

ESTIMATION OF DYNAMIC INVESTMENT
FUNCTIONS IN OIL EXPLORATION

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Abstract

This paper studies the dynamics of the discovery process for oil and gas pools in the province of Alberta, Canada from 1950 to 1979. A costs of adjustment, rational expectations model is specified in which current and future returns from exploration are random, and the main source of the uncertainty is the amount of discoveries. The parameters are estimated from the stochastic Euler equation using the Generalized Methods of Moments estimator proposed by Hansen (1982). We find that, at the appropriate level of aggregation, there is a stable, quantifiable relationship between exploratory investment and returns which is consistent with optimizing behavior.

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1. Introduction

Empirical models of exploration can be grouped into two categories. The first is supply models which focus on estimating the probability law governing the sequence of discoveries¹. Such models usually ignore the economic decisions which help determine the sample of drilling outcomes. The second is econometric models which purport to estimate the effects of economic incentives on the flows of drilling inputs and discoveries.² These usually involve an ad hoc specification of the forces determining supply in which measures of drilling inputs and discoveries are regressed on one-period lags of these variables, and proxies for prices and costs. An exception is the work by Eppler (1984), who develops an optimizing, dynamic model of investment behavior with rational expectations.

Both categories have their problems. The supply models can predict the probability of finding at least z more reserves if k more wells are drilled, but they cannot address the issue of whether the k wells should be drilled and if so, at what rate. Furthermore, their failure to account for the behavioral process which generates the data is likely to bias the forecasts. The econometric models have yielded notoriously unreliable estimates of supply elasticities. In his evaluation of three of these studies, Pindyck (1975) found that estimates of supply elasticities were highly unstable with respect to both model specification and time period. He attributed the instability to two factors: the absence of an optimizing framework and an inadequate specification of the dynamic structure of the discovery process. We would add a third factor, aggregation of investment paths and returns from diverse geographical regions.

The work by Eppler (1984) addresses some of these criticisms. He derives the firm's decision rule from a dynamic, stochastic optimization model

in which the firm chooses a contingent production plan to maximize the expected present value of resources. The distinguishing feature of nonrenewable resources like oil and gas is, of course, that they are depletable. This aspect is incorporated into Epple's model through the assumption that the costs of finding and developing new deposits in future periods depend upon cumulative production. Thus, in choosing its current output, the firm must take into account the effect of its decision on expected future extraction costs. Forecasts of ultimate recovery are calculated from the condition that expected marginal extraction costs of the last unit produced must be equal to the resource price.

Our modelling is in the same spirit as Epple's, but the approach is quite different. We focus on the decision of the individual firm to drill exploratory wells in a specific area, Alberta, Canada. The optimal drilling plan is characterized as a solution to a stochastic Euler equation which equates the expected returns of a exploratory well to expected marginal drilling costs. Uncertainty about returns is caused primarily by randomness in discoveries rather than in drilling costs. The effect of depletion is incorporated into the model through the stochastic process determining returns: average returns are assumed to decline over time. Our objective is to establish whether a stable, quantifiable relationship between exploratory investment and returns exists which is consistent with optimizing behavior. In particular, we are interested in determining whether a dynamic model is necessary to explain the data (i.e., are there any costs of adjustment), and how information flows from drilling activity influence expectations of current and future returns.

The presence of nontrivial investment dynamics has important implications for measuring the impact of changes in the economic environment. Previous studies have generally focused on short-run measures. But this is

valid only if exploratory investment is not a dynamic process. Otherwise, the appropriate comparison is between the time paths of investment and discoveries that are generated by alternative economic environments. In particular, one needs to recognize the possibility that a once-and-for-all increase in, say, the price of oil, is likely to "tilt" the drilling profile in a region to the present. As a result, much of the initial response in discoveries may be a transfer from the near future to the present, and does not represent finds that would not have occurred otherwise.

It is also important to determine the role of information in the exploration process. If information spillovers are substantial, the competitive allocation of resources is probably not efficient. Too much drilling may occur in areas where discoveries have been made, and too little in areas which are relatively unexplored.³ Learning may also help explain why estimates obtained from data which aggregate returns and investments from diverse regions are often so imprecise and unstable. The response of investment and discoveries in a region to changes in the economic environment may depend upon its stage of development. In that case, there is no reason to suppose that the relationship between aggregate investment and aggregate returns is a stable one if the vintage distribution of areas is not stationary.

Finally, we should note that, in contrast to the previous literature, we do not try to forecast reserves. The reason is that the model is partial equilibrium: it predicts the supply path of drilling inputs of a representative firm when that firm faces a fixed probability law for drilling returns. At the industry level, this probability law is almost certain to be endogenous, since depletion effects implies that current drilling returns should depend upon the past history of industry drilling inputs. This endogeneity of the returns process needs to be taken into account when making

forecasts about a region's reserves and stocks of wells.

The paper is organized as follows. In the next section, we introduce our model of the firm's investment problem. The estimation procedure is outlined in Section 3. Although it is possible to solve explicitly for the decision rule under appropriate assumptions on the stochastic processes, we have chosen instead to estimate the parameters of the objective function from the Euler equation using the Generalized Method of Moments estimator introduced by Hansen (1982). The main reasons for doing so are the relatively small number of observations, and our reluctance to impose the requisite restrictions on the stochastic process for returns. We test the model using the restrictions implied by the theory. Previous applications of this method include studies by Hansen and Singleton (1982) on stock returns data and by Pindyck and Rotemberg (1983, 1984) to energy demand.

The data is described in Section 4. A virtue of the data set is that exploratory wells are distinguished from development wells, and discoveries from revisions to initial estimates. This is important since the evidence strongly suggests that discoveries from exploratory wells generate information which affect drilling decisions on other prospects, but that revisions to reserves due to development wells do not. The estimation results are presented in Section 5, and conclusions follow in Section 6.

2. The Investment Model

We begin with a brief description of the exploration process in the province of Alberta, Canada.

The ownership of natural resources in Canada is, for the most part, invested in the provinces. In developing its hydrocarbon reserves, the Alberta government has generally followed a policy of retaining public ownership, and contracting with firms in the private sector to find and

produce the deposits. For example, in 1971, less than 12% of the total oil and gas acreage in the province of Alberta was privately owned. The rest of the acreage was leased by the government to oil and gas firms in a variety of tenure arrangements.

Three types of licenses control exploration for oil and gas in Alberta. Geophysical licenses give non-exclusive rights to conduct private geological, seismic, magnetometer, and other types of surveys almost anywhere in the province, including those areas subject to existing leases. The license does not permit the drilling of test wells. Anyone can obtain a geophysical lease, and it is renewable each year for a fee. Firms use the surveys to identify promising areas, or prospects, over which it desires to obtain exclusive rights to drill exploratory wells and to produce oil and gas.

Drilling and production rights are usually sold via a first price, sealed bid auction. Initially, firms bid for a petroleum and natural gas reservation, which gives the winner an exclusive right to drill in a specified area. The size of the area cannot exceed 156 square miles. The term of a reservation is four months, but it can be extended for several years with a pledge to conduct an exploration program that meets the approval of the government. If oil or gas is discovered, or prospects look promising, the reservation holder would normally apply for petroleum and natural gas leases, which confer long-term (at least 10 years) rights to produce oil and gas on designated parts of the reservation. A lease cannot exceed eight or nine square miles, or be less than a quarter-section. Furthermore, the reservation holder cannot apply for more than 50% of any township subject to the reservation, or for leases that are adjacent.⁴ This gives rise to a "checkerboard" pattern of lease ownership. Leases not claimed by the reservation holder are returned to the government as Crown reserves, and subsequently sold in first-price, sealed bid auctions. The reservation holder

is required to submit cores and logs from any wells drilled, and this information is made public one year after the application date.

The markets for land and drilling inputs into the exploration process were competitive. For any given year in the sample period from 1946 to 1979, the fraction of exploratory wells drilled by the top sixteen oil companies ranged from a low of 22 per cent to a high of 40 per cent. Thus, the individual firm's annual investment in exploratory wells was small relative to the total number of wells drilled by the exploration industry. Wellhead prices for the Alberta oil and gas products were clearly exogenous to the firms, since these prices were determined by the international market, and total output of oil and gas from Alberta was only a tiny fraction of the volumes transacted in this market.

Wellhead prices for oil were roughly constant in nominal terms until 1973, after which they rose according to a formula imposed by the federal government. Wellhead prices for gas increased steadily throughout the sample period, but were generally too low prior to the 1970's to make exploration and development of gas pools a profitable investment. Nominal drilling costs per well were also basically constant throughout the sample period, ranging between \$70,000 to \$130,000, depending upon the depth of the well. Thus, for our sample, the main source of uncertainty in returns was the sizes of the discoveries. Many prospects did not contain sufficient quantities of oil and gas to make production profitable and, as a result, the cost of drilling exploratory wells was not recovered.

We shall consider a somewhat stylized model of firm behavior. At the beginning of each period, each firm must decide on the number of prospects it should acquire and drill in that period, taking as given market prices of prospects and wellhead prices of oil and gas products. Prospects not drilled by the end of the period are returned to the government. We shall further

assume that prospects and drilling rigs are combined in fixed proportion to produce wells. Given these assumptions, the planning problem of the representative firm can be formulated in terms of choosing a contingent drilling program to maximize the expected present value of discoveries⁵:

$$(2.1) \quad E_t \sum_{j=0}^{\infty} \delta^{t+j} \{ (v_{t+j} a_{t+j} - b_{t+j}) x_{t+j} - \alpha_1 x_{t+j} - (1/2) \alpha_2 x_{t+j}^2 - (1/2) \alpha_3 (x_{t+j} - x_{t-1+j})^2 \}.$$

In the above notation: x_t is the number of exploratory wells drilled during period t , a_t is the amount of reserves discovered per well during period t , v_t is the after-tax, present value price of a unit of reserves discovered in period t , and b_t is the cost of period t drilling inputs. The vector of cost parameters, $\alpha = (\alpha_1, \alpha_2, \alpha_3)$, is nonnegative, and the discount parameter δ satisfies $0 < \delta < 1$. The operator E_t denotes the conditional expectation: $E_t y = E[y | \Omega_t]$ where y is a random variable, and Ω_t is the firm's information set at the beginning of period t . It includes, in particular, b_{t-i} and x_{t-i} , for all $i \geq 0$, current and lagged prices of oil and gas, and a_{t-i} for $i > 0$. It does not include a_t , which is random at the time the firm drills the wells.

Define $\pi_t \equiv v_t a_t - b_t$ to be the dollar return per well drilled net of drilling costs. We shall assume that the individual firm views the stochastic process governing π_t as lying outside of its control, and that it is of exponential order less than $1/\delta$. Then, differentiating (2.1) with respect to x_t yields the Euler equation:

$$(2.2) \quad E_t [\pi_t - \alpha_1 - \alpha_2 x_t - \alpha_3 (x_t - x_{t-1}) + \delta \alpha_3 (x_{t+1} - x_t)] = 0.$$

The transversality condition is

$$(2.3) \quad \lim_{T \rightarrow \infty} E_t \delta^T [\pi_t - \alpha_1 - \alpha_2 x_t - \alpha_3 (x_t - x_{t-1}) + \delta \alpha_3 (x_{t+1} - x_t)] = 0.$$

Our assumption on $(\pi_{t+j})_{j=0}^{\infty}$ ensures that equation (2.3) is satisfied. Also,

given our representative firm assumption, we shall assume that the solution to the Euler equation is interior (i.e., $x_t > 0$ for all t).

The strength of our model is that it captures two features which we believe are essential to understanding the exploration process. First, current and future returns are stochastic because of the randomness in discoveries. If learning is present, and depletion effects are anticipated, then expectations about these returns depend upon past discoveries and drilling efforts. Second, fluctuations in drilling activity are costly. One interpretation of this property is that there is a fixed factor, such as the stock of drilling rigs, which makes it costly for a firm to change its level of drilling activity from period to period. Note that if there are no costs of adjustment (i.e., α_3 is equal to zero), then there are no dynamics to explain, since drilling effort in each period would depend only upon the expected returns in that period.

The restrictive aspect of the model lies in the assumption that firms cannot hold inventories of prospects. Each firm acquires only as many prospects as it intends to drill in that period. This assumption has several important implications. It means that a firm cannot delay exploration of a prospect until after the outcomes of wells being drilled in the current period are known. Furthermore, information from current drilling efforts about prospects sold in subsequent periods is incorporated into the prices of these prospects since drilling outcomes are observable. Hence, a prospect has no informational value to its owner, and private drilling returns are determined solely by the amount of hydrocarbons which can be produced.

The purpose of the above assumption is to rule out any incentive firms might have to hold prospects for speculative purposes. Its necessity follows from limitations in the available data, which identifies the location of an exploratory well and the number of wells drilled each year, but not which

firms drilled them or their lease holdings. The empirical significance of the assumption is not clear. As mentioned previously, many tenure arrangements are in fact relatively short-lived, although in most instances, they seem to be longer than the period required to drill and evaluate core samples. Consequently, it is a restrictive assumption, and we shall have to bear it in mind when interpreting the estimation results.

Finally, we should point out that the model is partial equilibrium. The probability law governing drilling returns is assumed to be exogenous to the individual firm. In a more complete model, the amount of discoveries in each period would be determined simultaneously with investment in wildcat wells, and the supply paths of both drilling inputs and discoveries would be expressed as functions of the underlying cost technology and the probability law for discoveries.

3. Estimation Strategy

The literature suggests several procedures for estimating the model. The simplest method is the approach followed by McCallum (1982) and Kennan (1979). They use realizations of future variables in the Euler equation as proxies for their conditional expectations, and treat the estimation of the resulting model as an errors in variables problem. As Hansen and Sargent (1980,1982) point out, this method ignores a number of restrictions implied by the model. A major benefit of the linear-quadratic structure is that, given suitable restrictions on the stochastic environment, one can obtain closed form solutions for the decision rule. Hansen and Sargent exploit this property in developing full information estimators for models such as ours.

The methods of Hansen and Sargent involve estimation of a relatively large set of parameters. In addition to those in the decision rule, one must estimate the parameters of the vector autoregressive representation of the

instrumental variables, and the parameters of the linear projections of the forcing variable, π_t , and the decision variable, x_t , on the set of instruments. We have at most 33 observations, of which we lose at least two due to lags in the decision rule and instruments. This number seems too few to implement with any degree of confidence the Hansen-Sargent methods. In addition, we were somewhat reluctant to impose the requisite restrictions on the stochastic structure.

Hansen and Singleton (1982) suggest an alternative estimation strategy based upon the General Methods of Moments estimator proposed by Hansen (1982). This method interprets the Euler equation as implying a set of orthogonality conditions, which can be used to obtain estimates for the parameters, and to test the model. As the authors note, an important benefit of this approach is that it does not require strong a priori assumptions about the properties of the stochastic process of the forcing variable π_t . In particular, it permits conditional variances of the disturbances to depend on variables in the information set.

In describing the estimation procedure, it will be useful to express the Euler equation in the form:

$$(3.1) \quad E_t[x_t - \beta_0 - \beta_1 x_{t-1} - \beta_2 x_{t+1} - \beta_3 \pi_t] = 0,$$

where:

$$\beta_0 = -\alpha_1/[\alpha_2 + \alpha_3(1-\delta)]$$

$$\beta_1 = \alpha_3/[\alpha_2 + \alpha_3(1-\delta)]$$

$$\beta_2 = \delta\alpha_3/[\alpha_2 + \alpha_3(1-\delta)]$$

$$\beta_3 = 1/[\alpha_2 + \alpha_3(1-\delta)].$$

Our strategy will be to derive an estimator for $\beta = (\beta_0, \beta_1, \beta_2, \beta_3)$, and then obtain estimates for α from the four equations given in (3.1). Standard

errors will then be obtained using the δ method.

Following Hansen and Singleton (1982), we define the function

$$h(Y_{t+1}, \beta) = x_t - \beta_0 - \beta_1 x_{t-1} - \beta_2 x_{t+1} - \beta_3 \pi_t,$$

where $Y_{t+1} = (x_{t-1}, x_t, x_{t+1}, \pi_t)$ is the vector of variables in the decision rule. Deviations of $h(Y_{t+1}, \beta)$ from zero can be interpreted as arising from errors in expectations. Letting v_{t+1} denote this error, the Euler condition can then be written as

$$h(Y_{t+1}, \beta) + v_{t+1} = 0$$

or:

$$(3.2) \quad x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 x_{t+1} - \beta_3 \pi_t + u_{t+1},$$

with $u_{t+1} = -v_{t+1}$.

If expectations are rational, the residual in equation (3.2) should be unpredictable, given the available information: $E_t[u_{t+1}] = 0$. Furthermore, residuals are serially uncorrelated, since, letting E denote the unconditional expectations operator, $E[u_{t-s} \cdot u_{t+1}] = E[E_t[u_{t-s} \cdot u_{t+1}]] = E[u_{t-s} \cdot E_t u_{t+1}] = 0$ for all $s \geq 0$. Finally, although not required by the estimation procedure, we shall make the simplifying assumption that the errors are conditionally homoscedastic.

The Hansen-Singleton estimator is an instrumental variables procedure which minimizes the correlation between variables known at time t and the residuals of equation (3.2). Ideally, the vector of instruments is the expectation of the gradient vector $\partial h(Y_{t+1}, \beta) / \partial \beta$ conditional on the information set at time t . Hansen and Singleton note that, in general, choosing the optimal instruments requires a detailed specification of the economic environment. The task is greatly simplified if one is willing to

assume that conditional expectations coincide with linear projects. We make this assumption. Combined with the conditional homoscedsticity assumption, the estimation procedure then reduces to two step least squares.

The estimation method uses only four of the available orthogonality conditions to identify the parameter vector β . The remaining othogonality conditions can then be used as a specification test since, if the model and its behavioral restrictions are correct, the sample analogues of the cross-products of the residuals and the instruments should be close to zero. The appropriate test statistic for the hypothesis that these restrictions are satisfied has been derived by Hansen (1982). He has shown that the asymptotic distribution of T times the minimized value of the objective function is chi-square with $p-r$ degrees of freedom, where p is the number of instruments and r is number of orthogonality conditions actually used for identifying β .

To summarize: we propose to estimate the structural parameters of our model directly from the stochastic Euler equation using the Generalized Instrumental Variables estimator of Hansen and Singleton. The specification of the model will be tested by examining whether or not (1) the residuals are serially uncorrelated, (2) the set of overidentifying restrictions are satisfied, and (3) the coefficients β_1 and β_2 satisfy the linear restriction $\beta_1 = \delta\beta_2$ for reasonable values of δ . We also propose to test the hypothesis that there are no costs of adjustment (i.e., $\beta_1 = \beta_2 = 0$) by estimating the static model in which wildcat drilling is a linear function of $E_t\pi_t$.

4. Description of Data

The data were obtained from the study submitted by Uhler (with Eglington) (1983) to the Economic Council of Canada on the exploration for oil and gas deposits in the province of Alberta, Canada. An important feature of the data set is the geographical detail of the drilling and discovery

information.

Oil and gas pools in this study were indexed by areal location and geological horizon (depth). Horizons refer to the substrata associated with different geological periods. The province was divided into ten areas and ten horizons, generating one hundred possible (horizon,area) pairs. For each (horizon,area) pair, Uhler gives annual data on changes in oil and gas reserves separately, and on drilling activity. The sample period is basically from 1950 to 1979. Discoveries are reported separately from revisions, and the latter are reported by vintage. That is, annual revisions of pools discovered in year t are listed separately from those of pools discovered in year s , $s \neq t$.

The drilling data for each (horizon,area) pair reports the number of wells drilled annually, and the type of well: wildcat wells, which are drilled in search of new pools, and development wells, which are drilled in pools that have already been discovered. Wells were allocated to each category under the assumption that the deepest horizon penetrated is the target horizon.

Since the amount of drilling in many of the (horizon,area) pairs was quite small, Uhler (1983) reduced the number of categories to eight, mainly by aggregating across areas within an horizon. We combined the two Lower Devonian categories, and dropped one category from the sample due to incomplete drilling data. (It contained virtually no oil pools, and the number of wells drilled prior to the 1970's was quite small.) The remaining six categories are listed in Table 1. They are ordered by depth, with the Lower Devonian horizon representing the deepest horizon. Gas reserves are measured in billions of cubic meters, and oil reserves are measured in millions of cubic meters. Gas reserves in pools which contain both oil and gas are called associated gas (AG). Reserves in gas pools are called nonassociated gas (NAG). The Upper Cretaceous horizon includes only oil and

gas pools discovered in the central part of Alberta. The other horizons include all pools discovered in these horizons throughout the province.

The pools in the shallower horizons tend to have a higher gas content than oil. The exception is the Upper Cretaceous horizon, where 170 million cubic meters of oil were discovered. This figure is somewhat misleading, however, since most of the oil reserves, 130 million cubic meters to be precise, are in one pool, the gigantic Pembina pool. It was discovered in 1953, and accounts for approximately 11% of Alberta's oil reserves. Most of the remaining 89% of Alberta's oil reserves lie in the deeper Devonian horizons.

The differences in the ratio of the number of wildcat wells to the number of development wells across horizons reflect differences in the size distributions of pools. When a wildcat well discovers a pool, it is usually completed and turned into a producing well. Development wells are drilled only if the pool is sufficiently large that more than the one well is needed for optimal production. In the Lower Devonian and Mannville horizons, the oil and gas pools were relatively small, and frequently did not require any development wells. The gigantic Pembina pool, on the other hand, required a large number of development wells and, as a result, the ratio of wildcat to development wells in this horizon was relatively low. Many of the pools in the Upper Devonian horizon were also quite large.

What is the appropriate geographical and geological context for the firm's investment decision? One possibility is the horizon, on the grounds that the presence (or absence) of hydrocarbon deposits at one location in a horizon changes the probability of finding deposits elsewhere in that horizon. But, even if the formation of pools in different horizons are independent geological events, when a well is drilled to an horizon, it necessarily produces information about every horizon that lies above the target horizon.

Therefore, all horizons in a specific area is likely to be a better approximation to the decision environment than the individual horizon.

Unfortunately, the raw data that would permit this type of aggregation were not available. A crude alternative is to aggregate drilling investment in horizons where drilling occurred in approximately the same areas at the same time. Prior to the 1970's, most of the drilling activity in the Upper Cretaceous, Mannville, Viking, Mississippi, and Upper Devonian horizons took place in central Alberta. In the 1970's, drilling in the Mannville, Viking, and Mississippi horizons shifted to the southwestern part of Alberta, near the foothills, where most of the gas pools are located. Drilling in the Lower Devonian horizon was concentrated almost exclusively in northwestern Alberta. Moreover, almost all of the drilling in this area was targeted to the Lower Devonian horizon, since other horizons in this area contained few oil or gas pools. On the basis of these facts, we separated the Lower Devonian horizon from all other horizons, and aggregated the latter into one category, called Agg\LoDev. This classification represents our base case.

Our measure of exploratory drilling effort is the number of wildcat wells. We regard expenditures on development wells as part of the cost of developing the pool. Accordingly, the amount of oil and gas discovered by a wildcat well drilled in year t is given by the average size of pools discovered in that year, where size is measured by cumulative production. Revisions to initial estimates of discoveries are assumed to reflect reporting lags, and to add no new information about the prospect. We examine the validity of these assumptions in the appendix, and find that they are strongly supported by the data.

Annual prices for developed and undeveloped reserves of an "average" pool in Alberta were computed using annual data on wellhead prices for oil and gas, development costs, and taxes. The formula which Uhler used to calculate

the expected price of reserves in each year is based upon the following assumptions: a fifteen year production life, a constant rate of production, and price expectations which were static up to 1973⁶, and consistent with the government-controlled growth rate in prices after 1973. The price shock of 1973 was assumed to be unanticipated. After deducting various operating costs, taxes, and royalties from the wellhead price (measured in nominal dollars), the flow of expected net returns were discounted back to the date of discovery at the nominal rate of interest to yield a price for developed reserves. The price of undeveloped reserves was obtained by deducting development costs, which were assumed to occur in the first year of the production period.

Drilling costs were obtained from the Annual Report on exploration expenditures by Alberta Department of Energy and Natural Resources.

The empirical definitions of the variables introduced in Section 2 are listed in the top panel of Table 2. All dollar figures are expressed in millions of 1965 dollars. The gross return from a wildcat well is defined as the sum of oil and gas revenues, although in most horizons, and for most of the sample period, oil revenues was the main component. The net return is calculated by deducting exploratory drilling costs. It does not net out geophysical and land acquisition costs, since these data were available only for Alberta as a whole, and not for the individual horizons. Means and standard deviations for annual wildcat drilling, net returns, and profits are reported in the bottom panel of Table 2 for each horizon, and for each of the aggregates.

5. Estimation Results

The model was estimated by two step least squares. Instrumental variables for x_{t+1} and π_t were constructed using as instruments one lag each of

discoveries, discoveries per well, and wildcat drilling, and contemporaneous values of wildcat drilling, oil prices, and gas prices. The standard errors for β were calculated using a consistent estimate $\hat{\Sigma}$ of the asymptotic covariance matrix under heteroskedasticity, proposed by White (1982).⁷ Standard errors for α were derived using the covariance matrix $A \cdot \hat{\Sigma} \cdot A$, where A is the Jacobian of the mapping between α and β .

Table 3 gives the estimation results for the base case. The estimates in column U correspond to the model with no restriction on the coefficients. The estimates in column R are obtained under the restriction $\beta_2 = \delta\beta_1$ and a value for $\delta = .95$. T_1 gives the chi-squared statistic which tests the validity of this hypothesis. T_2 gives the chi-squared statistic for the overidentifying restrictions of the dynamic model, and T_3 gives this statistic for the restriction $\beta_1 = \beta_2 = 0$, which yields the static model. The numbers which appear in parentheses after the chi-squared statistics are their marginal significance levels.

The specification tests indicate that our model is consistent with the data. The overidentifying restrictions are not rejected in either of the two equations, although the marginal significance levels are not very high. The linear restriction on the coefficients (i.e., $\beta_1 = .95\beta_2$) was satisfied at conventional significance levels. The first order autoregressions for the residuals in each of the two equations revealed no evidence of serial correlation. We checked the simple and partial autocorrelation functions of the residuals, and did not find any contrary evidence. Of course, one should remember that the sample size is small, so the confidence intervals for the autocorrelations are quite wide (i.e., the tests surely have low power).

Most of the coefficients of the restricted model are significant at conventional confidence levels, have the correct sign, and are stable across the two classifications. Furthermore, the magnitudes of the associated cost

parameters do not appear to be implausible. The parameter α_1 can be interpreted as average geophysical costs and land acquisition costs per well over the sample period. For the province as a whole, these costs ranged between \$400,000 to \$1 million per well during the sample period, which does not differ too greatly from the point estimates of \$1.2 and \$1.5 million in the Agg/LoDev and Lower Devonian horizons, respectively. The cost of adjustment parameter α_3 is estimated to be about \$15,000 to \$28,000 per well, which also seems plausible. However, the estimates for α_2 are consistently negative, although not significantly different from zero.

The statistics for the set of overidentifying restrictions in the static version of the model indicate that the null hypothesis of no costs of adjustment (i.e., $\beta_1 = \beta_2 = 0$) is overwhelmingly rejected in both the Lower Devonian horizons and the unit called Agg/LoDev. Thus, the instantaneous response in exploratory drilling to the discovery of a large pool, or an unexpected rise in oil prices, can differ substantially from the long-term response.

A striking result that deserves emphasis is the similarity of the estimates across the two regions. In Figures 2 and 3, we have depicted the actual drilling paths for the Agg/LoDev horizons and the Lower Devonian horizons, respectively. The pattern of drilling activity in the latter case was clearly quite different from that in the non-Lower Devonian horizons, even after one accounts for differences in scale. Nevertheless, the same model is capable of explaining both patterns.

Furthermore, the model performs well. The dashed lines in Figures 2 and 3 depict the predicted drilling paths for the two regions. The overall fit in both cases is quite good. In the Agg/LoDev case, the model just misses the spike in 1973, but it adjusts to it in a short period of time. In the Lower Devonian case, the model captures quite well the turning points in 1960,

1964, and 1974.

Alternative Classifications of Pools

Table 4 reports the results of the restricted model when it is estimated by horizon. As expected, the model does quite poorly in horizons other than the Lower Devonian. The specification tests reveal inconsistencies between the model and the data. The overidentifying restrictions are rejected at the 5% significance level in the Upper Cretaceous and Mississippi horizons. Support for the linear constraint is relatively weak in the Upper Cretaceous, Viking and Upper Devonian horizons. Also, there is evidence of serial correlation in the residuals for the Mississippi and Upper Devonian horizons.

Additional evidence for rejecting the model as a plausible explanation of drilling behavior in non-Lower Devonian horizons is provided by the parameter estimates themselves. The estimates for β_0 and β_3 are generally not significantly different from zero, and in several instances, they have the wrong sign. The implied estimates for α_1 are implausibly large in the Upper Cretaceous and Upper Devonian horizons, and have the opposite signs to that expected in the other three non-Lower Devonian horizons. The estimates for α_2 and α_3 frequently have the wrong sign, although in most instances, they are not significantly different from zero.

These results indicate that firms did not choose their drilling programs to maximize the present value of discoveries within an horizon. Our interpretation of this negative conclusion, in conjunction with the positive conclusion obtained for the base case, is that the horizon was not the geographical context in which firms were making their investment decisions. Rather, the appropriate geographical context is one which aggregates different horizons in a region.

6. Concluding Remarks

In this paper we have formulated and estimated a dynamic model of exploratory drilling with rational expectations. The results provide some important insights about the nature of the exploration process, which are likely to be useful for policy-makers. In particular, we would emphasize the following implications.

First, the data strongly rejects the hypothesis of a static model of drilling behavior. This puts into question short-run measures of the impact of changes in the economic environment on drilling and discovery rates. Most of the initial response in discoveries may simply represent transfers from future periods to the current period as the drilling profile is "tilted" toward the present. Furthermore, they do not provide a measure of discoveries that would not have been made if prices had not increased. The correct approach to evaluating the impact of price and other policy changes is to recognize that alternative economic environments generate different paths of investment and discoveries, and to compare these paths.

Second, the data provides evidence that firms do not regard drilling returns as a sequence of independent random variables. Instead, they use the information from past realizations of returns and drilling efforts to forecast current and future values of returns. Furthermore, their forecasts appear to be consistent with the underlying probability law. The structure of this probability law and the firm's response to drilling outcomes indicates that information spillovers are likely to be an important determinant of drilling behavior. This raises several issues, which could not be addressed at the current level of aggregation, but could be studied if firm data were available. How important are "bandwagon" effects in determining the geographical pattern of drilling? What is the impact of different tenure arrangements on rates of exploration?

Third, classifying pools and wells according to the informational context in which drilling decisions are taken is important for obtaining precise and stable estimates of the decision rule. In the Alberta basin, this implied a classification that was based more on geographical than on geological criteria. It is also apparent that aggregating investments from regions which are in different stages of development, or have different probability laws for returns, is in general not a good idea, since the underlying probability laws and the expectations mechanism determining investment behavior are region-specific.

Our costs-of-adjustment interpretation of the dynamics is subject to an important caveat. It was derived under the assumption that the tenure of prospects was equal to the drilling period. The presence of information spillovers indicates that this assumption is crucial. Otherwise, firms have an incentive to behave strategically, and delay drilling marginal prospects until after drilling outcomes on prospects that are currently being drilled are observed. As a descriptive matter, there is some reason to believe that this assumption is not correct. Consequently, some of the dynamics detected in the data may reflect the desire of firms to delay in order to learn more. If so, the disaggregation of pools and wells must be extended to firms as well as location. This is clearly the next step in the research.

¹ Kaufman et. al. (1975, 1981), Meisner and Demirmen (1981) are good examples.

² Three examples of this literature are Erickson and Spann (1971), Khazzoom (1971), and MacAvoy and Pindyck (1973).

³ See Hendricks and Kovenock (1989) for a theoretical analysis of the efficiency issue.

⁴ These restrictions did not apply for those parts of the province designated as Block 'A' areas. In these areas, a firm did not have to surrender any of the leases covered by the initial permit to the government.

⁵ It can be shown that this maximization problem is equivalent to maximizing the present value of production if output in period t is assumed to be proportional to stock of reserves in period t .

⁶ Nominal prices were essentially constant during this period.

⁷ In most cases, a comparison of $\hat{\Sigma}$ to the covariance matrix obtained under the assumption of conditional homoskedasticity indicated that some heteroskedasticity was present.

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APPENDIX

In this appendix, we examine the hypothesis that development wells are primarily investments in production, and that revisions to initial estimates do not represent new information about reserves. In our evaluation, we ignore gas pools because, for most of the sample period, gas prices were too low to make exploration for gas a profitable investment. Gas pools tended to be produced only if it was discovered as a byproduct of the search for oil pools, after the costs of drilling had been sunk.

Let $z_{t,t}$ denote the initial estimate of the amount of oil reserves discovered in period t , and let $z_{s,t}$ represent the period s revision of oil reserve estimates for these pools, $s > t$. Final reserves of vintage t oil pools are given by:

$$Z_t = \sum_{s=t}^T z_{s,t},$$

where T is equal to 1981, the last year for which reserve data are available. Total changes in oil reserves in period t due to development drilling are given by:

$$R_t = \sum_{s=0}^{t-1} z_{t,s}$$

The Viking horizon contained virtually no oil pools. In the Upper Cretaceous, Mannville, and Mississippi horizons, oil pools were generally quite small, and the values of R_t were frequently zero. (The correlation coefficients between initial and final reserve estimates in these pools are 0.75, 0.97, and 1.00 respectively.) Yet, in many of these years, substantial numbers of development wells were drilled. In the Devonian horizons, the oil pools were much larger, and final reserve estimates were often four to eight times as large as initial reserve estimates. However, even in these cases, the revisions exhibited a predictable pattern. The correlation coefficients between initial and final reserves for the Upper Devonian horizon and the two Lower Devonian categories are 0.94, 0.97, and 0.95 respectively. This evidence supports our hypothesis.

If discoveries from wildcat drilling represent new information, and revisions of initial reserve estimates represent anticipated information, then the former should be an important determinant of wildcat drilling, and the

latter should not. To test this implication, we regressed the number of wildcat wells drilled in period t , x_t , on two lags of itself and one lag of each of the following variables: annual discoveries, Z_{t-1} , annual discoveries per wildcat well, a_{t-1} , annual revisions in reserves, R_{t-1} , and annual revisions per development well, r_{t-1} . If our hypothesis is correct, the set of coefficients on the lagged discovery variables should be significantly different from zero, and the coefficients on the lagged revision variables should not.

The estimation results for the base case, which are reported in Table A1, support the hypothesis. The F-statistics which tests the null hypothesis that the intertemporal correlations between wildcat drilling and the reserve revision variables are not significantly different from zero are equal to 0.689 for Lower Devonian and 0.162 for Agg\LoD. These figures indicate that the hypothesis can be maintained at the 99% confidence level. By contrast, the intertemporal correlations between wildcat drilling and the discovery variables are significantly different from zero at conventional significance levels.

We also estimated the equations on horizon data to verify that the results did not depend on our form of aggregation. The estimates for the Upper Cretaceous, Mannville, Mississippi and Upper Devonian horizons are reported in Table A2. (The Viking horizon was omitted since it contained virtually no oil pools.) Once again, there is no evidence that the reserve revision variables have any marginal explanatory power. Note that the evidence for a feedback effect of discoveries onto wildcat drilling is much weaker at this level of aggregation. The intertemporal correlations between the lagged discovery variables and wildcat drilling are generally not significantly different from zero at the 95% confidence level. One explanation of this result is that the horizon is not the appropriate decision environment.

TABLE 1

Cumulative Stocks of Wells and Reserves By Horizon

Category	Expl. Period	Total NAG	Total AG	Total Oil	Wildcat Wells	Devel. Wells
Upper Cretaceous	1953-79	53.9	0	170.4	1319	5733
Viking*	1950-79	88.7	24.3	11.8	1855	2970
Mannville*	1950-79	363.8	26.0	68.8	12733	12828
Mississippi*	1950-79	472.6	79.8	46.6	2427	2325
Upper Devonian	1947-79	252.2	111.1	553.0	4788	6326
Lower Devonian	1957-79	90.2	45.3	355.5	2816	2988

* Exploration in the Viking, Mississippi, and Mannville horizons began in the 1940's. However, 1950 is the first year for which annual data are available.

Table 2

Definition of Variables

x_t	: number of wildcat wells drilled in year t.
vo_t	: reserve price of an undeveloped cubic foot of oil discovered in year t.
vg_t	: reserve price of undeveloped thousand cubic feet of gas discovered in year t.
ao_t	: millions of cubic feet of oil discovered per wildcat well in year t.
ag_t	: billions of cubic feet of gas discovered per wildcat well in year t.
v_t	$= vo_t \cdot ao_t + vg_t \cdot ag_t$: gross return per wildcat well in year t, millions of 1965 dollars.
π_t	$= v_t - b_t$: return per wildcat well net of drilling cost in year t, millions of 1965 dollars.

Means and Standard Deviations of Wildcat Drilling and Returns*

Category	x_t	π_t	$\pi_t x_t$
1. Lower Devonian	122.4 (58.03)	.8000 (2.279)	47.373 (80.06)
2. Agg\Lower Devonian	762.9 (603.5)	.2339 (.6811)	66.119 (201.2)
Mississippi	80.90 (74.11)	.2733 (.7267)	6.2403 (18.25)
Mannville	424.4 (441.8)	-.03667 (.0801)	-17.999 (50.41)
Viking	61.8 (58.3)	-.0474 (.0961)	-2.6701 (5.680)
Upper Devonian	151.7 (48.2)	.3458 (.9885)	61.368 (164.02)
Upper Cretaceous	48.9 (18.5)	19.093 (98.79)	21.311 (99.197)
3. All Horizons	856.7 (632.1)	.2991 (.6748)	102.438 (204.843)

*Standard deviations are displayed in parantheses.

Table 3*

Instrumental Variable Estimates (2SLS) of Cost Parameters

Base Case

Indep. Variable	Agg\LD		Lower Devonian	
	U	R	U	R
Constant	-57.54 (-1.78)	-60.51 (-1.90)	-29.2 (-1.62)	-29.5 (-1.64)
x_{t-1}	0.49 (4.00)	0.56 (18.0)	0.49 (2.73)	0.61 (9.43)
x_{t+1}	0.59 (5.45)	0.53 (18.0)	0.71 (3.60)	0.58 (9.43)
π	29.64 (0.96)	38.10 (1.40)	12.93 (0.63)	21.5 (1.30)
α_1		1.59 (1.59)		1.37 (1.83)
α_2		-.0023 (.135)		-.0086 (-.86)
α_3		.0146 (1.39)		.028 (1.40)
δ		.95		.95
R^2	0.91	0.89	0.75	0.75
SEE	184.5	203.3	30.6	29.8
Q	8.16 (.881)	9.45 (.801)	3.54 (.968)	4.71 (.91)
T_1		.33 (.56)		.52 (.47)
T_2		8.81 (.18)		9.09 (.17)
T_3		1465 (.00)		234 (.00)
Period	1950 - 79		1957 - 79	

* t-statistics for the coefficients are displayed in parantheses. Q denotes the Box-Pierce statistic for serial correlation of any order. The number in paranthesis after the value of Q is its marginal significance level. T_1 is the chi-squared statistic for the hypothesis that $\beta_2 = \delta \cdot \beta_1$. T_2 is the chi-squared statistic for the overidentifying restrictions of the dynamic model, and T_3 is the statistic for the restriction that $\beta_1 = \beta_2 = 0$ (i.e., the static model). The numbers which appear in paranthesis after the chi-squared statistics are their marginal significance levels.

Table 4*

Instrumental Variable Estimates (2SLS) of Cost Parameters

Classification of Pools by Geological Horizon

Indep. Variable	Upper Cretaceous	Viking	Mississippi	Mannville	Upper Devonian	Lower Devonian
Constant	-15.0 (-1.63)	-4.7 (-.84)	-3.2 (-.60)	9.8 (.29)	-85.6 (-1.91)	-29.5 (18.0)
x_{t-1}	.66 (7.01)	.53 (8.26)	.54 (16.6)	.55 (19.7)	.82 (4.99)	0.61 (.06)
x_{t+1}	.63 (7.01)	.50 (8.26)	.51 (16.6)	.52 (19.7)	.78 (4.99)	0.58 (.06)
π	1.26 (.49)	-53.6 (-1.54)	-1.8 (-.25)	970.5 (1.82)	-.73 (-.21)	21.5 (16.6)
α_1	11.9 (.64)	-0.087 (-.97)	-1.76 (.19)	-.01 (.33)	38.6 (1.08)	1.37 (1.83)
α_2	-0.23 (.38)	.0005 (.25)	.03 (.27)	-.00006 (1.50)	-.27 (.67)	-.0086 (-.86)
α_3	0.53 (.52)	-.01 (1.67)	-.30 (.25)	.0005 (1.67)	.38 (1.00)	.028 (1.40)
δ	.95	.95	.95	.95	.95	.95
R^2	0.63	0.68	0.96	0.94	0.57	0.75
SEE	9.0	35.8	14.8	110.6	35.1	29.8
Q	19.4 (.08)	16.2 (.30)	17.4 (.23)	7.51 (.91)	9.79 (.78)	2.32
T_1	5.0 (.03)	1.02 (.31)	5.7 (.46)	.08 (.78)	.002 (.96)	.52 (.47)
T_2	17.4 (.01)	10.4 (.11)	6.9 (.01)	4.2 (.65)	9.3 (.16)	9.09 (.17)
T_3	104 (.00)	657 (.00)	963 (.00)	1065 (.00)	36.9 (.00)	234 (.00)
Period	1953-79	1950-79	1950-79	1950-79	1950-79	1957-79

*t-statistics for the coefficients are reported in parantheses. Q denotes the Box-Pierce statistic for serial correlation of any order. The number in paranthesis after the value of Q is its marginal significance level. T_1 is the chi-squared statistic for the hypothesis $\beta_2 = \delta \cdot \beta_1$. T_2 is the chi-squared statistic for the overidentifying restrictions of the dynamic model, and T_3 is the statistic for the restriction $\beta_1 = \beta_2 = 0$. The numbers which appear in paranthesis after the chi-squared statistics are their marginal significance levels.

TABLE A1*

Feedback Effects of Discoveries and Reserve Revisions on Wildcat Drilling

Base Case

Independent Variable	Agg\LoDev Horizons		Lower Devonian Horizons	
Constant	92.6 (1.40)	97.7 (1.24)	49.6 (2.32)	31.3 (1.10)
x_{t-1}	1.18 (5.92)	1.18 (5.62)	.604 (2.26)	.669 (2.28)
x_{t-2}	-.116 (-.49)	-.104 (-.42)	-.046 (-.194)	-.019 (-.063)
z_{t-1}	-34.8 (-3.41)	-37.2 (-3.26)	2.43 (2.07)	2.16 (1.77)
a_{t-1}	11826.3 (3.35)	12628.7 (3.21)	-186.1 (-1.72)	-167.8 (-1.50)
R_{t-1}		3.83 (0.56)		.702 (.976)
r_{t-1}		-3287.8 (-.56)		-14.2 (-.196)
R^2	.921	.922	.698	.725
Q	12.8 (.54)	13.6 (4.8)	8.73 (.56)	7.70 (.65)
SEE	185.1	192.3	33.0	33.7
Period	1950-79		1959-79	

*t-statistics for the coefficients are displayed in parantheses. Q denotes the Box-Pierce statistic for serial correlation of any order. The number of lags used in the computations is $\min[T/2, 3T^{1/2}]$. The number in paranthesis below the value of Q is its marginal significance level. Marginal levels below .05 means that the null hypothesis of no serial correlation is rejected at the 95% confidence level.

TABLE A2*

Feedback Effects of Discoveries and Reserve Revisions on Wildcat Drilling
Individual Horizons

Independent Variable	Upper Cretaceous		Mannville		Mississippi		Upper Devonian	
Constant	24.3 (2.14)	24.7 (2.03)	80.5 (1.56)	59.1 (1.05)	9.87 (1.35)	8.63 (.95)	98.0 (2.68)	109.6 (2.26)
x_{t-1}	1.03 (5.36)	1.06 (4.75)	1.39 (5.14)	1.35 (4.83)	1.11 (5.31)	1.12 (5.06)	1.04 (4.29)	1.07 (4.26)
x_{t-2}	-.453 (-2.25)	-.478 (-1.97)	-.42 (-1.53)	-.32 (-1.06)	-.135 (-.60)	-.143 (-.61)	-.733 (-3.09)	-.818 (-2.44)
z_{t-1}	-8.39 (-1.61)	-8.93 (-1.59)	-42.7 (-1.02)	-97.1 (-1.51)	-4.03 (-1.53)	-4.13 (-1.48)	-4.93 (-1.08)	-5.36 (-.95)
a_{t-1}	262.7 (1.59)	280.8 (1.58)	4016.3 (.70)	8311.2 (.85)	61.9 (1.22)	64.3 (1.01)	971.5 (1.13)	1025.6 (.94)
R_{t-1}		-.023 (-.10)		-.77.5 (-1.13)		.384 (.373)		.591 (.37)
r_{t-1}		-7.58 (-.18)		9002.1 (.75)		-3.88 (-.21)		-144.6 (-.73)
R^2	.597	.601	.911	.916	.925	.926	.564	.582
Q	10.98 (.53)	10.48 (.57)	7.01 (.93)	6.14 (.96)	8.77 (.85)	7.36 (.91)	8.26 (.87)	8.70 (.84)
SEE	11.06	11.60	144.5	146.8	22.04	22.98	34.9	35.9
Period	1953-79		1950-79		1950-79		1950-79	

*t-statistics for the coefficients are displayed in parantheses. Q denotes the Box-Pierce statistic for serial correlation of any order. The number of lags used in the computations is $\min[T/2, 3T^{1/2}]$. The number in paranthesis below the value of Q is its marginal significance level.

FIGURE 1

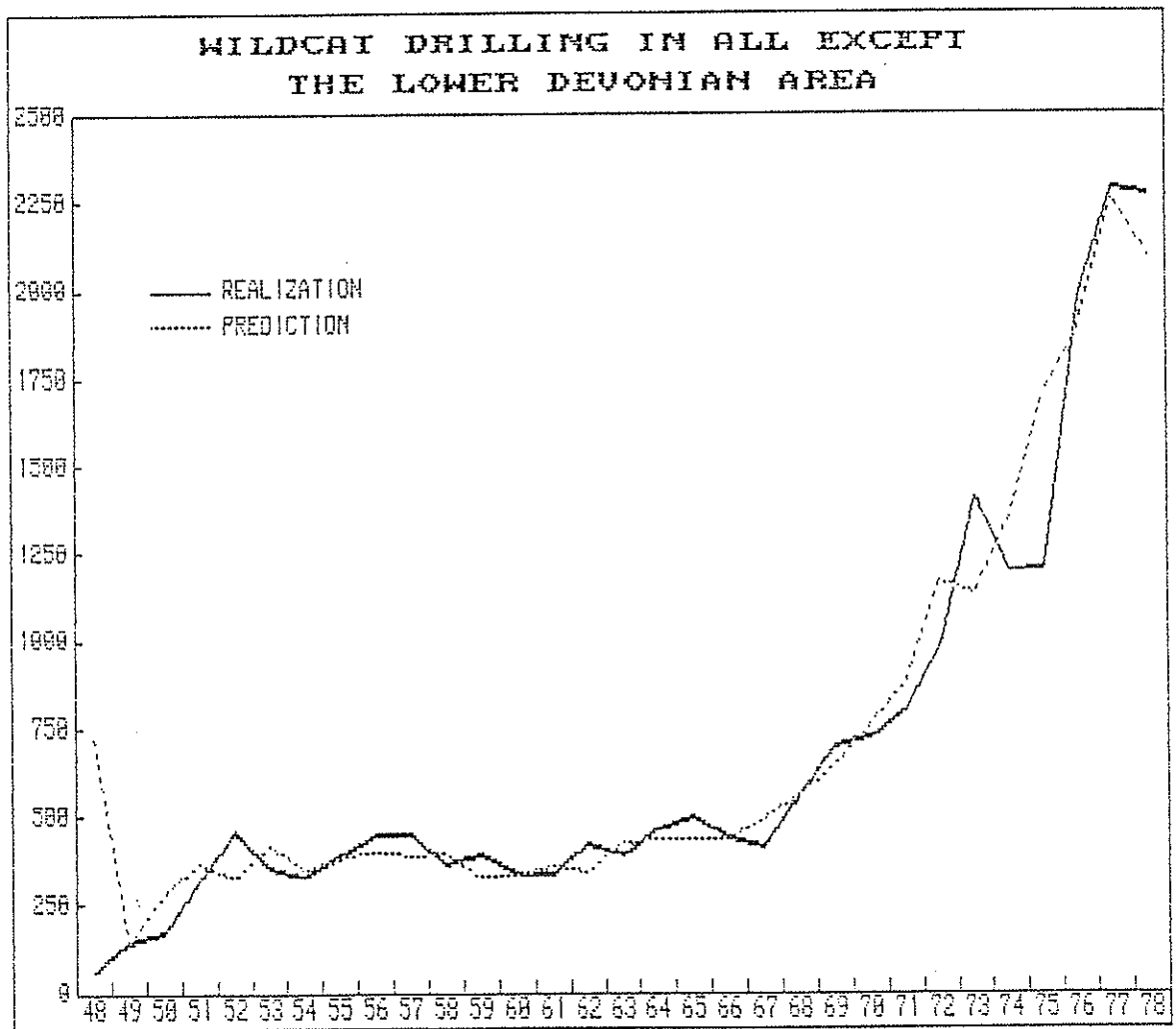


FIGURE 2

