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and the average age of capital\***

**by**

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# Creative Destruction, Investment Volatility, and the Average Age of Capital\*

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## Abstract

We analyse the ability of the Ramsey vintage capital growth model to propagate past fluctuations, the so-called “replacement echoes.” The key assumption in vintage capital technology is that technical progress is embodied in new machines, giving rise to an endogenous process of creative destruction during which investment is mainly a replacement activity. We propose a procedure to compute numerically the equilibrium of a vintage capital growth model with non-linear utility, where the scrapping time is non-constant. We show that equilibrium investment and output converge *non-monotonically* to the balanced growth path due to replacement echoes. We also find that the average age of capital is inversely related to output, which is consistent with recent micro evidence confirming the importance of the embodied question.

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# 1 Introduction

The vintage capital model has become increasingly popular among economists, especially because it provides an appealing framework for the analysis of investment volatility. The key assumption in vintage capital technology is that technical progress is embodied in new machines, which gives rise to an endogenous process of creative destruction. At equilibrium it might be optimal to scrap old machines in order to replace them by new and more productive equipment. It is very important to notice that, throughout the simultaneous process of creation and destruction, investment takes the form of a “replacement activity.” This would result in lumpy investment paths following the well-known “echo principle.”

As stressed by Benhabib and Rustichini [3], lumpy investment activities cannot arise in standard growth models with homogenous capital. In such models, the convergence to the balanced growth path is monotonic and investment solution paths are markedly smooth. In a vintage capital setting, however, investment paths may be non-monotonic even under stationary environments. This key property can be obtained under two different assumption sets. Indeed, as shown by Benhabib and Rustichini [3], this property does arise in vintage capital growth models incorporating non-exponential physical depreciation rules for the capital stock. In this paper, we are interested in another potential source of investment lumpiness, the replacement activities of obsolescent capital goods (Benhabib and Rustichini [4] and Boucekine et al. [5]). In particular, we will provide a more comprehensive analysis of the adjustment process that occur when investment takes the form of an economic replacement activity. It is worth pointing out that we are concerned with a completely endogenous fluctuation mechanism in sharp contrast to most of the recent contributions which have stressed the role of vintages in the propagation of the business cycles. For example, Caballero and Hammour [7] have obtained business fluctuations from a vintage capital model through an exogenous periodic aggregate demand.<sup>1</sup> Our paper sheds light on the role of replacement echoes as an endogenous source of fluctuations, which is likely to interact with other sources.<sup>2</sup>

To undertake this task, we will consider a Ramsey growth vintage capital

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<sup>1</sup>The role of vintages in economic fluctuations has also been investigated using a Real Business Cycle approach by Gilchrist and Williams [13].

<sup>2</sup>This feature is pointed out by Caballero and Hammour [8].

model (hereafter RVCM) with both non-linear utility and Leontieff technology. It is extremely important to motivate the choice of such a framework so as to obtain an immediate idea about the motivations and the contributions of this paper. First, we consider RVCMs, bearing in mind that Solow vintage capital growth models, i.e. with a constant saving rate, are particularly unsuitable for the analysis of replacement echoes. As numerically shown in Boucekkine et al. [6], Solow vintage models do not generate echo effects either in the long run (as analytically demonstrated in Solow et al. [18]) or in the short run. Indeed, when the saving rate is constant, investment is no longer guided by replacement activity. Secondly, the choice of the Leontieff technology is made fundamentally in order to have the simplest endogenous replacement rule. Actually, as shown by Benhabib and Rustichini [4], we could have chosen any production function with complementary production factors (labor and capital). It is worth pointing out that almost all recently published papers dealing with endogenous creative destruction incorporate a Leontieff technology for the same reason mentioned above (see Aghion and Howitt [1] and Caballero and Hammour [7] and [8] among others). Last but not least, we will numerically solve Ramsey models with non-linear utility functions in contrast to most of the recent contributions on the same topic.<sup>3</sup>

The main contribution of this paper comes from the analysis of the RVCM with non-linear utility, which yields non-constant optimal scrapping rules. When the utility function is linear, the RVCM with Leontieff technology yields periodic solution paths for detrended investment beginning at a finite date, as theoretically shown by Boucekkine et al. [5]. We argue here that this result comes mainly from the fact that the optimal scrapping rule is constant (beginning at a finite date) under linear utility. Actually while the scrapping rule is set equal to an arbitrary constant, investment paths are non-monotonic independently of the production technology and of the utility functional form (see Benhabib and Rustichini [4] who only consider the case of arbitrarily constant scrapping rules). In this paper, we show that, in contrast to the findings of the latter authors, the optimal scrapping rules are no longer constant when the utility functions are non-linear. As the scrapping rules are non-constant in the non-linear utility case, the resulting dynamics are much richer with respect to the linear utility case: In addition to echo effects on investment, we have to interpret and analyze the scrapping rule

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<sup>3</sup>For example, in contrast to Aghion and Howitt [1] and Caballero and Hammour [7].

dynamics and their interaction with the echo effects. Strictly concave utility induces consumption smoothing, which forces replacement echoes to vanish asymptotically. A non-monotonic pattern of the scrapping age is necessary to allow echoes to vanish. In this complex dynamics context, we will focus on the interactions between investment, output and the average age of capital.

There is a recent literature stressing the empirical importance of embodied technical progress. Greenwood et al. [15] found that around 60% of growth in the US aggregate output is due to investment-specific technological progress. One of the main results in Doms and Dunne [12] emphasises the lumpiness of investment activity, i.e., investment at the plant level occurs infrequently and in bursts. Doms et al. [13] found that plants employing advanced technologies have higher growth rates and are more likely to survive. Finally, Cooper et al. [10] found that the frequency of lumpy investment activity is higher during periods of high economic activity and more likely the older is the capital stock.

As stated by Denison [11], the embodied question implicit in the vintage capital literature matters if the average age of machines is negatively correlated with output. At the present time, there exists very little empirical and/or theoretical research dealing with the behavior of the average age of capital throughout the business cycle. Most of the empirical literature tackling this issue uses a plant level analysis, showing that there exists a strong negative correlation between output and the average age of capital. Bahk and Gort [2] found that a one-year change in the average age of capital is associated with a 2.5 to 3.5 percent change in output.<sup>4</sup> Indeed, it is far from easy to conduct rigorous empirical studies on the average age of aggregate capital due to obvious technical and accounting problems. For this reason, it is necessary to resort to a more theoretical appraisal and this is the approach we take here within a creative destruction framework. We believe that the model presented in this paper is particularly useful to address this question.

We organize this paper as follows. Section 2 gives some technical details on echo effects and the average age of capital when the scrapping rules are constant. Section 3 describes the RVCN considered in this paper and Section 4 analyzes the dynamics of this model. Section 5 concludes.

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<sup>4</sup>These authors use data for the U.S. on 2150 plants belonging to 40 industries over a period of 14 years.

## 2 Replacement Echoes with Constant Scrapping Time

As mentioned in the introduction section, vintage capital growth models yield non-monotonic investment paths if the scrapping time is constant. We can illustrate quite easily this property on the following vintage technology:

$$y(t) = \int_{t-T(t)}^t i(z) dz \quad (1)$$

$$\int_{t-T(t)}^t i(z) \exp\{-\gamma z\} dz = 1, \quad (2)$$

where  $y(t)$  is production,  $i(t)$  is investment and  $T(t)$  represents the age of the oldest operating machines or *scrapping time*. Machines from *vintage*  $t$ ,  $\forall t$ , are supposed to produce one unit of output each, and to require  $\exp\{-\gamma t\}$  units of labor. The parameter  $\gamma$  is supposed to be strictly positive and represents (Harrod neutral) technical progress. Total labor resources are assumed to be constant and equal to one.

If the scrapping time is constant, i.e., if  $T(t) = T$ , we obtain by differentiating (2):

$$i(t) = i(t - T) \exp\{\gamma T\}.$$

This condition implies that creation (on the left hand side of the equation above) should compensate for destruction (on the right hand side) in order to employ labor resources fully. Indeed, investment behavior in vintage capital models does depend crucially on two different factors. First, investment is mainly guided by replacement activity, in the sense that the economy invests at time  $t$  in order to employ all the workers dismissed from the scrapped machines. Secondly, since the labor requirement is decreasing at rate  $\gamma$ , investment should increase at the same rate in order to fully employ workers. If we normalize investment with respect to the labor requirement, i.e., we define  $\hat{i}(t) = i(t) \exp\{-\gamma t\}$ , we get:

$$\hat{i}(t) = \hat{i}(t - T).$$

It implies that, under a constant scrapping age, detrended investment might be purely periodic. We call “replacement echoes” this type of periodic behavior of detrended investment. Indeed, echoes may arise in vintage models

because when equilibrium investment is mainly guided by replacement activity, past fluctuations are reproduced again and again in the future.

To study the implications of constant scrapping rules on the average age of capital, denoted  $A(t)$ , we define it in the most natural way, given the specifications above:

$$A(t) = \int_{t-T(t)}^t (t-z) \frac{i(z)}{y(t)} dz.$$

This definition is in line with the one proposed by Nelson [17]: The average age of capital at  $t$  is simply a weighted average of the ages of the active vintages, the weights  $i(z)/y(t)$  being equal to the relative participations of the successive active vintages in the total operating capital stock at  $t$ . It is very easy to prove that the average age is periodic, beginning at a finite date, if the scrapping rule is constant. Indeed, as our technology is Leontieff, the periodicity of investment, demonstrated above, implies that output is periodic, beginning at a finite date.<sup>5</sup> It is then straightforward to show that, using a trivial variable change and taking advantage of the fact that detrended investment and output are periodic with the same period  $T$ , the average age is periodic, beginning at a finite date, i.e.,  $A(t) = A(t-T)$ .<sup>6</sup>

Additionally, under the assumption of a constant scrapping age it can be easily shown that, beginning at a finite date,

$$A(t) = \frac{(1 - e^{-\gamma T})^{-1}}{y(t)} \int_{t-T}^t y(z) dz - \frac{T}{(e^{\gamma T} - 1)},$$

which implies that there exists a negative relation between the average age of capital and contemporaneous output.

This section has been devoted to establishing that, under Leontieff technology and a constant scrapping age, detrended investment, detrended output and the average age of capital are all periodic, beginning at a finite date.

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<sup>5</sup>If investment is periodic for all  $t \geq 0$ , output is periodic beginning at  $t = T$ .

<sup>6</sup>Alternatively and using the labor requirements as weights, the average age of capital could be defined as:

$$\tilde{A}(t) = \int_{t-T(t)}^t (t-z) i(z) e^{-\gamma z} dz.$$

In which case, it is very easy to show that, if  $T(t) = T$  for all  $t \geq 0$ , then  $\tilde{A}(t) = \tilde{A}(t-T)$  for all  $t \geq 0$ .

As shown by Boucekine et al. [5], in the RVCN, with linear utility and Leontieff technology, the scrapping age becomes constant at a finite date. In the next section we develop a RVCN with strictly concave utility function, in which the endogenous scrapping rules are unlikely to be constant at a finite distance, in order to analyze the persistence of replacement echoes.<sup>7</sup>

### 3 The Ramsey Vintage Capital Model with Non-linear Utility

#### 3.1 Optimization Problem

Let a central planner solve the following problem:

$$\max \int_0^\infty u[c(t)] \exp\{-\rho t\} dt$$

subject to

$$y(t) = \int_{t-T(t)}^t i(z) dz \quad (1)$$

$$\int_{t-T(t)}^t i(z) \exp\{-\gamma z\} dz = 1 \quad (2)$$

$$y(t) = c(t) + i(t) \quad (3)$$

$$0 \leq i(t) \leq y(t)$$

given  $i(t) = i(t)$  for all  $t < 0$ .

$c(t)$  is consumption and the utility function  $u(c)$  is assumed to be  $CD^1$ , increasing in both arguments and strictly concave. Parameter  $\rho$  is strictly positive and represents time preferences.

In order to solve this control problem we write down the Lagrangian function, denoted by  $\mathcal{L}(t)$ . Following Malcomson [15], after changing the order of integration, and some algebra, we get:

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<sup>7</sup>Another factor that induces everlasting fluctuations could be the Leontieff specification of technology. Actually, Benhabib and Rustichini [4] have shown that if the scrapping rule is constant, both investment and output paths are non-monotonic, at least for production functions with gross complementarity.

$$\begin{aligned}
\mathcal{L}(t) = & \int_0^\infty (u[y(t) - i(t)] + w(t) - \phi(t)y(t)) e^{-\rho t} dt + \\
& \int_0^\infty i(t) \int_t^{t+J(t)} (\phi(z) - w(z)e^{-\gamma t}) e^{-\rho z} dz dt + \\
& \int_{-T(0)}^0 i(t) \int_0^{t+J(t)} (\phi(z) - w(z)e^{-\gamma t}) e^{-\rho z} dz dt
\end{aligned}$$

where

$$J(t) = T(t + J(t)). \quad (4)$$

$\phi(t)$  and  $w(t)$  are the Lagrangian multipliers associated with constraints (1) and (2) respectively. Equation (4) is just a definition, i.e., the expected life time for new machines  $J(t)$  is equal to the scrapping time  $T(\cdot)$  evaluated at  $t + J(t)$ , which corresponds to the date at which these newly installed machines will be scrapped in the future. The first order necessary conditions for this problem are,  $\forall t \geq 0$ :

$$\phi(t) = u' [y(t) - i(t)] \quad (5)$$

$$w(t) = e^{\gamma(t-T(t))} \cdot u' (y(t) - i(t)) \quad (6)$$

$$u' (y(t) - i(t)) = \int_t^{t+J(t)} (1 - e^{\gamma(z-t-T(z))}) u' (y(z) - i(z)) e^{-\rho(z-t)} dz \quad (7)$$

with initial conditions  $i(t) = i(t)$ ,  $\forall t < 0$ .

Equation (6) states that the marginal value of labor, the real wage at the decentralized equilibrium, is equal to the marginal productivity of labor, the inverse of the labor requirement of the oldest operating machines evaluated at the marginal utility of consumption. Finally, (7) corresponds to the optimal investment rule and states that the marginal cost of investing, on the right hand side, must be equal to the marginal revenue, which depends on the future scrapping time of new machines.

The major difference with respect to the linear utility case resides in the fact that in the general case the system of equations above is simultaneous:

We cannot solve first for  $T(t)$  and then deduce the solution paths of the remaining variables. In the linear case, (7) depends only on  $T(t)$  and  $J(t)$ . Together with equation (4), it allows  $T(t)$  and  $J(t)$  to be computed independently of any other endogenous variable. As shown in Boucekkine et al. [5], if the instantaneous utility function is linear, the system (4)-(7) implies that the optimal scrapping rule is constant after a certain time, which generates everlasting echo effects on investment as explained in Section 2. Accordingly, when the utility function is linear, the economy cannot converge to any balanced growth path. For strictly concave utility functions, since the optimal scrapping rule is likely to be time-varying, one has to address the issue of the existence of balanced growth paths. The next subsection is devoted to the computation of the balanced growth paths of the model.

### 3.2 Stationary Equilibrium and Comparative Statics

To be able to compute the balanced growth path, we need to specify the form of the utility functions. In this section and in our numerical experiments, we consider very simple utility functions of the form:  $u(c) = c^\theta$  with  $0 < \theta < 1$ .

Let us define a balanced growth path:  $i(t) = i_0 \exp\{\gamma t\}$  and  $T(t) = T$ , where both  $i_0$  and  $T$  are positive constants. From (2),  $i_0 = 1/T$ . From (1), we know that  $y(t) = y_0 \exp\{\gamma t\}$ , where  $y_0 = (1 - \exp\{-\gamma T\}) \frac{i_0}{\gamma}$ . From (4)  $J(t) = T$ . Finally, from (7) we can compute the stationary value for the scrapping age:

$$1 = \frac{e^{[\gamma(\theta-1)-\rho]T} - 1}{\gamma(\theta-1) - \rho} - \frac{e^{[\gamma(\theta-1)-\rho]T} - e^{-\gamma T}}{\gamma\theta - \rho}.$$

For comparative statics purposes, it is more useful to rewrite the previous equation as follows:

$$1 = \int_0^T e^{(\theta\gamma - \rho)\tau} (e^{-\gamma\tau} - e^{-\gamma T}) d\tau. \quad (8)$$

It can be shown that the latter equation admits a (positive) solution for  $T$  if and only if the stationary value for the interest rate is smaller than one, i.e.,  $\rho + (1 - \theta)\gamma < 1$ . Using the value of  $T$ , one can compute  $i_0$ ,  $y_0$  and particularly the stationary value  $A$  of the average age of capital given by

$$A = \frac{1}{\gamma} \left( 1 - \frac{\gamma T}{e^{\gamma T} - 1} \right).$$

Using the formulas above, especially equality (8), we can establish the following comparative statics results:<sup>8</sup>

1. An increase in the individual discount factor  $\rho$  should increase the life time of machines. Indeed, an increase in  $\rho$  reduces the value of the integrand appearing in (8). To reestablish the equality (to 1),  $T$  should increase. The economic interpretation is straightforward, when individuals become more impatient the replacement cost is increased and it becomes optimal to postpone the scrapping of old machines.
2. A decrease in  $\theta$  has the same effect as an increase in  $\rho$ : Both reduce the integrand appearing in (8) and require an increase in  $T$  to reestablish the equality. Indeed, both increase the cost of substituting consumption today for consumption tomorrow, which results in reducing investment and postponing the scrapping of old machines.
3. An increase in the rate of technical progress should reduce the life-time of machines, since replacement should become more profitable. However, this property is not checked for all parameterizations of the model, even if  $\rho + (1 - \theta) \gamma < 1$  and  $\rho > \theta\gamma$ .<sup>9</sup> In contrast to previous exercises, the integrand of (8) is not a monotonic function with respect to  $\gamma$ . However, we have analytically found out a sufficient condition on the parameters of the model ensuring that  $T$  decreases when  $\gamma$  increases. This condition is:  $\rho + (1 - \theta) \gamma < 0.3$ . This condition is checked for small  $\rho$  and  $\gamma$ , which corresponds to the parameter values usually considered in economics.<sup>10</sup>
4. The comparative statics for the average age of capital can be deduced easily from those for the scrapping rule just above. Indeed, by definition, the average age of capital is altered by a parameter change if this change affects the scrapping time (which measures the age of the oldest

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<sup>8</sup>We do not detail all the (tedious) calculations required for this exercise.

<sup>9</sup>As stated before the former condition is required to have a positive solution for  $T$ . The latter is a necessary condition in order to have a bounded objective.

<sup>10</sup>One can numerically check that our condition  $\rho + (1 - \theta) \gamma < 0.3$  is not necessary. However, we can also numerically check that conditions  $\rho + (1 - \theta) \gamma < 1$  and  $\rho > \theta\gamma$  are not sufficient to ensure that  $T$  decreases when  $\gamma$  increases. Numerous counter-examples can be found for  $\rho$  close to 0.9.

machine still in use) and/or the relative participations of the new vintages in the total active capital stock with respect to those of the old vintages. For example, a parameter change that increases the scrapping time without altering the relative participations of new vintages will increase the average age of capital. This is typically what should occur when  $\rho$  increases or when  $\theta$  decreases: Both changes only increase  $T$  and do not affect the relative participations of new vintages.<sup>11</sup> As before, the only complicated case arises when there is a change in  $\gamma$  as such a change will affect both  $T$  and the relative participations of new vintages. Fortunately, even in this case, there is no ambiguity if we assume as above that  $\rho + (1 - \theta) \gamma < 0.3$ . Then, an increase in  $\gamma$  will diminish  $T$  and increase the relative participations of the newest vintages. Both effects will decrease the average age of capital  $A$ .

The obtained balanced growth paths are therefore economically consistent. Obviously, it remains to be seen if for a given initial investment profile, the economy does converge to these balanced growth paths. If the scrapping rules are time-varying, the latter property is likely to hold. However, as argued in the next section, it is impossible to establish this result analytically, so we will resort to numerical assessment.

## 4 Dynamics

Unfortunately, we cannot solve (1) to (7) in general since the equilibrium conditions for this economy give rise to a mixed-delay differential equation system with endogenous leads and lags. As stated by Boucek et al. [6], until our days, this type of dynamic systems cannot be in general solved either mathematically or numerically. In this section we use a simple waveform relaxation algorithm to deal with the planner problem directly (see the appendix for a detailed description of the algorithm).

The RVCN with strictly concave utility differs from the standard Ramsey model in two main aspects. The first difference, as stated in the introduction, comes from the technological assumptions. Secondly, as a consequence

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<sup>11</sup>The relative participation in the total capital stock of a vintage  $\tau'$  with respect to a vintage  $\tau$  is equal to  $e^{\gamma(\tau' - \tau)}$  along the balanced growth path. This implies that only a change in  $\gamma$  will alter these relative weights.

of the vintage structure, pointwise initial conditions (like the usual initial conditions at the initial time) are no longer sufficient to determine a unique solution path. Instead, we need to define the initial conditions over an interval of non-zero measure. Relying on these two differences, in the RVCМ fluctuations could arise for two different formal reasons: i) because initial conditions could be non-monotonic, and ii) because the transition path from a balanced growth path to another is non-monotonic. We will concentrate on the analyses of the later situation, since the former could be seen as a particular case of it. For this reason, the initial investment profile is supposed to be growing at the same rate as technical progress, i.e., for all  $t < 0$ ,  $i(t) = z e^{\gamma t}$ ,  $z > 0$ .

As stated in the previous section, the RVCМ admits a balanced growth path. If initial conditions are on the balanced growth path, the economy will stay on forever. Otherwise, all the simulations we have performed show convergence to the balanced growth path, with two possible oscillating, and in some way symmetric, convergence paths. In the first of these two situations the initial investment profile is above the balanced growth path. In other terms, the economy starts with a relatively high initial stock of machines. In the alternative situation the economy starts with a relatively low initial stock of machines.

Let us analyze the case of a low initial stock of machines (figures 1 and 2).<sup>12</sup> As in the standard Ramsey model, in order to increase the stock of machines, the planner starts by investing more than in the stationary equilibrium. The main difference with the standard Ramsey model is that, as we can see in figure 1, small past investment and the initial boom will be partially reproduced in the future due to replacement echoes. This generates an oscillatory convergence path with a periodicity length given by the scrapping age. Additionally, since initial investment conditions are relatively low, at the beginning the scrapping age is relatively large because very old machines must be operated to clear the labor market. As we can see in figure 2, the scrapping age converges non-monotonically to its stationary level. Investment fluctuations are reflected in production, but as reported in the RBC literature, investment fluctuates more than production. The convergence of

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<sup>12</sup>All figures were drawn for the following parameters values:  $\rho = .05$ ,  $\gamma = .03$ , and  $\theta = .8$ . On the balanced growth path  $T$  is around 9.3 and  $i(t) \approx 0.1 e^{\gamma t}$ . Initial conditions, for all  $t < 0$ , are  $i(t) = 0.03 e^{\gamma t}$  in figures 1 and 2, and  $i(t) = 0.2 e^{\gamma t}$  in figures 3 and 4.

an economy with a relatively high initial stock of machines is symmetric to the case just analyzed (see figures 3 and 4).

Why does the economy converge optimally to a constant scrapping rule? As is standard in growth theory, consumption smoothing implies that the interest rate converges to a constant value, which is equal to  $\rho + (1 - \theta) \gamma$ . From equations (6) and (9), as shown by Boucekkine et al. [5] using a rational expectations argument, under a constant interest factor the scrapping rule must be constant. In a stationary environment the optimal life-time of machines converges to a constant value.

It is important to notice that on the convergence path, two opposite forces operate at the same time: consumption smoothing and the convergence to an optimally constant scrapping rule. The main reason is that it is not feasible to have from the beginning both a smooth path for consumption and a constant life-time of machines. As shown by Boucekkine et al. [5], under linear utility the planner would like to implement a constant scrapping rule, and the economy optimally converges to it at a finite distance. As a consequence, and mainly because the planner does not care about the timing of consumption, investment and consumption fluctuate due to replacement echoes. However, under concave utility, everlasting fluctuations in investment are not optimal since the planner would like to smooth consumption. The only way the planner has to reduce replacement echoes from the beginning is to allow fluctuations in the scrapping age of machines. The planner is thus obliged to play simultaneously in these two opposite directions. First, the planner needs to permit some echoes in investment to allow the life-time of machines to converge. At the same time, the convergence of the scrapping rule should be asymptotic and by oscillations, in order to smooth consumption.

Concerning the average age of capital, it depends by definition on both the life-time of equipments and the distributions of machines across active vintages. Since both components are fluctuating on the convergence path, the average age converges non-monotonically too (see figures 2 and 4). Obviously, the dynamics of the average age is not a simple combination of the dynamics of the scrapping age and those of investment, because the whole distribution of active vintages matters. In figures 3 and 4, the scrapping age is increasing and investment is zero at the very beginning. In this extreme case, the average age of capital must increase and that is what reflects figure 4. However, after a short adjustment period, investment starts increasing

altering significantly the distribution of active vintages (i.e., with a large proportion of the “younger” vintages). Although the scrapping age meanwhile keeps on increasing, the average age of capital may fall down after a while. That is exactly what happens in figures 3 and 4.

More important, and consistently with Bahk and Gort [2] findings, the average age of machines is negatively correlated with output, as we can see by comparing figures 1 and 2 or figures 3 and 4. This property is robust to changes in the set of admissible parameters and in the initial investment profile.

## 5 Conclusions

Two main conclusions can be drawn. First, the vintage structure of capital generates oscillatory convergency to the balanced growth path. In a growth model with vintage capital, investment fluctuations and the corresponding fluctuations in output are the optimal reaction to a change in the environment. The initial boom (resp. recession), needed to adjust from a low (resp. high) to a high (resp. low) stock of machines, will be reproduced again and again in the future due to replacement echoes. Consumption smoothing generates simultaneously an oscillatory response of the scrapping age of machines such that replacement echoes vanish in the long run. Second, output and the average age of capital moves in opposite directions on the convergency path. This second prediction of the model is consistent with recent microeconomic evidence showing that there is a negative relation between the average age of capital and output and thus reinforcing the importance of the embodied question.

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## Appendix: The Numerical Algorithm

Instead of solving the system of first order conditions corresponding to the central planner optimization problem, we apply a simple relaxation algorithm to the objective function, taking advantage of the optimization constraints for updating purposes. Such techniques have been used for solving plain differential equations, in cases where it is convenient to consider the solution on the whole time interval. This suits us, as equations (1)-(7) relate  $i(t)$  to past as well as future values. The technique is known in numerical analysis literature as the waveform relaxation method (Miekkala et al. [17]). There is room for very sophisticated convergence acceleration possibilities, which we have not yet used (we merely used a Gauss-Seidel method). First, we rewrite the optimization problem as follows:

$$\max \int_0^\infty u[y(t) - i(t)] \exp\{-\rho t\} dt$$

subject to

$$y(t) = \int_{t-T(t)}^t i(z) dz \quad (1)$$

$$\int_{t-T(t)}^t i(z) \exp\{-\gamma z\} dz = 1 \quad (2)$$

$$0 \leq i(t) \leq y(t)$$

given  $i(t) = i(t)$  for all  $t < 0$ .

This formulation of the optimization problem allows to iterate only on investment values, since constraints (1) and (2) can be used to compute both output and scrapping time series from an investment series. Note also that the maximization above makes sense if and only if the maximal value of the improper integral appearing in the objective function is finite. This means that the integrand of this improper integral should tend to zero when  $t$  goes to infinity. This allows us to truncate the integral and consider finite time approximations of our structurally infinite time model. From the algorithmic point of view, ensuring a good finite time approximation requires a loop on the solution time support, say on  $T_{max}$  with  $[0; T_{max}]$  the solution time

support. We abstract away from this loop here since it does not involve any technical sophistication. Hereafter, we assume that a convenient choice of  $T_{max}$  has already been made.

To solve the maximization problem on  $S = [0; T_{max}]$ , we iterate on the investment variable  $i(t)$  for any  $t$  in  $S$ . Obviously, we need to discretize the problem so as to apply the relaxation algorithm. We initialize the algorithm by a convenient choice of an initial investment vector  $I^0$ ,  $I^0 = [i^0(k)]'$ ,  $k$  denoting the  $k$ th component of the discretized initial investment series. Obviously, the range of  $k$  depends on the discretization step for a fixed  $T_{max}$ . To  $I^0$  corresponds a series for the output, say  $Y^0$ , for the scrapping rule, say  $T^0$  and an initial value for the integral to be maximized. The iterations on the investment vector are conducted as follows:

(i) For a fixed  $k$ , consider some values uniformly distributed around  $i^0(k)$ , say a vector  $[i^0(k, j)]'$ ,  $j$  denoting the  $j$ th component of the chosen vector of values around  $i^0(k)$ .  $j_0$  exists such that  $i^0(k, j_0) = i^0(k)$ .

(ii) For each  $j$ , substitute  $i^0(k)$  by  $i^0(k, j)$  in  $I^0$ . Using constraints (1) and (2), update  $Y^0$  and  $T^0$ . Compute the resulting integral value. Store the integral value.

(iii) Once steps (ii) are performed, we can update the component  $i^0(k)$ . Denote by  $i^1(k)$  the updated component. We set  $i^1(k) = i^0(k, j_{max})$ , with  $i^0(k, j_{max})$  the component of the vector  $[i^0(k, j)]'$  yielding the maximal value for the integral.

Obviously, steps (i) to (iii) have to be performed for all  $k$  in an increasing order. Therefore, we can obtain an updated vector  $I^1$ , with the corresponding updated output vector  $Y^1$ , updated scrapping time vector  $T^1$  and updated integral value. We keep on iterating on the investment vector in this way until reaching a fixed point, as in any relaxation algorithm.

Figure 1: The case of a low initial capital stock

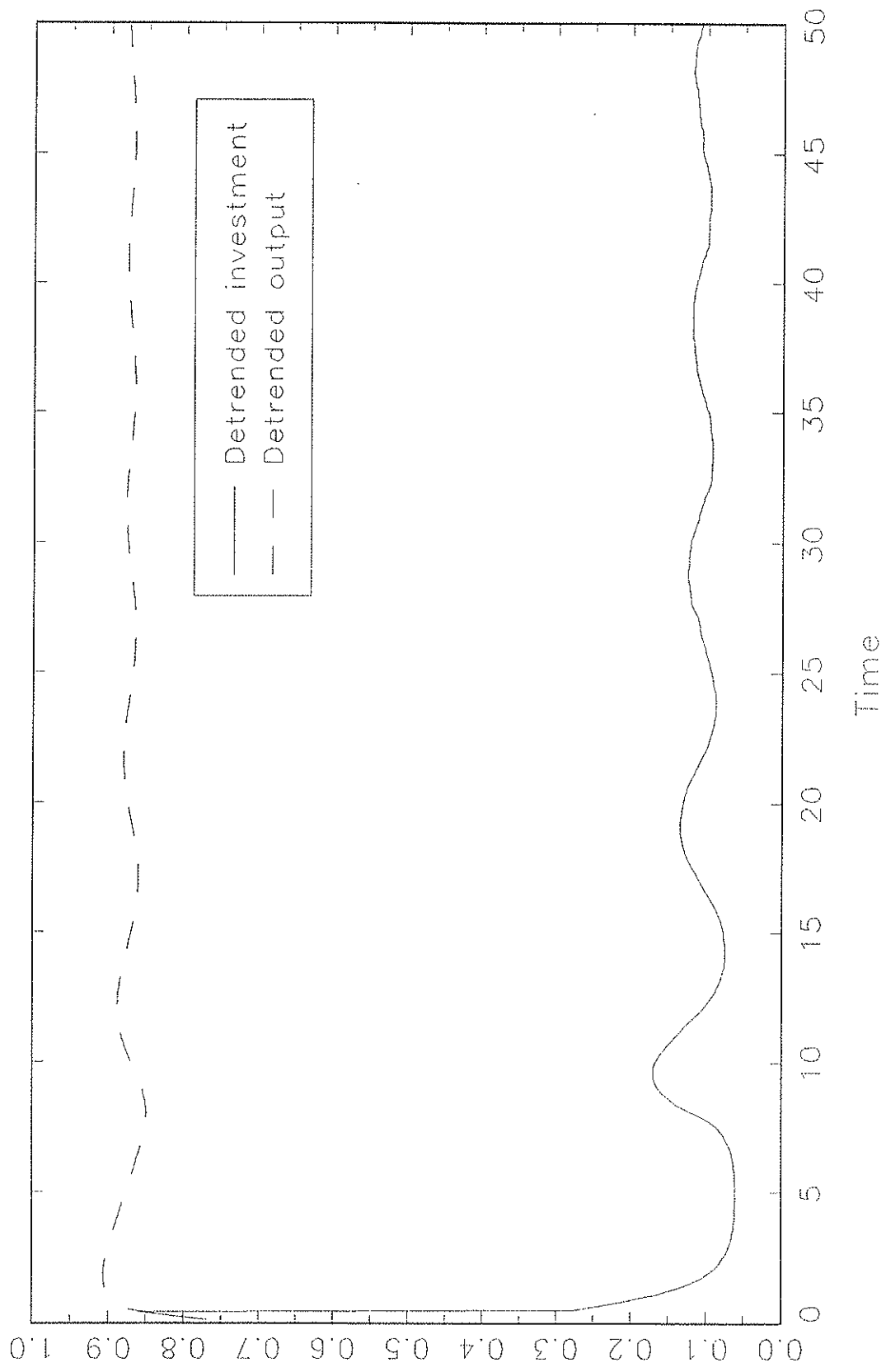


Figure 2: The case of a low initial capital stock

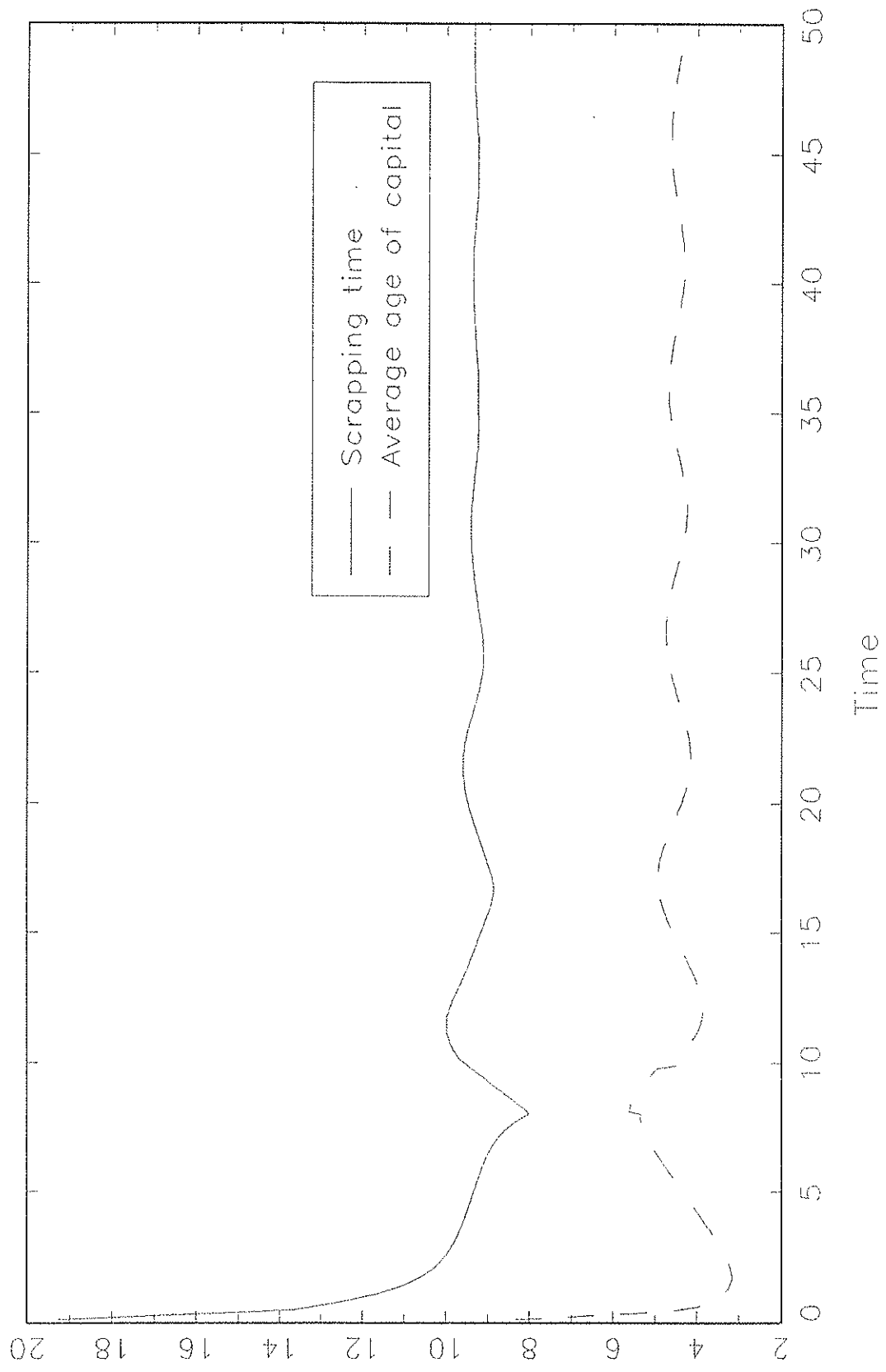


Figure 3: The case of a high initial capital stock

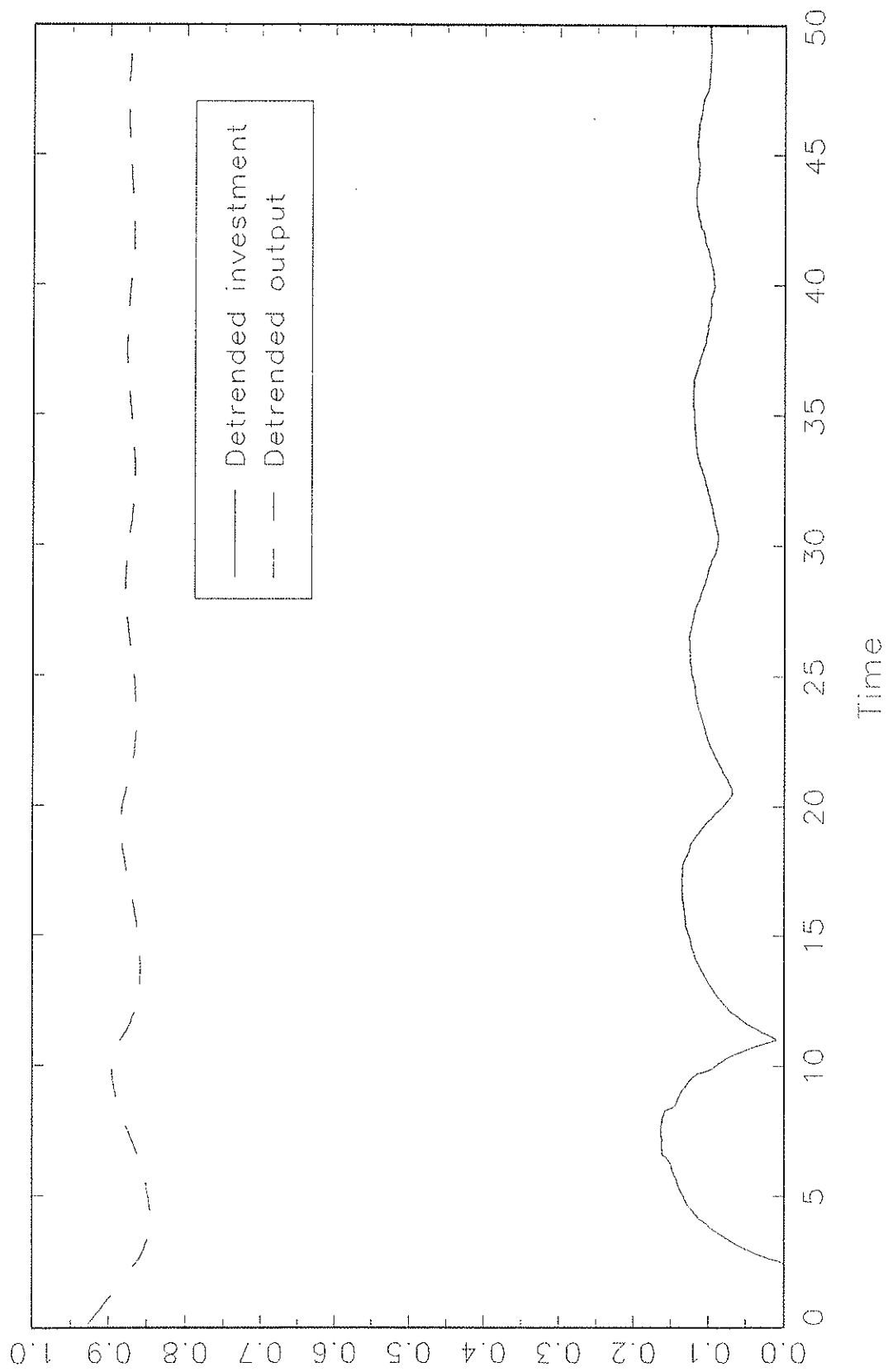
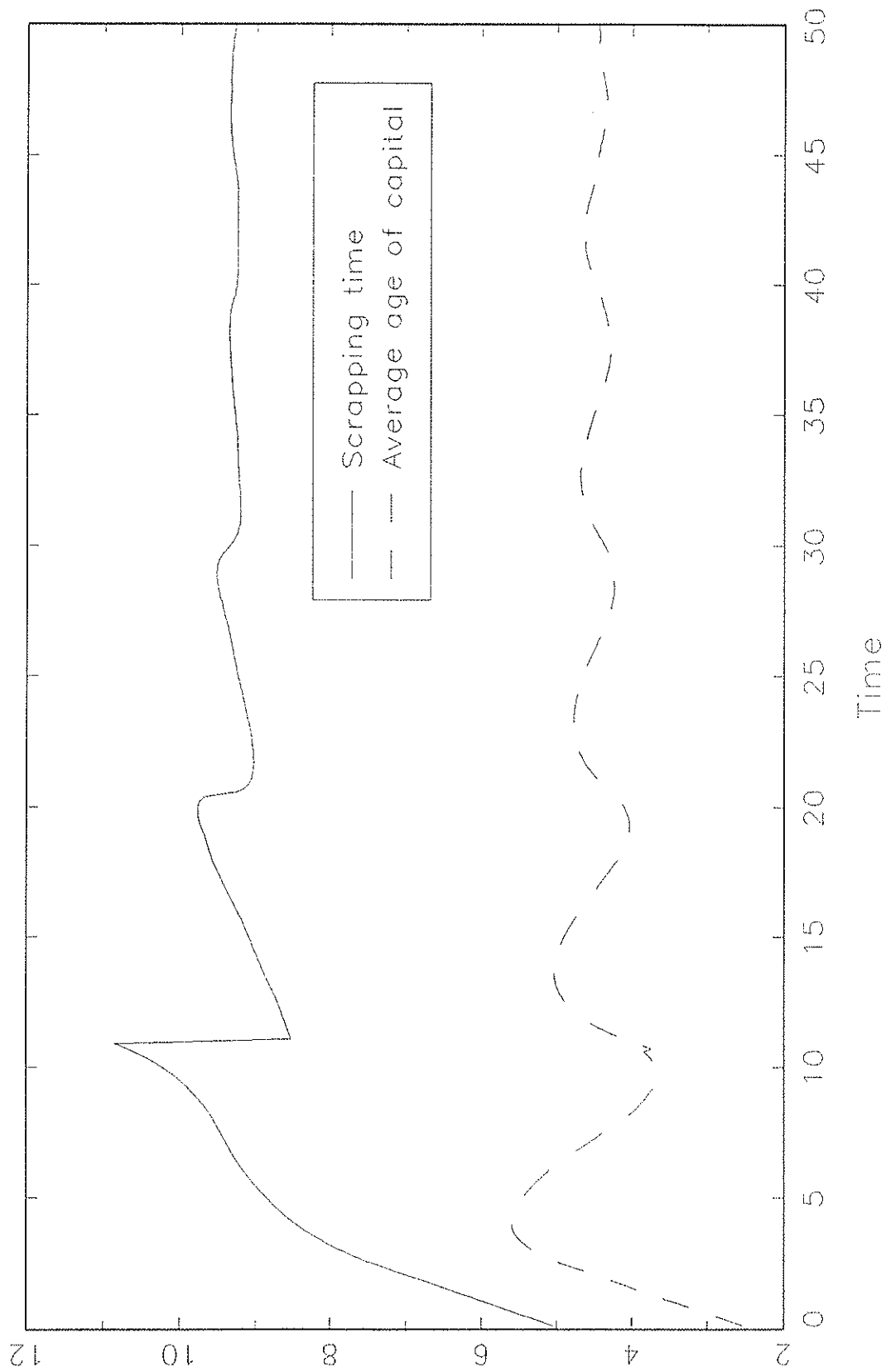


Figure 4: The case of a high initial capital stock



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