

"EMPLOYMENT, INVESTMENT AND BANKRUPTCIES
WHEN DISMISSAL COSTS ARE PRESENT".

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Documenta 87-13

INTRODUCTION

This paper is concerned with the dynamics of employment, investment and bankruptcies in a model where firms face dismissal costs in laying off workers and increasing labor costs. It is shown that such a model predicts an immediate decrease in investment, producing decreased employment opportunities for new entrants, and a delayed response in layoffs and bankruptcies. The dismissal costs play a major role in those results. In the identical model without dismissal costs there are no bankruptcies and layoffs respond immediately to any increase in labor costs. The timing described above matches that displayed by the aggregate economic time series for Spain beginning in the middle 1970's.

THE MODEL

Each period there is a distribution of investment projects. An investment project is a discrete entity; either the firm invests in it or it doesn't; there is no choice of capacity. Each project is characterized by a project specific production function. Each project is characterized by a one period production function.

$$\lambda f(n)$$

where n is employment and λ is the project specific factor. It is assumed that $f'(0) = \infty$. Once in place a project lasts forever; there is no depreciation of capital. Firms maximize discounted present value of profits over the lifetime of a project. The discount factor is β . Firms may hire labor in each period at the cost of w_t per worker. Two features of the model which are not standard are that there are dismissal costs associated with layoffs and a firm is allowed to declare bankruptcy at any point in time. Dismissal costs are represented by the factor cw_t per worker laid off in period t . The constant c represents the number of periods of pay which must be given to a dismissed worker as compensation. Bankruptcies are modelled in the following manner. In period t , if a firm wishes to declare bankruptcy then it loses all claims to capital and profits from period t onward will be zero. All earnings prior to period t are assumed to have been distributed to shar-

holders and hence are not affected by the bankruptcy decision.

Note that projects become available in each period. If a project is passed by in period t it is not available in any future period, although there will be a new project with the same value of λ in each future period.

THE FIRM'S PROBLEM

We shall begin by studying the firm's problem without allowing for the possibility of bankruptcy. This possibility will be considered later in the section. The firm has two different kinds of decisions to make. The first is whether or not to make the (discrete) investment in a particular project and the second is a time path for labor given that the investment takes place. Consider the second problem. If a firm decides to invest in a project characterized by the parameter λ at time t then the optimal path for labor is given by solving the following maximization problem:

$$\begin{aligned}
 (P-1) \text{ Maximize } & \lambda f(n_t) - r - wn_t + \\
 & (n_{t+1}) \\
 & + \sum_{j=1}^{\infty} [\beta^j f(n_{t+j}) - w_{t+j} n_{t+j} \\
 & - \max((n_{t+j-1} - n_{t+j}) w_{t+j} - c, 0)]
 \end{aligned}$$

The interpretation of this problem is quite straightforward but there are a few points worth

noting. The final term in this expression represents the dismissal costs that the firm must pay if it decides to lay off any workers. If employment increases between periods $t+j-1$ and $t+j$ then there is no cost. This one sided nature of adjustment costs is what makes it necessary to have the maximum operator in the last term. The second point to be noted is that when the firm is starting up there are no adjustment costs in the first period. For this reason the terms representing profits in the first period are written separate from the infinite sum. Finally, note the indexing: the investment takes place in period t and j indexes the number of periods that the projects has been in existence.

Let $(n^*_{t+j}(*))$ be the solution to problem (P-1), and let $\pi_t^*(\lambda)$ be the discounted present value of profits which results from carrying out the investment and letting the time path of employment follow $[n^*_{t+j}(\lambda)]$. Then $\pi_t^*(\lambda)$ is the maximum discounted present value of profits corresponding to undertaking a project with parameter λ in period t . The firm now needs to decide whether or not to undertake this project. If the firm doesn't invest in the project it receives discounted present value of profits of zero. Hence the decision of the firm is straightforward: Invest in the project with parameter λ at time t if $\pi_t^*(\lambda) \geq 0$ and don't invest otherwise.

Note that a project cannot be saved for future periods. In period $(t+1)$ there will be another potential project with parameter λ but the project with parameter λ which becomes available in period t cannot be saved up for future periods.

Although we have used the notation $\pi^*(\lambda)$ it should be noted that profits also implicitly depend upon wages and other parameters of the problem.

Before proceeding to allow for the possibility of bankruptcies we will study some properties of the problem without bankruptcies. The first two results are immediate.

Result One: The Function $\pi^*(\lambda)$ is strictly increasing in λ .

Proof: Fix λ . Consider $\lambda < \lambda$. If a firm uses the same employment path for λ as is optimal for λ then a strictly greater profit than $\pi^*(\lambda)$ will result because revenues are strictly greater and costs are the same. Hence the maximum profit must also be strictly greater.

Result Two: The function $\pi^*(\lambda)$ is strictly decreasing in each u_{j+1} for $j \geq 0$.

Proof The proof is analogous to that used above. It is important to note that under the boundary condition assumed on $f'(0)$ each of the n^*_{j+1} will be strictly positive.

One of the implications of result one is that in each period there will be a reservation value of λ , say $\bar{\lambda}_t$, with the property that only projects with $\lambda \geq \bar{\lambda}_t$ will be undertaken in period t . Projects with $\lambda < \bar{\lambda}_t$ will be passed by in period t .

Consider a wage path given by:

$$w_t = w$$

labor force in period two in response to the higher wages.

We first show that if any adjustments are to be made then they will all be carried out in period two. Let n_t^* be the optimal first period employment. Using $n_t^* = n_t^*$ for all t as a starting point consider decreasing employment by Δn at some point t where $t > 1$. Compare profit of these two alternative paths. Each path gives identical profit for the first $t - 1$ periods so we need only compare the terms after this. With no adjustment there is a profit of

$$\lambda f(n_t^*) - (\omega + \epsilon) n_t^*$$

in each period, so viewed from period one the contribution of these terms to the discounted present value of profits is

$$\beta^t \frac{1}{1 - \beta} [\lambda f(n_t^*) - (\omega + \epsilon) n_t^*]$$

If the adjustment is carried out in period t then profits are

$$\lambda f(n_t^* - \Delta n) - (\omega + \epsilon) (n_t^* - \Delta n)$$

in each period except in period t where there is an additional cost of $c(\omega + \epsilon) \Delta n$. The contribu-

tion of these terms to the discounted present value of profits is given by:

$$\beta^{t-1} \left[\frac{1}{1-\beta} (\lambda f(n_t^* - \Delta n) - (w+E)(n_t^* - \Delta n)) - c (w+E)\Delta n \right]$$

If the adjustment of labor in period t increases profits then the term in square brackets in the second expression is larger than the term in square brackets in the first expression. And the change in the discounted present value of profits is given by β^t times the difference between the two expressions in square brackets. since this difference is multiplied by β^{t-1} it follows that the adjustment should be made as early as possible, since β^{t-1} is decreasing in t , the period in which the adjustment is made.

Returning to the main line of argument, for the given wage sequence (w_t) it follows that any adjustment to be made to employment should be made in period two. If Δn is small then the change in profits resulting from this adjustment is given by

$$\beta \left[\frac{\Delta n (w+E)}{1-\beta} - \frac{\lambda f'(n_t^*) \Delta n}{1-\beta} - \Delta n c (w+E) \right]$$

The first term is the saving which results from a lower wage bill. The second term is the output sacrificed by laying off workers and the

last term is the dismissal cost which must be paid. As ϵ goes to zero this expression becomes

$$\beta \Delta n \left[\frac{w}{1 - \beta} - \frac{\lambda f'(n_t^*)}{1 - \beta} - c w \right]$$

Since n_t^* is the optimal choice of labor it must be the case that

$$w \leq \lambda f'(n_t^*)$$

since the firm would never hire labor past the point where the wage equals the marginal product of labor, given that wages are always going to be higher in the future. It follows that when ϵ is small the change in profits is negative, independently of the magnitude of Δn . Hence, there exists some $\bar{\epsilon}$ such that for $\epsilon < \bar{\epsilon}$ employment adjustments are not optimal.

The reasoning behind this result is that for small wage changes the increase in wage payments is of the order of magnitude of the change in wages. On the other hand, dismissal costs per worker are of the order of magnitude of wages. This should be contrasted with the case of quadratic adjustment cost which have received much attention in the literature. In these models the per unit adjustment cost for small changes is of the magnitude of the change. Small changes are zero marginal cost. This produces a gradual res-

ponse to one time changes in the level of wages. But there is always an initial response to any change in wages, however small. With the dismissal cost, which is a linear adjustment cost, the adjustment process is discrete. There may or may not be an adjustment, but if there is it takes place in one period.

Generalizing the arguments used above it is possible to show that if there is a wage sequence with gradually increasing wages the typical response of a firm is to have a period of no adjustment followed by a discrete adjustment once the wage change has reached a certain threshold.

We now turn to the question of bankruptcy. First we show via an example how bankruptcy may arise in this model. The example is somewhat contrived but it illustrates the forces which are at work. Let wages follow the path.

$$\begin{aligned}w_t &= w \\w_t &= 11w \quad t = 2\end{aligned}$$

From the above analysis it follows that if c is large enough then there will be no layoffs in response to the wage change. Hence, all the firm needs to choose is n^* , the initial level of employment. This is chosen so as to:

$$\text{Max } f(n_t) - wn_t + \frac{\beta}{1 - \beta} f(n_t) - \frac{\beta}{1 - \beta} 11w \quad n_t$$

The first order condition for this problem is:

$$f_1(n_1) = w + 10 \beta w$$

Assume that

$$f(n) = n^{1/2}$$

$$\beta = .1$$

The solution to the first order condition is

$$n_1 = \left[\frac{1}{4w} \right]^2$$

First period profits are

$$n_1^{1/2} - wn = \frac{1}{4w} - \frac{11}{16bw} > 0$$

Now, as long as $r < \frac{1}{4w} - \frac{1}{16bw}$ it is optimal

for this project to be undertaken, but the firm should declare bankruptcy at the end of the first period. Note that this will never happen if there are no dismissal costs. If $c = 0$, the firm simply equates the marginal product of labor with the real wage. Under the standard assumption that $f'(0) = \infty$, a firm would never choose to become bankrupt. In the presence of dismissal costs this is no longer true because the firm may find itself a position where the dismissal costs are too large

to make layoffs profitable, but that in the presence of wage increases the payroll becomes too large to make operation profitable. The result is that it is optimal for the firm to declare bankruptcy.

The next point we want to establish is that a small increase in wages in period two will never cause firms which started operating in period one to declare bankruptcy. Let wages follow

$$\begin{aligned}w_1 &= w \\w_2 &= w + \epsilon \quad \epsilon > 0\end{aligned}$$

Where $\epsilon > 0$. We will show that for ϵ less than some $\bar{\epsilon}$ the firm will not declare bankruptcy. We will prove this by way of contradiction. Suppose that it is optimal for the firm to declare bankruptcy in period two. Then n_1^* is the only choice variable. Because the project was undertaken in period one it must yield non-negative profit. This implies,

$$\lambda [f(n_1^*) - wn_1^* - r] \geq 0$$

or

$$\lambda [f(n_1^*) - wn_1^*] \geq r$$

Suppose that instead of declaring bankruptcy the firm continued to operate with the same level of employment. Then profits in each subsequent period would be

As long as $E < \frac{r}{n_1^*}$ this expression will be positive. Hence it cannot be optimal for the firm to declare bankruptcy.

The intuition behind this result is that capital serves as a fixed cost when the bankruptcy decision is made. Even if the project initially had zero discounted present value profit, when wages increase in period two the project does not become unprofitable to operate. As a new project it would not be profitable to start it up, but as an existing project the cost of capital is sunk and the small increase in wages does not make bankruptcy optimal. Note that for this result it is not essential that all capital costs be borne in the first period. It is only important that some of the costs are paid out in the first period.

Generalizing the argument used above it can be shown that if wages are increasing gradually over time there will be a period where no bankruptcies occur followed by one where the wage increase is sufficient to potentially bring about some bankruptcies. Note that firms with lower values of λ are more likely to declare bankruptcy since profits are an increasing function of λ .

DISCUSSION

In this section we illustrate how the preceding analysis relates to the observations mentioned in the introduction. In particular, we are

concerned with the timing of certain changes. Starting in the mid 1970's. There was a pattern of increasing unit labor costs. There simultaneously was a pattern of decreasing investment. Initially there was some increasing unemployment among youths but virtually no change in employment. Several years later large scale bankruptcies and employment losses resulted.

The model of the preceding section predicts exactly this type of qualitative response. As noted, as labor costs gradually increase the only initial response is a decrease in investment. There is no immediate response in either bankruptcies or layoffs. As there is less investment taking place this will decrease the ability of new entrants in the labor market to find unemployment. However, at some point the wage increases may be sufficient to cause layoffs by some firms and force some less profitable firms completely out of business. This is exactly the qualitative pattern experienced in Spain.

Characterizing solutions quantitatively in this model is a very difficult task and will not be attempted here. In other papers I have shown the potential empirical significance of increased labor costs and of dismissal costs in much simpler frameworks. The analysis performed here simply confirms that increasing labor costs and costly labor adjustment may have been important factors in the development of the Spanish economy since the mid 1970's.

Although the analysis has concentrated on increasing costs of labor as the only changing factor in the economy it should be clear that there are other sources of change which would

cause similar results. Changes to technology which lower productivity will have an almost identical impact as changes in wages, since what matters in the firms optimization problem is the relationship between wages and productivity. A second source of change is interest rates. A temporary increase in real wages accompanied by a temporary increase in interest rates will have effects that are similar to a permanent wage increase. If the increase in wages is substantial then the current period profits may be negative. But if interest rates also rise, future profits have a lower present value and it may be sufficient to cause firms to declare bankruptcy.