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Replacement Echoes in Durable Goods Purchases*

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Abstract

In this note, we formalize durable goods' purchases as a replacement problem in line with the recent vintage capital investment literature. We analytically characterize the optimal scrapping rule associated with the economy under consideration and we show that the optimal durable purchases pattern is purely periodic due to replacement echoes.

Keywords: Durable goods, Vintage models, Replacement echoes

Journal of Economic Literature classification numbers: C52, E22

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1 Introduction

Durable goods consumption involves fundamentally a replacement problem. As durable goods depreciate over time, replacing the old durable goods by the new ones is indeed a key economic decision. Much attention has been paid to this problem in the optimal control literature (see Rust, 1985, for example). In this paper, we argue that the durable goods replacement problem can be formalized in line with vintage capital investment models, and to this end we follow Boucekkine *et al* (1997-a). The basic outcome of this paper is the following: If durable goods purchases are guided mainly by replacement, then the equilibrium could be periodic. This property is traditionally referred to as *replacement echoes*, i.e., purchases today are high (resp. low) when we replace a high (resp. low) stock of old durables.

Empirical and computational evidence of such a cyclical behaviour have been recently provided by Bar-Illan and Blinder (1992) who investigated the implications of (S,s) durable consumption rules, and by Adda and Cooper (1997) who used the dynamic programming approach proposed by Rust (1985). As these approaches are obviously computational to a large extent, the mechanisms of echo fluctuations are not theoretically analyzed. This paper proposes a fully analytical illustration of echo dynamics in durable goods markets. To this end, we adopt a continuous time deterministic formulation of the replacement problem (Section 2) and use a non-standard fixed-point argument to characterize explicitly durable goods scrapping over time (Section 3).

2 The Economy

Our economy is basically a continuous time version of the discrete choice exchange economy described by Adda and Cooper (1997). We consider a continuum of individuals with measure one. Each individual is endowed with a flow y of a non-durable good. The individual can consume the non-durable good or transform it into a durable good at the transformation rate α . Investment in durables is partially irreversible, in the sense that it can be transformed again into the non-durable good at a transformation rate $\beta < \alpha$. Services provided by the durable good depreciate at the instantaneous rate $\delta > 0$, implying that older vintages are less productive than younger ones. We assume that the individuals of this economy hold one and only one unit of the durable good.

Finally, we assume that the consumer's instantaneous utility is linear, and separable between durable services and non-durable consumption. This assumption will allow us to get an analytical solution for the associated optimal control problem and therefore to characterize explicitly the dynamics of the durable goods market under consideration. Under all the assumptions above, the competitive equilibrium can be written as the solution of the following planner problem

$$\max \int_0^\infty [c(t) + s(t)] \exp\{-\rho t\} dt \quad (1)$$

subject to

$$s(t) = \int_{t-T(t)}^t i(z) \exp\{-\delta(t-z)\} dz \quad (2)$$

$$\int_{t-T(t)}^t i(z) dz = 1 \quad (3)$$

$$y(t) = c(t) + (\alpha - \beta)i(t) \quad (4)$$

$$0 \leq i(t) \leq \frac{y(t)}{\alpha - \beta} \quad (5)$$

given $i(t) \geq 0 \forall t < 0$. $c(t)$ represents non-durable consumption and $i(t)$ durables purchases. $s(t)$ is the service flow of durables and $T(t)$ is the lifetime of the oldest durables still in use at time t .

By equation (2), the flow of durable goods' services depends on the stock of durables still in use. Restriction (3) states that all people own one and only one unit of the durable good. The resource constraint is given by restriction (4). Finally, the inequality constraints (5) state that consumption and investment must be non-negative.

In order to solve the problem at $t = 0$, initial conditions on investment are needed for $t < 0$. Clearly, $T(0)$ is undetermined if the past investment profile is not known (by equation (3) at $t = 0$). Moreover, to ensure that equation (3) has a solution at $t = 0$, we assume that: $\int_{-\infty}^0 i(\tau) d\tau > 1$. The investment past profile corresponds to the distribution by vintages of the initial stock of durable goods. Indeed, the economy is characterized by a distribution by vintages of "active" durables at any date.

We denote $\mu(t)$ the Lagrangian multiplier associated with the constraint (3). We also define $J(t)$, the expected age of new durables bought at time t , as follows:

$$J(t) = T(t + J(t)). \quad (6)$$

Taking advantage of the linearity of the problem with respect to $s(t)$ and $i(t)$, we use standard techniques to characterize the interior and corner solutions (see Malcomson, 1975, for technical details). The optimality conditions for this problem are, $\forall t \geq 0$:

$$\mu(t) = \exp\{-\delta T(t)\} \quad (7)$$

$$i(t) \begin{cases} = 0 & \text{if } \Phi(t) < \alpha - \beta \\ = \frac{y(t)}{\alpha - \beta} & \text{if } \Phi(t) > \alpha - \beta \\ \in [0, \frac{y(t)}{\alpha - \beta}] & \text{if } \Phi(t) = \alpha - \beta \end{cases} \quad (8)$$

where: $\Phi(t) = \int_t^{t+J(t)} (\exp\{-\delta(z-t)\} - \exp\{-\delta T(z)\}) \exp\{-\rho(z-t)\} dz$.

Equation (7) is the optimal scrapping condition: the marginal value of durables at t is given by the services provided by the oldest durables in use. Equation (8) describes the three possible regimes the economy can experience. In the interior solution optimal investment is feasible, and the marginal cost $\alpha - \beta$ of a new durable must be equal to its marginal revenue, which is given by the integral over its lifetime of the difference between provided services, $\exp\{-\delta(z - t)\}$, and the opportunity cost $\mu(z)$. Otherwise, a) $i(t) = 0$, if marginal costs are greater than the marginal value of investing; or b) $i(t) = \frac{y(t)}{\alpha - \beta}$, if marginal costs are smaller than the marginal value of investing.

3 Optimal scrapping and echo dynamics

We first derive the optimal scrapping rule corresponding to the interior solution defined above, assuming that this solution is implementable beginning at $t = 0$. To this end, we focus on the dynamics of $T(t)$, since the block recursive structure of our problem allows us to solve first equations (6) and (8) on $T(t)$ and $J(t)$. Provided that $T(t)$ and $J(t)$ are in the set of differentiable functions, we can state the following proposition:

Proposition 1 *For any $t > 0$, $J(\cdot)$ differentiable at t , $J(t)$ and $T(t)$ are such that*

$$T(t) = F(J(t))$$

with: $F(x) = -\frac{1}{\delta} \ln \left[\frac{\rho}{\delta + \rho} - \rho(\alpha - \beta) + \frac{\delta}{\delta + \rho} \exp\{-(\delta + \rho)x\} \right]$.

Given that $J(t)$ is differentiable at t , we can differentiate (8) and easily show Proposition 1 after some elementary manipulations.

In the following we use a fixed-point argument proposed by van Hilten (1991) in order to solve the system (6)-(8). This argument requires function $F(\cdot)$ to be strictly increasing and to admit a unique strictly positive fixed-point. It is easy to show that $F(\cdot)$ fulfills the latter requirements if the parameters check the condition: $\alpha - \beta < \frac{1}{\delta + \rho}$. The latter condition means that durable goods costs must be smaller than the flow of services that durables provide if they are used forever. If not, it will be never profitable to buy durables.

Proposition 2 *Assume that $\alpha - \beta < \frac{1}{\delta + \rho}$ and denote T^* the positive fixed-point of $F(\cdot)$. The unique differentiable interior solutions $T(t)$ and $J(t)$, $t \geq 0$, are given by*

$$T(t) = J(t) = T^*.$$

Proof : The condition on the parameters' values ensures that $F(\cdot)$ is increasing and admits a unique positive fixed-point. By Proposition 1, we know that $T(t) = F(J(t))$,

$\forall t \geq 0$. Since $J(t) \geq 0$ and $F'(x) > 0$, we have $F(0) \leq T(t) \leq \bar{F}$, for every $t \geq 0$, with $\bar{F} = \lim_{x \rightarrow \infty} F(x)$

Applying the latter inequalities at $t + J(t)$ and using (6) yields $F(0) \leq J(t) \leq \bar{F}$, $\forall t \geq 0$. As $F'(x) > 0$, for $x \geq 0$, we get $F(F(0)) \leq T(t) \leq F(\bar{F})$, $\forall t \geq 0$.

As before, we can apply the latter inequalities at $t + J(t)$, find new lower and upper bounds for $J(t)$ and then for $T(t)$. Repeating this reasoning, we can construct a sequence of lower bounds (X_n) and upper bounds (Y_n) for $T(t)$, $\forall n \geq 0$ and $\forall t \geq 0$, such that:

$$X_n \leq T(t) \leq Y_n$$

where $X_0 = F(0)$ and $X_n = F(X_{n-1})$, $n \geq 1$. Y_n is defined by $Y_0 = \bar{F}$ and $Y_n = F(Y_{n-1})$, $n \geq 1$.

It is then easy to show that (X_n) is an increasing bounded sequence and that (Y_n) is a decreasing bounded sequence. Both of them converge by construction to the fixed-point of $F(\cdot)$, T^* . $\square$

Notice that the interior solution $T(t) = T^*$ is implementable beginning at $t = 0$ if and only if $T(0)$ is equal to T^* , which is not true for any investment past profile. Under similar conditions, Boucekkine *et al* (1997-a) show that if the economy starts with $T(0) \neq T^*$, the optimal investment solution is feasible after a finite time adjustment period involving the corner solutions defined in Section 2. Since the interior solution $T(t) = T^*$ is the unique permanent regime for our economy, we abstract away from this problem and we assume that $T(0) = T^*$.

Once established the constancy of the optimal scrapping rule, replacement echoes on durable purchases can be immediately derived from (3). Indeed by differentiation of this equation, we obtain $\forall t \geq 0$:

$$i(t) = i(t - T^*).$$

In our model, the optimal scrapping rule is constant and equilibrium durable goods' purchases are periodic. In particular, the (efficient) equilibrium is characterized by the following property: The initial purchases profile on the interval $[-T^*, 0[$ will be reproduced forever. As such, our model yields a "pure" case of replacement echoes occurrence. It shows that replacement echoes may occur and may be everlasting in an efficient economy. Echoes may be smoothed out under non-deterministic environments when the utility function is strictly concave (as in Boucekkine *et al*, 1997-b), but as our simple model shows, they seem to be a fundamental source of fluctuations in economies involving optimal replacement decisions.

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