^s = P_{x_r}^s\; {}_{x-x_e}p_{x_e} \prod_{t=x_r}^{x}(1+\alpha_t^s) \quad [10]$$

P^s_{xr}: Initial pension (at retirement age x_r), obtained (see Equation 3) according to the notional capital accumulated under scenario "s".

α_t^s : Arithmetic rate used to increase pensions under scenario "s".

The determination of the IRR for each scenario "s" can also be expressed directly with the following equation:

$$\sum_{x=x_e}^{x_r-1}\left[CR_x\; W_x^s\; {}_{x-x_e}p_{x_e}\right](1+IRR^s)^{-(x-x_e)} = \sum_{x=x_r}^{\omega}\left[P_{x_r}^s\; {}_{x-x_e}p_{x_e}\prod_{t=x_r}^{x}(1+\alpha_t^s)\right](1+IRR^s)^{-(x-x_e)} \quad [11]$$

VALUE AT RISK (VAR)

In the analysis carried out, VaR is understood as the minimum value of IRR at a given confidence level. For a probability of ε %, and as long as the conditions included in the scenario generation model used are maintained, the minimum value of IRR for each model is:

$$VaR_\varepsilon\,(IRR) = F^{-1}_{IRR^s}\,(1-\varepsilon) = Sup\,[IRR^s\!: F_{IRR^s}\,(IRR^s) \le (1-\varepsilon)] \quad [12]$$

with $F^{-1}_{IRR^s}(1-\varepsilon)$ being the inverse of the distribution function of the random variable IRR for an accumulated probability of (1 - ε).

MARKOWITZ FUNCTION

In order to carry out an overall analysis of the risk, it is necessary to include the subjectivity of its valuation by the beneficiary through his risk aversion. This can be done either by means of the Markowitz function or directly with a utility function. Levy and Markowitz (1979) and Kroll *et al.* (1984) show that the expected utility of the return can be approximated through a function that relates the mean and the variance. The function used, based on the theory of Markowitz, is the following:

$$\mathrm{FM}(1+\mathrm{IRR}) = \mu_{(1+IRR)} - \frac{\gamma}{2}\sigma^2_{(1+\mathrm{IRR})} \qquad \textbf{[13]}$$

with:

$\mu_{(1+\mathrm{IRR})}$: Mean value of the capitalization factor associated with the IRR.

$\sigma^2_{(1+\mathrm{IRR})}$: Variance of the capitalization factor associated with the IRR.

γ : Parameter that quantifies the risk aversion.

If $\gamma = 0$, the individual is neutral to risk.

If $\gamma > 0$, the individual is averse to risk (the higher γ, the higher the risk aversion).

UTILITY OF THE PENSION

To achieve a fuller analysis, the risk is analyzed not only in terms of IRR, but also in terms of the consumption (pension) that a pensioner can obtain:

$$\mathrm{EU}^{s,m}_{x_r} = \sum_{t=x_r}^{w} \frac{U(P^{s,m}_t)}{(1+\delta)^{t-x_r}} \, {}_{t-x_r}p_{x_r} \qquad \textbf{[14]}$$

where:

$\mathrm{EU}^{s,m}_{x_r}$: Expected utility at retirement age (as a function of the individual's survival probability, attitude to consumption, and degree of risk aversion) for the consumption deriving from the pension obtained for model "m" under scenario "s" at the retirement age.

δ : Pure preference rate in time, which reflects the impatience of the pensioner to consume.

${}_{t-x_r}p_{x_r}$: Probability that an individual of age x_r survives until age $1+x_r$.

$P^{s,m}_t$: Retirement pension under scenario "s" and the notional accounts model "m". The hypothesis adopted is that the amount of the pension is converted into consumption (there is neither savings nor a draw-down on savings). This analysis of the utility function of the pension allows a measure of both the objective economic risk for the beneficiary when the individual is neutral to risk, and the subjective risk taking into account the different potential degrees of aversion.

FIGURES AND TABLES

Table 1: Formulae for the calculation of the initial pension and its subsequent variation

Model	Notional rate for contributions	Notional rate for pensions
1	VGDP	Constant in real terms
2	VAEI	Constant in real terms
3	VGDP	Constant in real terms ± differential VGDP
4	VGDP	Constant in real terms ± differential VAEI
5	VAEI	Constant in real terms ± differential VGDP
6	VAEI	Constant in real terms ± differential VAEI
7	VGDP	VGDP
8	VGDP	VAEI
9	VAEI	VGDP
10	VAEI	VAEI

Table 2: Expected mean replacement rate as a function of the average wage, for various ages and AH (2003) projection.

Fórmula	RR (60)	AC	RR (65)	RR (70)	AC
2, 5, 6	44.84%	0.723	61.99%	87.24%	1.407
1, 3, 4	41.45%	0.752	55.14%	76.53%	1.388
9, 10	37.07%	0.703	52.74%	76.37%	1.448
7, 8	34.26%	0.730	46.92%	67.00%	1.428

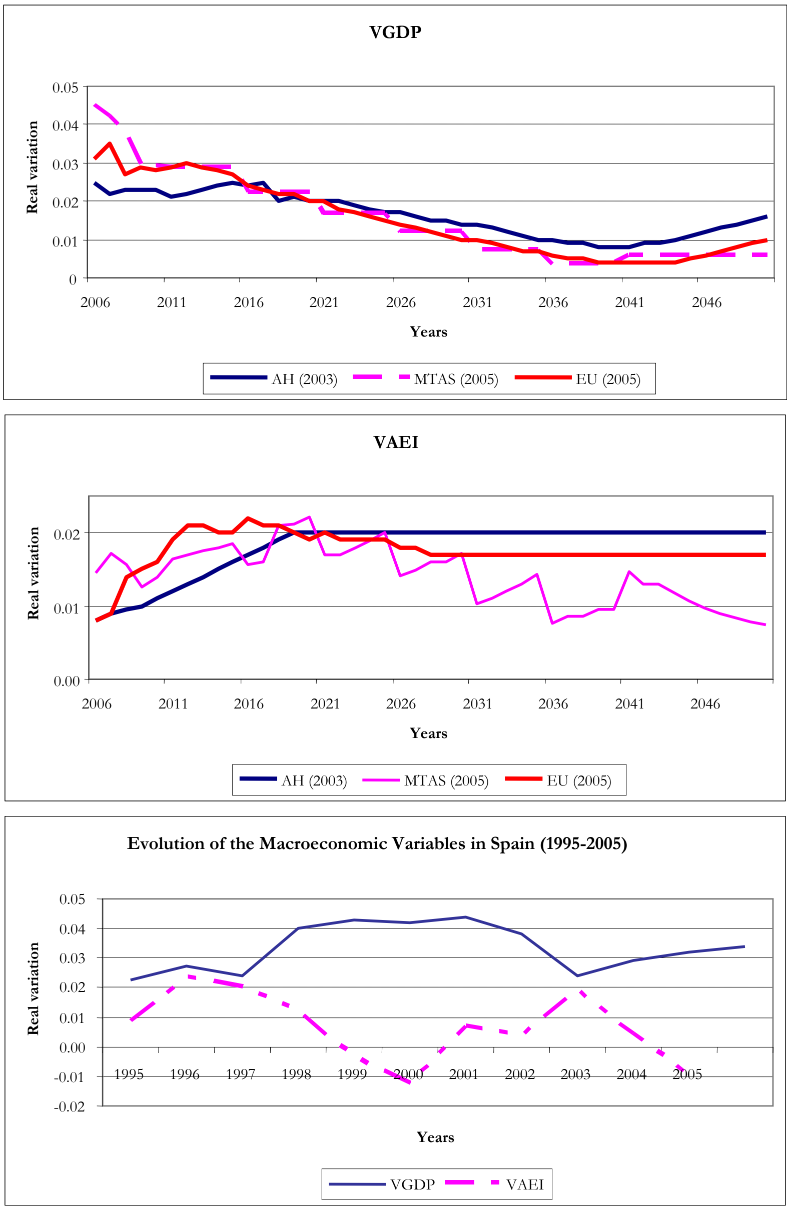

Figure 1: Evolution of the macroeconomic variables (VGDP and VAEI) in real terms, using the mean value projections of Alonso and Herce (2003), MTAS (2005), and EU (2005), and the real historical variations of the last eleven years.

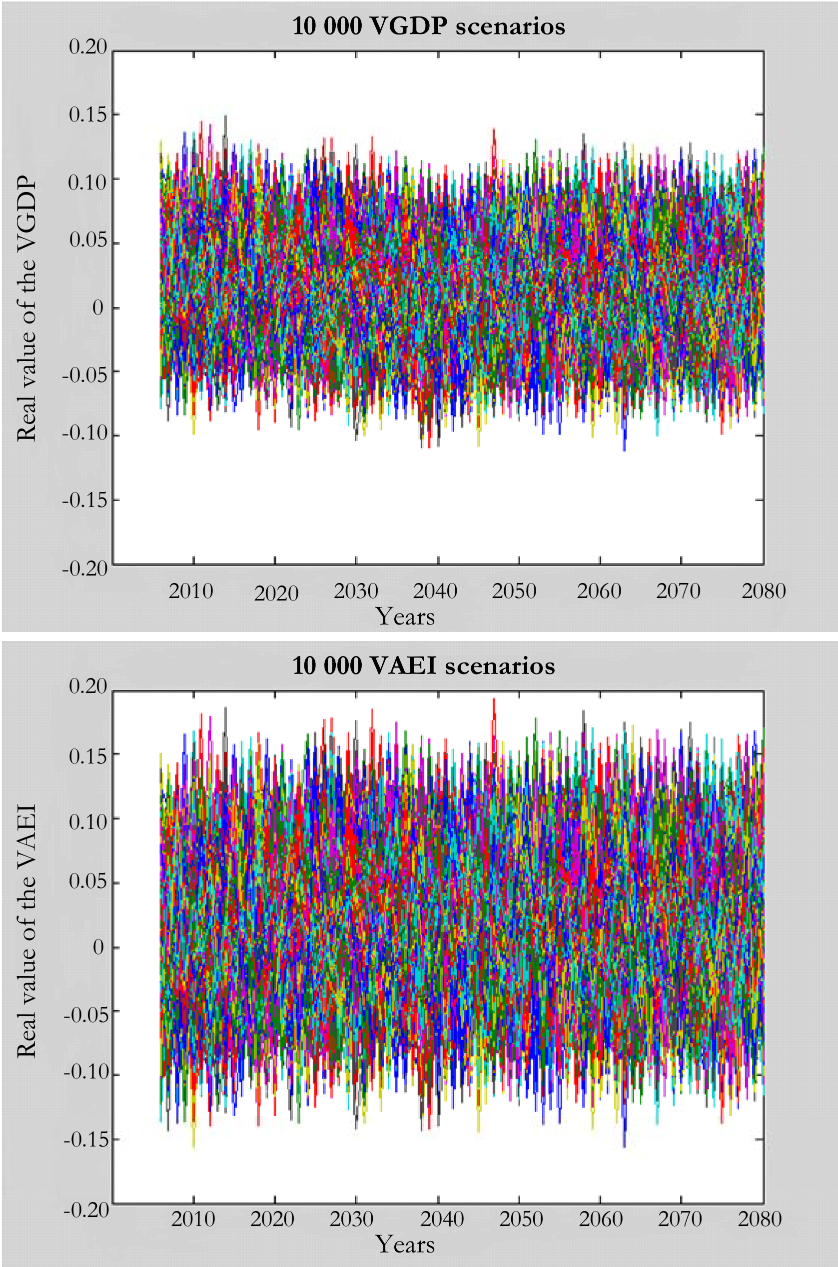

Figure 2: 10 000 scenarios of VGDP and VAEI for the period 2005-2080, based on the mean scenario of Alonso and Herce (2003).

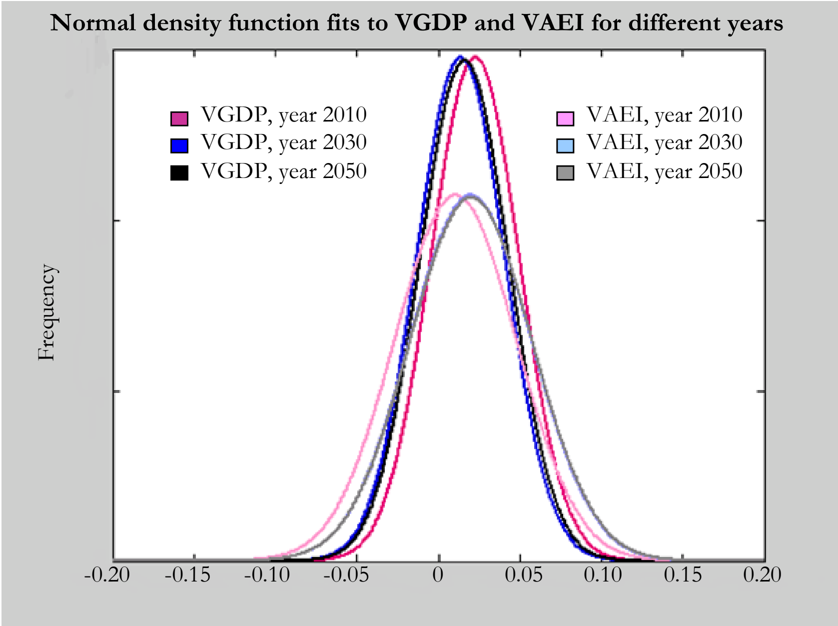

Figure 3: Normal density function fits to VGDP and VAEI for different years, taking as reference the mean scenario of Alonso and Herce (2003).

Table 3: Average IRR and standard deviation of the IRR for men and women of 65 years, AH (2003) projection.

Formula	IRRM	Dev	%dev	Formula	IRRW	Dev	%dev
10	0.01472	0.00545	37.03%	10	0.02455	0.00539	21.94%
2	0.01393	0.00473	33.94%	2	0.02324	0.00464	19.98%
5	0.01388	0.00518	37.28%	5	0.02319	0.00510	22.01%
6	0.01382	0.00555	40.15%	6	0.02312	0.00548	23.72%
9	0.01319	0.00507	38.45%	9	0.02299	0.00500	21.76%
8	0.01093	0.00447	40.91%	8	0.02095	0.00446	21.29%
1	0.01001	0.00341	34.08%	1	0.01951	0.00339	17.37%
3	0.00996	0.00404	40.50%	3	0.01946	0.00402	20.65%
4	0.00990	0.00452	45.70%	4	0.01939	0.00451	23.26%
7	0.00937	0.00397	42.39%	7	0.01936	0.00396	20.44%

Table 4: Average IRR and standard deviation of the IRR for men and women of 60 years, AH (2003) projection.

Formula	IRRM	Dev	%dev	Formula	IRRW	Dev	%dev
10	0.01745	0.00551	31.55%	10	0.02589	0.00543	20.99%
2	0.01652	0.00459	27.76%	2	0.02436	0.00449	18.43%
5	0.01645	0.00516	31.39%	5	0.02428	0.00508	20.92%
6	0.01637	0.00563	34.42%	6	0.02419	0.00556	22.98%
8	0.01488	0.00462	31.03%	8	0.02345	0.00460	19.60%
9	0.01478	0.00503	34.01%	9	0.02320	0.00494	21.31%
1	0.01384	0.00332	23.96%	1	0.02180	0.00327	15.01%
3	0.01377	0.00409	29.73%	3	0.02172	0.00406	18.70%
4	0.01369	0.00469	34.24%	4	0.02163	0.00466	21.55%
7	0.01219	0.00401	32.90%	7	0.02073	0.00398	19.20%

Table 5: Average IRR and standard deviation of the IRR for men and women of 70 years, AH (2003) projection.							
Formula	**IRRM**	**Dev**	**%dev**	**Formula**	**IRRW**	**Dev**	**%dev**
10	0.01111	0.00538	48.42%	10	0.02266	0.00532	23.49%
2	0.01051	0.00482	45.87%	9	0.02164	0.00502	23.22%
5	0.01043	0.00516	49.47%	2	0.02164	0.00475	21.96%
6	0.01037	0.00545	52.61%	5	0.02156	0.00510	23.64%
9	0.01013	0.00509	50.23%	6	0.02149	0.00540	25.11%
8	0.00692	0.00433	62.52%	8	0.01870	0.00433	23.13%
1	0.00621	0.00348	56.01%	7	0.01765	0.00392	22.20%
3	0.00613	0.00396	64.69%	1	0.01757	0.00348	19.79%
4	0.00606	0.00436	71.97%	3	0.01749	0.00396	22.67%
7	0.00590	0.00392	66.41%	4	0.01742	0.00436	25.04%

Table 6: $\mathrm{VaR}_{0.95}$ **for men and women, various ages and AH (2003) projection.**											
For	**VarM (60)**	**For**	**VarM (65)**	**For**	**VarM (70)**	**For**	**VarW (60)**	**For**	**VarW (65)**	**For**	**VarW (70)**
2	0.0090	2	0.0061	2	0.0025	2	0.0170	10	0.0156	10	0.0139
1	0.0084	10	0.0057	10	0.0022	10	0.0168	2	0.0155	2	0.0137
10	0.0082	5	0.0053	5	0.0019	1	0.0165	5	0.0147	9	0.0133
5	0.0079	9	0.0047	9	0.0017	8	0.0158	9	0.0147	5	0.0130
8	0.0073	6	0.0046	6	0.0013	5	0.0158	6	0.0140	6	0.0126
6	0.0069	1	0.0043	1	0.0004	9	0.0150	1	0.0139	1	0.0118
3	0.0069	8	0.0035	8	-0.0002	3	0.0149	8	0.0135	8	0.0116
9	0.0064	3	0.0032	3	-0.0004	6	0.0149	3	0.0128	7	0.0112
4	0.0059	7	0.0028	7	-0.0006	7	0.0141	7	0.0128	3	0.0110
7	0.0055	4	0.0024	4	-0.0011	4	0.0139	4	0.0119	4	0.0103

Table 7: Ranking of the formulas according to the Markowitz function criterion, for men and women of 65 years, with different values of risk aversion γ **and AH (2003) projection.**						
γ **=0**	γ **=50**	γ **=100**	γ **=150**	γ **=200**	γ **=300**	γ **=400**
Men						
10	10	10	10	10	2	2
2	2	2	2	2	10	10
5	5	5	5	5	5	5
6	6	6	6	6	9	9
Women						
10	10	10	10	10	10	2
2	2	2	2	2	2	10
5	5	5	5	5	5	9
6	6	9	9	9	9	5

Table 8: Preferred formula for each age according to the Markowitz function, for men and women, with different values of risk aversion (γ) and AH (2003) projection.							
Age	γ=0	γ=50	γ=100	γ=150	γ=200	γ=300	γ=400
60 M	10	10	10	10	2	2	2
65 M	10	10	10	10	10	2	2
70 M	10	10	10	10	10	2	2
60 W	10	10	10	10	10	10	2
65 W	10	10	10	10	10	10	2
70 M	10	10	10	10	10	10	2

Table 9: Ranking of the formulas according to the utility of the pension, for men and women of 65 years, with different values of risk aversion (β) and AH (2003) projection.				
β=0	β=1	β=2	β=3	β=5
Men				
10	10	2	2	2
6	2	5	5	5
5	5	10	6	6
2	6	6	10	10
9	9	9	9	9
Women				
10	10	10	10	2
6	2	2	2	5
5	5	5	5	10
2	6	6	6	6
9	9	9	9	9

Table 10: Preferred formula for each age according to the utility of the pension, for men and women, with different values of risk aversion (β) and AH (2003) projection.					
Age	β=0	β=1	β=2	β=3	β=5
60 M	10	10	2	2	2
65 M	10	10	2	2	2
70 M	10	10	2	2	2
60 W	10	10	10	2	2
65 W	10	10	10	10	2
70 W	10	10	10	10	2

Table 11: Average RR, average IRR, and $VaR_{0.95}$ of the IRR with GRMF-95 and with AH (2003) for men and women of 65 years. Comparison with PEMF-98-99.												
	RR (65)			IRRM(65)		IRRW(65)		VarM(65)		VarW(65)	
For.	GR	PE	For.	GR	PE	GR	PE	GR	PE	GR	PE
2	51.89%	61.99%	10	0.01506	0.01472	0.02522	0.02455	0.00658	0.00568	0.01695	0.01556
5	51.89%	61.99%	2	0.01423	0.01393	0.02351	0.02324	0.00715	0.00608	0.01664	0.01551
6	51.89%	61.99%	5	0.01413	0.01388	0.02341	0.02319	0.00613	0.00526	0.01561	0.01468
1	46.16%	55.14%	6	0.01405	0.01382	0.02332	0.02312	0.00534	0.00458	0.01480	0.01401
3	46.16%	55.14%	9	0.01334	0.01319	0.02343	0.02299	0.00559	0.00473	0.01584	0.01466
4	46.16%	55.14%	8	0.01162	0.01093	0.02203	0.02095	0.00457	0.00350	0.01509	0.01354
9	42.09%	52.74%	1	0.01062	0.01001	0.02014	0.01951	0.00549	0.00433	0.01511	0.01386
10	42.09%	52.74%	3	0.01052	0.00996	0.02004	0.01946	0.00418	0.00324	0.01382	0.01279
7	37.45%	46.92%	4	0.01043	0.00990	0.01994	0.01939	0.00325	0.00276	0.01285	0.01277
8	37.45%	46.92%	7	0.00987	0.00937	0.02021	0.01936	0.00369	0.00237	0.01412	0.01186

Table 12: Average RR, average IRR, and $VaR_{0.95}$ of the IRR with unanticipated changes in the survival rate and with AH (2003) for men and women of 65 years. Comparison with PEMF-98-99.											
	RR (65)			IRRM(65)		IRRW(65)		VarM(65)		VarW(65)	
For.	GR	PE	For.	GR	PE	GR	PE	GR	PE	GR	PE
2	61.99%	61.99%	10	0.02184	0.01472	0.03155	0.02455	0.01333	0.00568	0.02322	0.01556
5	61.99%	61.99%	9	0.02019	0.01319	0.02982	0.02299	0.01230	0.00473	0.02214	0.01466
6	61.99%	61.99%	2	0.01984	0.01393	0.02878	0.02324	0.01258	0.00608	0.02171	0.01551
1	55.14%	55.14%	5	0.01976	0.01388	0.02869	0.02319	0.01162	0.00526	0.02077	0.01468
3	55.14%	55.14%	6	0.01968	0.01382	0.02860	0.02312	0.01094	0.00458	0.02005	0.01401
4	55.14%	55.14%	8	0.01842	0.01093	0.02835	0.02095	0.01133	0.00350	0.02138	0.01354
9	52.74%	52.74%	7	0.01673	0.00937	0.02659	0.01936	0.01049	0.00237	0.02044	0.01186
10	52.74%	52.74%	1	0.01625	0.01001	0.02541	0.01951	0.01096	0.00433	0.02020	0.01386
7	46.92%	46.92%	3	0.01616	0.00996	0.02531	0.01946	0.00977	0.00324	0.01904	0.01279
8	46.92%	46.92%	4	0.01607	0.00990	0.02522	0.01939	0.00888	0.00276	0.01809	0.01277

Table 13: Expected mean replacement rate as a function of the average wage for AH (2003) and their variations for men of 65 years.			
Formula	AH(2003)	AH(2003)x1.5	AH (2003)x2
2, 5, 6	61.99%	73.23%	85.47%
1, 3, 4	55.14%	61.51%	67.88%
9, 10	52.74%	62.31%	72.72%
7, 8	46.92%	52.34%	57.76%

Table 14: Average IRR with AH (2003) and variations for men and women of 65 years.								
	AH (2003)			AH (2003) x1.5			AH (2003) x2	
Formula	M	W	Formula	M	W	Formula	M	W
10	0.01472	0.02455	10	0.02449	0.03440	10	0.03426	0.04426
2	0.01393	0.02324	9	0.02217	0.03203	9	0.03115	0.04107
5	0.01388	0.02319	2	0.02066	0.02992	2	0.02741	0.03662
6	0.01382	0.02312	5	0.02061	0.02987	5	0.02736	0.03657
9	0.01319	0.02299	6	0.02055	0.02980	6	0.02730	0.03650
8	0.01093	0.02095	8	0.01873	0.02894	8	0.02654	0.03693
1	0.01001	0.01951	7	0.01633	0.02648	7	0.02329	0.03361
3	0.00996	0.01946	1	0.01457	0.02412	1	0.01906	0.02868
4	0.00990	0.01939	3	0.01452	0.02407	3	0.01901	0.02862
7	0.00937	0.01936	4	0.01445	0.02400	4	0.01894	0.02855

Table 15: $VaR_{0.95}$ with AH (2003) and variations for men and women of 65 years.											
AH (2003)				AH (2003) x 1.5				AH (2003) x 2			
For	VarM	For	VarW	For	VarM	For	VarW	For	VarM	For	VarW
2	0.0061	10	0.0156	10	0.0154	10	0.0254	10	0.0252	10	0.0352
10	0.0057	2	0.0155	9	0.0136	9	0.0237	9	0.0226	9	0.0327
5	0.0053	5	0.0147	2	0.0127	2	0.0221	2	0.0194	8	0.0295
9	0.0047	9	0.0147	5	0.0119	8	0.0215	8	0.0191	2	0.0288
6	0.0046	6	0.0140	8	0.0113	5	0.0213	5	0.0186	5	0.0279
1	0.0043	1	0.0139	6	0.0112	6	0.0206	6	0.0179	6	0.0272
8	0.0035	8	0.0135	7	0.0097	7	0.0199	7	0.0166	7	0.0270
3	0.0032	3	0.0128	1	0.0088	1	0.0184	1	0.0133	1	0.0230
7	0.0028	7	0.0128	3	0.0078	3	0.0173	3	0.0122	3	0.0218
4	0.0024	4	0.0119	4	0.0068	4	0.0164	4	0.0113	4	0.0209

Table 16: Expected mean replacement rate as a function of the average wage for men of 65 for various projections.			
Fórmula	AH (2003)	MTAS(2005)	UE (2005)
2, 5, 6	61.99%	55.52%	59.88%
1, 3, 4	55.14%	52.46%	52.02%
9, 10	52.74%	47.24%	50.95%
7, 8	46.92%	44.64%	44.26%

Table 17: Average IRR for men and women of 65 years for various projections.								
AH (2003)			MTAS(2005)			EU (2005)		
Formula	M	W	Formula	M	W	Formula	M	W
10	0.01472	0.02455	2	0.00990	0.01932	2	0.01269	0.02206
2	0.01393	0.02324	5	0.00985	0.01926	5	0.01264	0.02201
5	0.01388	0.02319	6	0.00978	0.01920	6	0.01258	0.02194
6	0.01382	0.02312	1	0.00808	0.01759	10	0.01254	0.02239
9	0.01319	0.02299	3	0.00803	0.01754	9	0.01002	0.01980
8	0.01093	0.02095	4	0.00797	0.01747	1	0.00795	0.01755
1	0.01001	0.01951	10	0.00682	0.01659	8	0.00793	0.01801
3	0.00996	0.01946	9	0.00624	0.01600	3	0.00790	0.01749
4	0.00990	0.01939	8	0.00502	0.01488	4	0.00784	0.01742
7	0.00937	0.01936	7	0.00443	0.01429	7	0.00534	0.01535

Table 18: $VaR_{0.95}$ for men and women of 65 years for various projections.								
AH (2003)			MTAS(2005)			EU (2005)		
Formula	**M65**	**W65**	**Formula**	**M65**	**W65**	**Formula**	**M65**	**W65**
2	0.00608	0.01551	1	0.00237	0.01194	2	0.00483	0.01433
10	0.00568	0.01556	2	0.00202	0.01160	5	0.00400	0.01349
5	0.00526	0.01468	3	0.00132	0.01087	10	0.00343	0.01339
9	0.00473	0.01466	5	0.00120	0.01076	6	0.00332	0.01282
6	0.00458	0.01401	6	0.00052	0.01010	1	0.00227	0.01191
1	0.00433	0.01386	4	0.00043	0.00994	9	0.00150	0.01142
8	0.00350	0.01354	7	-0.00219	0.00770	3	0.00117	0.01080
3	0.00324	0.01279	9	-0.00232	0.00761	8	0.00048	0.01060
7	0.00276	0.01277	10	-0.00235	0.00755	4	0.00029	0.00988
4	0.00237	0.01186	8	-0.00244	0.00743	7	-0.00130	0.00878

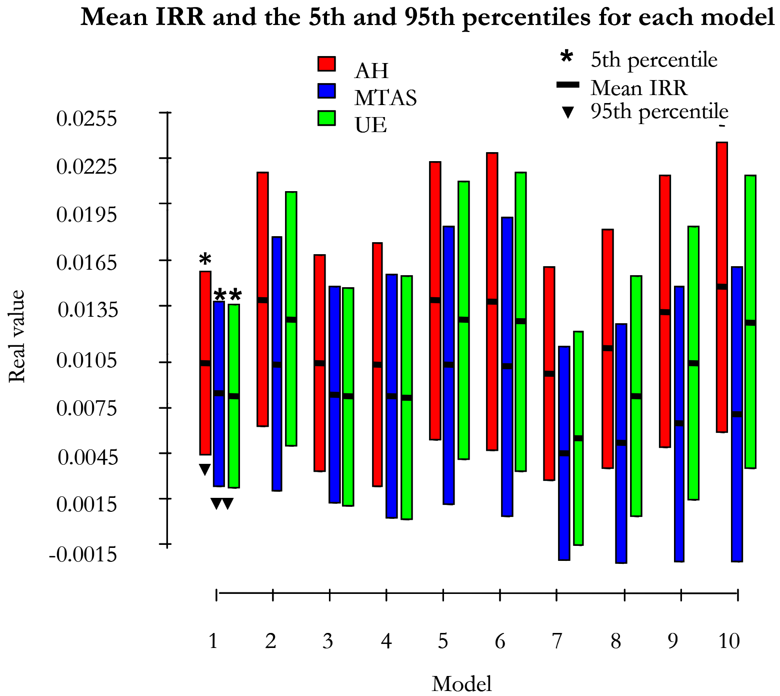

Figure 4: Mean IRR, and the 5th and 95th percentiles for each model, for men aged 65, using the mean value projections of Alonso anda Herce (2003), MTAS (2005), and EU (2005)