FEDEA
Fundación de Estudios de Economía Aplicada

**Numerical solution by iterative methods
of a class of vintage capital models***

by

Raouf Boucekkine , Marc Germain***
Omar Licandro**** and Alphonse Magnus*******

DOCUMENTO DE TRABAJO 97-26

December, 1997

* The numerical computations have been performed on the Convex Exemplar SPP-1600 of the Université Catholique de Louvain at Louvain-la-Neuve.

** Universidad Carlos III de Madrid.

*** C.O.R.E.

**** FEDEA and Universidad Carlos III de Madrid.

***** Université Catholique de Louvain.

This paper is available through Internet:

<http://www.fedea.es/hojas/publicaciones.html#Documentos de Trabajo>

Numerical solution by iterative methods of a class of vintage capital models ^{*}

Raouf Boucekkine, Marc Germain,
Omar Licandro and Alphonse Magnus[†]

December 5, 1997

Abstract

In this paper, we build up an iterative numerical procedure in order to solve a class of vintage capital growth models. More precisely, the numerical method can be applied to solve optimal growth vintage capital models with nonlinear utility functions and Leontieff technologies. Such models have been intensively used in the economics literature since the early 90's. The numerical procedure is of the relaxation type and uses a step by step maximization scheme for updating. The method is applied on an example model. We conclude by some remarks on the relation of our method to the waveform relaxation computational mathematics literature.

Keywords: Vintage Capital Models, Relaxation, Optimization, Waveform Relaxation.

JEL classification: C63; E32; O40

^{*}The numerical computations have been performed on the Convex Exemplar SPP-1600 of the Université Catholique de Louvain at Louvain-la-Neuve.

[†]Universidad Carlos III de Madrid, C.O.R.E., FEDEA and Universidad Carlos III de Madrid, and Université Catholique de Louvain, respectively. **Send mail to:** Raouf Boucekkine, Universidad Carlos III de Madrid, 28903 Getafe (Madrid), Spain; e-mail: barouf@eco.uc3m.es.

1 Introduction

The analysis of vintage capital growth models has regained interest since the early 90's. Indeed, this kind of models allow to address quite conveniently many of the current key economic issues like investment volatility, equipment replacement and the general consequences of the Schumpeterian creative destruction process.¹ Investment volatility and equipment replacement are analyzed using vintage capital models by Benhabib and Rustichini (1993) and by Boucekine *et al.* (1997a, 1997c). Using the same framework, Aghion and Howitt (1994) and Caballero and Hammour (1996), among others, have analyzed the effects of creative destruction on unemployment and job reallocation. Except in Benhabib-Rustichini's paper in which a general CES production function is adopted, all the previous contributions use additionally a Leontieff technology. That is this type of technologies with complementary production factors ensures an endogenous determination of the equipment replacement decision, while gross substitutability may directly induce infinite optimal lifetimes for equipments, which sounds unrealistic. Moreover, when the utility functions are linear, Leontieff technologies allow to bring out some analytical results as in Boucekine *et al.* (1997a).

When the utility functions are non-linear, numerical resolution is unavoidable. The major difficulty in handling numerically optimal growth vintage capital models with the latter feature and with Leontieff technologies, results in the treatment of the endogenous leads appearing in the associated optimality necessary conditions. Precisely, these conditions together with Leontieff specifications yield systems of differential-difference or integro-differential-difference equations with state dependent lags and leads. As long as only state dependent lags or only state dependent leads are involved, the resulting systems can be solved using, directly or with some adaptation work, some well-known algorithms (see Baker and Paul, 1996, and Boucekine *et al.*, 1997b). However, optimal vintage capital growth models do involve simultaneously state dependent leads and lags. So far, no robust numerical solver for such systems has been proposed. In the economic literature, a worthwhile attempt at solving a system of this type is due to Caballero and Hammour (1996). However, the treatment of these authors cannot be applied

¹Creative destruction reflects the idea that economic growth is mainly guided by successive innovations leading to the "death" of the resulting less productive techniques and equipments.

to a general system of equations with endogenous leads and lags since it is based on some very particular assumptions. Especially, having introduced a periodic forcing variable in their model, the authors assume that the solution paths are also periodic and succeed at locating them using some standard algorithms in the literature of two-point boundary value systems.

Because the latter treatment seems at best only implementable on differential-difference systems driven by periodic forcing functions, it is not general. One way to overcome the problem arising from the simultaneous occurrence of state dependent leads and lags consists in tackling the optimization problem directly without using the optimality conditions (where precisely the endogenous leads appear). The optimization work can be performed step by step within a standard fixed-point relaxation algorithm. This strategy is applied by Boucekine *et al.* (1997c) for example. Indeed, this device can be applied to a large class of vintage models of the recent economic literature, namely those including a Leontieff technology (like those quoted above for example). This paper is designed to explain in a simple way the principles of this method so as to make definitely clear the latter claim. In fact, the method has three important advantages:

- i) As explained just above, it allows to avoid the numerical difficulties coming from the simultaneous presence of endogenous leads and lags.

- ii) As theoretically and numerically shown in Boucekine *et al.* (1997a, 1997c), vintage capital growth models may give rise to corner and non-differentiable solutions. These features are obviously likely to be captured by an optimization algorithm based on the objective function. The relaxation algorithms so far used in the economic literature cannot do so because they usually handle the optimality necessary conditions corresponding to the interior solutions of the considered optimization problems.

- iii) Last but not least, our device can be related to the more general waveform relaxation schemes, as explained in the last section of this paper. As such, our method can be improved in different respects using the recent developments in the field of waveform relaxation.

To illustrate the method, we apply it to the vintage capital model analyzed in Boucekine *et al.* (1997c). This model is presented in the next section. Section 3 presents the numerical setting and shows how it is related to the original optimization problem. Section 4 illustrates the working of the method through a very simple example. The concluding section 5 shows to

which extent our method is related to the more general waveform relaxation setting and points at further potential improvements of the method.

2 Optimization Problem

Consider a centrally planned closed economy characterized by a clay-clay vintage capital technology, i.e., technical progress is embodied in new machines. These new machines are of the Leontieff type, and the scrapping of old machines is endogeneous.

The central planner is supposed to maximize social welfare by solving the following problem:

$$W = \max \int_0^\infty u[c(t)] \exp\{-\rho t\} dt \quad (1)$$

subject to

$$y(t) = \int_{t-T(t)}^t i(z) dz \quad (2)$$

$$\int_{t-T(t)}^t i(z) \exp\{-\gamma z\} dz = 1 \quad (3)$$

$$y(t) = c(t) + i(t) \quad (4)$$

$$0 \leq i(t) \leq y(t)$$

given $i(t)$ for all $t < 0$.

$c(t)$ is consumption, $y(t)$ is production, $i(t)$ is investment and $T(t)$ represents the age of the oldest operating machines, or *scrapping time*. The utility function $u(c)$ is assumed to be CD^1 , increasing and strictly concave. Parameter ρ is strictly positive and represents the rate of time preference.

Machines from *vintage* $t, \forall t$, are supposed to produce one unit of output each, and do require $\exp(-\gamma t)$ units of labor. The parameter γ is positive and represents (Harrod neutral) technical progress. Total labor resources are assumed to be constant and equal to one. (2) is the Leontieff production function and (3) is the equilibrium condition on the labor market.

A necessary condition for an interior solution of this optimization problem to exist is:²

²All the properties of the model stated in this section are taken from Boucekline *et al.* (1997c).

$$u'(c(t)) = \int_t^{t+J(t)} (1 - e^{\gamma(z-t-T(z))}) u'(c(z)) e^{-\rho(z-t)} dz \quad (5)$$

with $J(t) = T(t + J(t))$. $J(t)$ is the expected lifetime of the equipments bought at t . The optimality condition above is the optimal (interior) investment rule: The marginal cost of investing, on the left hand side, must be equal to the marginal revenue, which depends on the future scrapping time of new machines. Indeed, this condition captures the *forward-looking* component of the model while (2) and (3) capture its backward-looking dimension. Differentiating all these equations with respect to time yields an integro-differential-difference system with an endogenous lag $T(t)$ and an endogenous lead $J(t)$. No robust solver is so far available to handle this kind of systems. The next section presents an iterative method allowing to solve the considered optimization problem. The optimization work relies directly on the objective function, so it does not use explicitly the necessary conditions that cause the simultaneous presence of endogenous leads and lags to occur.

Before, let us briefly present the major dynamic property of this kind of models in order to allow the reader to understand the results of our experiments in section 4. Because new machines are more productive in our model, older machines are optimally scrapped to be replaced by the former. The resulting dynamics follow the so-called “echo principle”, i.e the ability of an economy to reproduce its own past history (see Benhabib and Rustichini, 1993, for a comprehensive analysis of this feature). In addition to that, the example model considered in this paper admits a balanced growth path. When the economy starts with a too high (Resp. low) past investment profile with respect to the balanced growth paths values, the adjustment to these balanced growth paths optimally induces an initial depression (Resp. expansion) of investment, which will be reproduced in the future according to the “echo principle”. These echo oscillations are indeed damped to allow convergence to the balanced growth paths.

3 Maximization by iteration: Numerical setting and theoretical justifications

3.1 The numerical setting

We decide to perform the approximate maximization of (1), with $c(t) = y(t) - i(t)$ by (4), by replacing the unknown functions i and y by piecewise constant functions on the intervals $(0, \Delta)$, $(\Delta, 2\Delta)$, $\dots$. Let $i_0, i_1, \dots; y_0, y_1, \dots$ be the unknown values. Piecewise constant functions would be crude and not very satisfactory approximations to well-behaved smooth functions, but we want to be ready to cope with possibly discontinuous solutions, and prefer to make a robust and unsophisticated choice. Then, $y - i$ and $u(y - i)$ still are piecewise constant functions, the values of the latter one being $u(y_0 - i_0), u(y_1 - i_1), \dots$

The integral (1) may be performed exactly as

$$\sum_{k=0}^{\infty} u(y_k - i_k) \frac{e^{-k\rho\Delta} - e^{-(k+1)\rho\Delta}}{\rho}.$$

Actually, only a finite sum, from $k = 0$ to $k = N$, is performed, where N is tried to be kept as large as possible, in order to limit the discrepancy (there will be other discrepancies anyhow).

Most integrals will however be estimated by the midpoint rule:

$$\int_0^{(N+1)\Delta} F(t) dt \approx \Delta \sum_{k=0}^N F((k + 1/2)\Delta).$$

We maximize the (discretized) integral by iteration. At each iteration, we use a step by step maximization device. Precisely, at each step, we look at the influence of a single value i_k associated to a time $t_0 = (k + 1/2)\Delta$ on the integral W , keeping unchanged the other investment ordinates. The whole algorithm can be described briefly as follows:

a) Initialize the investment vector $[i_0 i_1 \dots i_N]'$, the base of the relaxation.

b) *Maximization step by step*

i) **step 0** Maximize the discretized integral (1) with respect to i_0 keeping unchanged all the subsequent investment ordinates with respect to the base. Update i_0 with the resulting maximizand.

ii) **step k** For $k = 1, \dots, N$, maximize the discretized integral (1) with respect to i_k keeping unchanged the posterior investment ordinates, if any, with respect to the base, with the anterior investment ordinates i_l , $0 \leq l \leq k - 1$, updated thanks to the anterior maximization steps. Update i_k using the resulting maximizand.

c) Compare the relaxation base with the updated investment vector resulting from step b). If they are sufficiently close, stop the iterations. Otherwise, run another step b) with the updated investment vector as a base. The latter type of iteration should be repeated until locating a fixed-point investment vector.

Obviously, the computations require also the updating of the state variables y and T . In effect, the maximization with respect to a single ordinate i_k involves all the $y_j = y((j+1/2)\Delta)$'s, which have to be computed from (2), for $j = 1, 2, \dots$. And as (2) shows that $y(t)$ depends on $T(t)$, we have to compute all the $T((j+1/2)\Delta)$'s as well. The equation (3) allows to compute each $T((j+1/2)\Delta)$ on the basis of a piecewise constant function $i(z) \exp(-\gamma z)$:

We look for the area of $i(z) \exp(-\gamma z)$ enclosed between the abscissa $(j+1/2)\Delta - T((j+1/2)\Delta)$ and the abscissa $(j+1/2)\Delta$ to have unit value.

Remark that by (3), we have only to change the values of T at points $t_1 = (j+1/2)\Delta$ in the future of t_0 such that $t_1 - T(t_1) \leq t_0$, i.e., at points $t_1 \in [t_0, t_0 + J(t_0)]$. The new trial values of y also have to be recomputed in the same interval. Therefore, maximization on the value of i on a single time step Δ asks for the computation of fresh values of y and T on a future interval of length say J , whence about J/Δ steps for updating a single value of i . And these operations should be done for all time steps, from $t = 0$ to the chosen solution time horizon with a Δ increment. Note also that these

operations are not so elementary: They include the computation of integrals by equations (2)-(3).

The latter comments give an idea about the complexity of steps b). The computational burden of the whole algorithm depends on the number of steps c), i.e, iterations on the whole investment vector, required to achieve convergence to a given tolerance level. We have right now still no satisfactory hints on the behaviour of the error through iterations c). We will come back to this issue in the concluding section. Indeed, in our most refined tests, we took $\Delta = .1$, about 1% of the average value of the scrapping time T , and a time solution horizon equal to 100, about 10 times the average T . So, we had about 10^3 one-dimensional maximizations to perform per iteration. Finally, at least 100 iterations c) are needed to get a reasonable accuracy (around 10^{-4}).

Although the convergence properties of step c) seem rather intractable analytically, we can find some strong theoretical foundations for the algorithm. We present them just below.

3.2 Theoretical justifications of the numerical setting

The following concavity proof validates the step by step optimization strategy followed in our numerical setting. Using this proof, we can also deduce a differential relation that shows clearly how and why the solutions provided by the algorithm are related to the optimality conditions of the original optimization problem.

Proposition. *The discretized integral (1) is a concave function, when considered as a function of a single ordinate $i(t_0)$, for any fixed $t_0 \geq 0$.*

Proof. Indeed, let $W(\alpha)$ be the value of (1) in a situation where $i(t_0) = \alpha$. The integral is estimated by the mid-point rule, as it is made in actual computations:

$$W(\alpha) = \Delta \sum_{j=0}^N u[c((j + 1/2)\Delta)] \exp(-\rho(j + 1/2)\Delta),$$

where $t_0 = (k + 1/2)\Delta$, Δ a sufficiently small number and k is some nonnegative integer.

We now decide to give some new value $i(t_0) = \beta$ on the interval of width Δ and center t_0 , with $i(t)$ unchanged in all the other intervals. The value of

$W(\beta)$ depends on all the changes induced in the various values $c((j+1/2)\Delta)$. Let $t_1 = (j+1/2)\Delta$ be another time abscissa. We have $c(t_1) = y(t_1) - i(t_1)$, so we have only to appreciate the change in $y(t_1)$, which depends on $T(t_1)$. From (3),

$$1 = \int_{t_1-T(t_1)_\beta}^{t_1} i(z)_\beta e^{-\gamma z} dz = \int_{t_1-T(t_1)_\beta}^{t_1} i(z)_\alpha e^{-\gamma z} dz + (\beta - \alpha)\Delta e^{-\gamma t_0},$$

only if $t_1 - T(t_1) \leq t_0 \leq t_1$, as (3) will be kept unchanged in other cases. Let us subtract the “old” equation (3):

$$\begin{aligned} 0 &= \int_{t_1-T(t_1)_\beta}^{t_1-T(t_1)_\alpha} i(z) e^{-\gamma z} dz + (\beta - \alpha)\Delta e^{-\gamma t_0} \\ &\approx [T(t_1)_\beta - T(t_1)_\alpha] i(t_1 - T(t_1)) e^{-\gamma(t_1 - T(t_1))} + (\beta - \alpha)\Delta e^{-\gamma t_0}, \end{aligned} \quad (6)$$

if β is chosen to be close to α , so that the change of $T(t_1)$ is small.

We now look at the new value of $y(t_1)$:

$$y(t_1)_\beta = \int_{t_1-T(t_1)_\beta}^{t_1} i(z)_\beta dz = \int_{t_1-T(t_1)_\beta}^{t_1} i(z)_\alpha dz + (\beta - \alpha)\Delta,$$

$$y(t_1)_\beta - y(t_1)_\alpha = \int_{t_1-T(t_1)_\beta}^{t_1-T(t_1)_\alpha} i(z)_\alpha dz + (\beta - \alpha)\Delta \approx [T(t_1)_\beta - T(t_1)_\alpha] i(t_1 - T(t_1)) + (\beta - \alpha)\Delta,$$

which simplifies into

$$y(t_1)_\beta - y(t_1)_\alpha = (\beta - \alpha)\Delta [1 - \exp(\gamma(t_1 - T(t_1) - t_0))],$$

thanks to (6). We just got the increase of $c(t_1) = y(t_1) - i(t_1)$, as far as $t_1 - T(t_1) \leq t_0 < t_1$. At t_0 , the change is of course

$$c(t_0)_\beta - c(t_0)_\alpha = y(t_0) - \beta - [y(t_0) - \alpha] = \alpha - \beta,$$

and the change for W is

$$\begin{aligned} W(\beta) - W(\alpha) &= \Delta [u(c(t_0)_\alpha + \alpha - \beta) - u(c(t_0)_\alpha)] e^{-\rho t_0} \\ + \Delta \sum_{t_1-T(t_1) \leq t_0 < t_1} \{ &u(c(t_1)_\alpha + (\beta - \alpha)\Delta [1 - e^{\gamma(t_1 - T(t_1) - t_0)}]) - u(c(t_1)_\alpha) \} e^{-\rho t_1}, \end{aligned} \quad (7)$$

for β close to α and for small Δ .

As (7) is a linear combination of concave functions of β , $W(\beta)$ is indeed a concave function too, for β in a small interval, but concavity must only be established locally to hold everywhere. $\square$

As W is now known to be a concave function of a single i ordinate, we are therefore sure that $W(x)$ has a unique maximum in the admissible range, i.e., from $x = 0$ up to the value such that $x = i(t_0) = y(t_0)$.

An interesting consequence of (7) is the differential relation when $\beta \rightarrow \alpha = x$:³

$$W'(x) = \Delta \left[-u'(c(t_0))e^{-\rho t_0} + \int_{t_0}^{t_0+J(t_0)} u'(c(t_1))[1 - e^{\gamma(t_1-T(t_1)-t_0)}]e^{-\rho t_1} dt_1 \right], \quad (8)$$

and $W'(x) = 0$ for all $t_0 \geq 0$ yields the well known optimality condition (5), solving problem (1) with constraints (2-4). This shows how the numerical procedure is linked to this optimality conditions of the original optimization problem.

4 An example

Let us solve the problem with $\rho = 0.05$, $\gamma = 0.03$, $u(c) = c^\theta$, $\theta = 0.85$ close to 1. So we have a weakly concave function to maximize, and we expect rather a rough behaviour of the solution paths. A perfect exponential solution is $i(t) = 0.1078 \exp(0.03t)$: if this law holds for $t < 0$, it will still hold when $t > 0$. As one expects a perturbation of this law, one makes the computations with i_k approximating $i(t) \exp(-\gamma t)$ at $t = (k + 1/2)\Delta$, $k = 0, 1, \dots$

One decides to take $i(t) = 0.3 \exp(0.03t)$ on $t < 0$. This is much too high! At $t = 0$, the economy is allowed to react. What will happen?

Let us look at the record. One starts with trial values $i(t) = 0.1078 \exp(0.03t)$ at $t > 0$, i.e., $i_k = 0.1078$ for all $k \geq 0$. Let us perform the approximate calculations with a (much too large) time step $\Delta = 1$. We look at the integral (1) for several test values i_0 , keeping i_1, i_2 , etc. unchanged:

	i_0	W
→	0	32.7516401
	0.05	32.7313349
	0.1	32.7089774
	0.15	32.6845603
	0.2	32.6588714

³Assuming that all the differentiations are valid, or in other words that an interior solution exists for the optimization problem at t_0 .

The integral is the largest when $i_0 = 0$, which is not surprising, as $i(t)$ was too large at $t < 0$. We keep $i_0 = 0$, and make i_1 move:

	i_1	W
→	0	32.778625
	0.05	32.7673855
	0.1	32.7539278
	0.15	32.7379688
	0.2	32.7206465

Again, $i_1 = 0$ is best. And we get the same answer for i_2 . Setting $i_0 = i_1 = i_2 = 0$, we maximize with respect to i_3 :

	i_3	W
	0	32.7801023
	0.05	32.7851276
	0.1	32.787636
→	0.11	32.7876691
	0.12	32.7875742
	0.15	32.786856

Some change at last, the best i_3 appears to be close to 0.11. We know look at i_4 :

	i_4	W
	0	32.7758335
	0.05	32.7825734
	0.1	32.7873551
	0.15	32.7884092
	0.16	32.7884577
	0.17	32.7884902
→	0.18	32.7885064
	0.19	32.7884807

Here, the best value of i_4 is about 0.18. One goes on with next values (in order not to be bothered by truncated series effects in the estimation of the integral, we took a big upper bound, $N = 500$).

These successive maximizations of the integral (1) (step b) using the terminology of subsection 3.1) lead to the following first approximation to $i(t) \exp(-\gamma t)$:

Most of the values stay close to the initial 0.1078. One sees slight depressions at 11 and 19, faint echoes of the big depression near the origin.

The job is not finished yet! We just performed maximization of (1) on each i_k , keeping unchanged the i_j 's for $j < k$. This is only a limited way to see how i_k interacts with its neighbours, and we start again a whole run of maximizations following exactly the same principles as before (step c) in the terminology of section 3.1).

Such a process is often called “relaxation”, as it was first used in the calculation of complicated mechanical devices, with lots of interacting parts. The calculation considered only a limited number of links, ignoring (i.e., *relaxing*) the other ones. Of course, when such a run of simplified calculations was achieved for all parts, there was still no exact matching, and the whole process was restarted again, and again. . . Given the concavity of our problem with respect to the investment variable, the amount of discrepancy between the solutions of two successive iterations tends to diminish along this process

On our example, a second iteration (or a second run of maximizations) does not change the values of i_0, i_1 , and i_2 , which are still kept at the zero level. However, the best i_3 is found to be no more 0.11, but 0.04:

i_3	W
0	32.7895023
0.02	32.7912463
0.03	32.7916228
→ 0.04	32.7916472
0.05	32.7916362
0.1	32.7905948

Things hardly changed:

After about 20 iterations, the final shape (at this poor $\Delta = 1$ precision) appears:

Indeed, much more work (150 iterations with $\Delta = 0.2$) is needed for disclosing with finer detail the solution:

The solution settles in a regime of slowly damped oscillations about the limit value of 0.1078 (indicated by a thin horizontal line).

5 Concluding: Our method as an elementary waveform relaxation scheme

Actually, our method can be seen as an elementary implementation of the general waveform relaxation scheme, and as such there is a room for substantial improvements of our method using the recent computational mathematics developments on the latter general scheme (see for example Bjorhus and Stuart, 1997, Vandewalle and Piessens, 1993, and Miekkala and Nevanlinna, 1987). Initially, waveform relaxation is an algorithmic device used for solving a large number of neighbouring ordinary differential equations. Of course, no iterative method is needed for the numerical solution of ordinary differential equations, but if one knows the time history of the solution of one equation, a small inexpensive correction may be enough to get a satisfactory approximation to the solution of a nearby equation.

We kept the idea of successive corrections to a known (or guessed) time history, as it seems to work in our problem. As explained in section 3, the corrections are achieved through local maximization of (1).

Now, when the needed corrections are not merely infinitesimal, the process appears to be very slow. Progress in waveform relaxation deals with the possibility of acceleration. A quite abstract writing of what is going on is

$$i = \mathcal{K}(i), \tag{9}$$

where $\mathcal{K}$ summarizes all the amount of work needed in a complete iterative step. Note that $\mathcal{K}$ is an operator, as i is here a function. The equation (9) is simply the fixed-point functional equation, as it relates to a particular function i kept unchanged by application of the operator $\mathcal{K}$.

A typical scheme for our problem could be $\mathcal{K}(i) = i + \omega W'(x)$ with the $W'(x)$ of (8), ω a fixed positive parameter. Such a scheme (Gauss-Seidel, SOR, i.e., *Successive Over Relaxation*) is examined, for instance in Miekkala and Nevanlinna (1987) and Vandewalle and Piessens (1993) for differential problems. Theoretical study of convergence is heavily linked to spectral properties of the operator $\mathcal{K}$. Unlike in the traditional waveform relaxation application fields, the operator $\mathcal{K}$ is fundamentally implicit in our case, that is why we could not find any analytical characterization of the convergence speed of the algorithm (see subsection 3.1). However, the acceleration devices

mentioned above can be incorporated into our algorithm. We did not resort to them in this paper. Nonetheless, there is no doubt that acceleration can be highly useful in many circumstances, especially when the iteration work has to be performed on more than one function. Reducing the computational burden of the method would result crucial in such cases.

6 References

- Aghion, Ph. and P. Howitt, 1994, Growth and unemployment, *Review of Economic Studies* 61, 477-494.
- Baker, C. and C. Paul, 1996, A global convergence theorem for a class of parallel explicit Runge-Kutta methods and vanishing lag delay differential equations, *SIAM Journal of Numerical Analysis* 33, 1559-1576.
- Benhabib, J and A. Rustichini, 1993, A vintage capital model of investment and growth, in R. Becker *et al.* Eds, *General Equilibrium, Growth and Trade. II The Legacy of Lionel McKenzie* (Academic Press, New York) 248-301.
- Bjorhus, M. and A.M. Stuart, 1997, Waveform relaxation as a dynamical system, *Mathematics of Computation* 66, 1101-1117.
- Boucekkine, R., M. Germain and O. Licandro, 1997a, Replacement echoes in the vintage capital growth models, *Journal of Economic Theory* 74, 333-348.
- Boucekkine, R., O. Licandro and C. Paul, 1997b, Differential-difference equations in economics: On the numerical solution of vintage capital growth models, *Journal of Economic Dynamics and Control* 21, 347-362.
- Boucekkine, R., M. Germain, O. Licandro and A. Magnus, 1997c, Creative destruction, investment volatility and the average age of capital, DT 97-08 (FEDEA, Madrid).
- Caballero, R. and M. Hammour, 1996, On the timing and efficiency of creative destruction, *Quarterly Journal of Economics* 111, 805-851.
- Miekkala, U. and O. Nevanlinna, 1987, Convergence of dynamic iteration methods for initial value problems, *SIAM Journal of Scientific and Statistical Computing* 8, 459-482.
- Vandewalle, S. and R. Piessens, 1993, On dynamic iteration methods for solving time-periodic differential equations, *SIAM Journal of Numerical Analysis* 30, 286-303.

RELACION DE DOCUMENTOS DE FEDEA

COLECCION RESUMENES

97-01: "Geografía económica y crecimiento", **Juan J. de Lucio.**

TEXTOS EXPRESS

97-02: "II Encuesta sobre la UEM InterMoney- FEDEA: Resultados", **C. Arenillas, J. A. Herce, J. A. Ketterer, S. Sosvilla y D. Vegara.**

97-01: "La cuestión de las pensiones", **José A. Herce.**

DOCUMENTOS DE TRABAJO

97-26: "Numerical solution by iterative methods of a class of vintage capital models", **R. Boucekkine, M. Germain, O. Licandro y A. Magnus.**

97-25: "Statistical modeling of fishing activities in the North Atlantic", **C. Fernández, E. Ley y F. J. Steel.**

97-24: "Asymmetry in the EMS: New evidence based on non-linear forecasts", **O. Bajo-Rubio, S. Sosvilla-Rivero y F. Fernández-Rodríguez.**

97-23: "Life expectancy and endogenous growth", **David de la Croix y Omar Licandro.**

97-22: "Los efectos de los planes Renove y Prever sobre el reemplazo de turismos", **Omar Licandro y Antonio R. Sampayo.**

97-21: "Geografía Económica: Aglomeración, localización y externalidades", **Juan J. de Lucio.**

97-20: "Divide et impera: Bargaining and incentives within a hierarchy", **Diego R. Palenzuela y Justo Jorge Moreno.**

97-19: "Union effects and the reform of labor relations, with consensus", **Diego R. Palenzuela.**

97-18: "Pay determination in the Spanish public sector", **Cecilia Albert, Juan F. Jimeno y Gloria Moreno.**

97-17: "Provision of private health insurance under public insurance captivity", **Diego R. Palenzuela.**

97-16: "Inversión directa extranjera y especialización comercial en los países periféricos, **Salvador Barrios.**

97-15: "Replacement echoes in durable goods purchases", **Raouf Boucekkine y Omar Licandro.**

97-14: "Credibility in the EMS: New evidence using nonlinear forecastability tests", **F. Fernández-Rodríguez, S. Sosvilla-Rivero, J. Martín-González.**

97-13: "Replacement investment, endogenous fluctuations and the dynamics of job creation and job destruction", **Raouf Boucekkine, Fernando del Rio y Omar Licandro.**

97-12: "La demanda de automóviles en España: Un análisis de la evolución y variabilidad de las tasas de reemplazo", **Omar Licandro, Antonio R. Sampayo.**

97-11: "Respuesta de los tipos de interés nominales españoles a *shocks* de inflación esperada y de tipos de interés real *ex-ante*: Una aplicación VAR estructural", **Vicente Esteve.**

97-10: "Convergence in fiscal pressure across EU countries", **Vicente Esteve, Simón Sosvilla y Cecilio Tamarit.**

97-09: "Evaluación de los efectos macroeconómicos del fondo de cohesión en España", **Juan Carlos Císcar.**

97-08: "Creative destruction, investment volatility, and the average age of capital", **R. Boucekkine, M. Germain, O. Licandro y A. Magnus.**

97-07: "Spatially and intertemporally efficient waste management: The costs of interstate flow control", **Eduardo Ley, Molly K. Macauley y Stephen W. Salant.**

97-06: "Are there any special features in the Spanish business cycle?", **Luis Puch y Omar Licandro.**

97-05: "Los factores específicos del paro en Andalucía" **Juan F. Jimeno.**