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Minimum Consumption, Transitional Dynamics and the Kuznets Curve*

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Abstract

This paper investigates the evolution of wealth in a one sector growth model since the early stages of development to its steady state. A key feature of the model is that a household's consumption cannot fall below a positive level each period. This requirement introduces a positive association between the intertemporal elasticity of substitution and household wealth. Households only differ in their initial holdings of capital. The model is calibrated to match some key statistics of the US economy. The level of inequality in the wealth distribution of our artificial economy increases in the early stages of development and declines as the economy approaches the steady state. The level of wealth inequality and its evolution resembles that of the US economy. Additionally our model illustrates that observing a time series Kuznets curve does not imply we should observe such relationship in cross-section data, and vice-versa.

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1 Introduction

Kuznets in his seminal 1955 paper suggested, observing the experience of the US and some Western European countries, that income inequality widened in the early stages of industrialization and declined afterwards. Then he further suggested that this inverted U relationship between inequality and level of income could be a pattern of the process of development. Since then, many researchers have investigated this issue. The research has taken two approaches: In the applied area, there has been an effort to gather data to test this inverted U relationship —the Kuznets curve— and in the theoretical approach, authors have built dynamic models where the Kuznets curve arises endogenously.

There has been much applied work trying to test whether the Kuznets hypothesis is rejected or not by the data. See, for instance, Deininger and Squire (1996, 1998), who have extensive references in the area. They conclude in their 1998 paper that there is little evidence for the Kuznets hypothesis, whereas Williamson (1991) is a firm supporter of it. Most of the theoretical work in which the Kuznets curve arises endogenously rely on some type of friction, market imperfection or a non-convexity in the technology. See, for instance, Greenwood and Jovanovic (1990), Galor and Tsiddon (1992), Banerjee and Newman (1993) or Glomm and Ravikumar (1998).

Our paper takes a completely different approach from the above mentioned work. Here we investigate how much inequality can be generated by the sheer process of capital accumulation. We study the evolution of the distribution of wealth in a one sector model along its transition path. A key feature of the model is that a household's consumption cannot fall below a positive level each period. This requirement introduces a positive association between the intertemporal elasticity of substitution (IES, hereafter) and household's wealth. Individuals only differ in their initial holdings of capital. There is no uncertainty of any source in this model. We assume that there exists a perfect capital market. Therefore, the level of wealth inequality does not affect the aggregate dynamics of the model, but this dynamics does influence the evolution of the distribution of wealth. The model is calibrated to match some key statistics of the US economy. We find that the level of inequality in the wealth distribution of our artificial economy increases at low levels of per capita income and declines as the economy approaches the steady state. Thus, we find a Kuznets curve for wealth. The level of inequality and its evolution resembles that of the US economy.

We find that the initial level of inequality in a country is very persistent over time. This also means that cross-country differences in inequality are persistent over time, too. We think these results are at the heart of the next result: Observing an inverted U relationship between wealth inequality and the level of per capita income along the transition for a group of countries does not say anything about the cross-country relationship between inequality and level of income, nor vice-versa. Therefore, we cast doubts about the validity of using cross-section data to test a time series phenomenon.

This paper is very close to Chatterjee (1994). The main difference between his setup and ours is that he assumes that the individual's share of total labor earnings is always the same that the individual's share of capital earnings. Under that assumption, he proves

that wealth inequality is either invariant with respect to income, or increases monotonically if a minimum consumption requirement is introduced. We drop that assumption and assume that all individuals have the same human capital and, hence, receive the same wage per hour worked.

Caselli and Ventura (1996) use a framework similar to ours. They build a model in which there is a minimum consumption requirement and assume that there are perfect capital markets. They find a similar result: The evolution of the wealth distribution exhibits the inverted U shape. The main difference between their model and ours is that they use a C.E.S. production function with a elasticity of substitution between capital and labor less than one, while we use the standard Cobb-Douglas technology. Thus, our model is able to replicate the standard long-run stylized facts.

Chatterjee and Ravikumar (1999) also study the link between capital accumulation and inequality along the transition path and introduce a minimum consumption requirement. They also generate a Kuznets curve. As opposed to us, they use a linear technology and calibrate their model to match some estimates for the Indian economy.

The key departure of our model from the standard neoclassical framework is that we introduce a minimum consumption requirement. A minimum consumption requirement introduces a positive association between household wealth and the intertemporal elasticity of substitution (IES). Wealthier households have a higher IES and, therefore, a higher savings rate. Using panel data on Indian villagers, Atkeson and Ogaki (1996, 1997) find economically significant differences in the IES across rich and poor households. Rosenzweig and Wolpin (1993) also use Indian data and find a minimum consumption requirement to be statistically significant and to amount to a sizable fraction of consumption expenditures of the average household. Moreover, Rebelo (1992) and Ogaki, Ostry and Reinhart (1996) argue that low savings rates and low elasticity of savings to the interest rate point to the existence of a minimum consumption requirement. Thus, we think that a minimum consumption requirement is important to understand the process of capital accumulation and growth.

The rest of the paper is organized as follows: Section 2 describes the economic environment. Section 3 shows some theoretical results and establishes the connection between level of per capita income and wealth inequality. In section 3 we present the calibrated version of our model and the results of our simulations. In section 4 we study whether cross-section data can be used to test the existence of the Kuznets curve. Section 5 concludes.

2 The model economy

The model economy is a discrete time infinite horizon economy, populated by a measure one of individuals that live forever. Each period, individuals obtain utility from consuming a commodity, c_t , that is produced using physical capital and labor. Individuals are endowed each period with one unit of labor. They do not value leisure and differ in their initial holdings of capital, k_0^i , where i is just an index to order the types of individuals ac-

cording to their level of initial wealth (physical capital). There are I types of individuals. We assume all types have the same measure. K_0 denotes the initial aggregate capital of the economy. Population grows at the rate η ,

$$N_t = (1 + \eta)^t \quad (1)$$

All households have identical preferences defined over consumption at every date,

$$U = \sum_{t=0}^{\infty} \beta^t (1 + \eta)^t u(c_t), \quad \beta \in (0, 1). \quad (2)$$

The one period utility function is a variant of the Stone-Geary form,

$$u(c_t) = \frac{(c_t - \alpha_t)^{1-\sigma}}{1-\sigma}, \quad \sigma > 1,$$

where α_t denotes the subsistence level of per capita consumption at period t . We assume that this level varies over time. The representative firm uses a Cobb-Douglas technology to produce the consumption good,

$$F(K_t, N_t) = A_t K_t^\theta N_t^{1-\theta}, \quad \theta \in (0, 1), \quad (3)$$

where K_t denotes aggregate capital, N_t denotes aggregate labor. A_t is the exogenous technological progress factor that grows at rate γ . Thus, the growth rate of per capita income in the balanced growth path is

$$1 + g = (1 + \gamma)^{\frac{1}{1-\theta}}. \quad (4)$$

We assume that the subsistence level of consumption grows at the balanced growth rate of per capita income. This is a way of capturing the notion that the level of subsistence consumption grows at a rate less than or equal to that of the economy,

$$\alpha_t = (1 + g)^t \alpha. \quad (5)$$

The firm faces a series of static one-period profit maximization problems,

$$\max_{K_t, N_t} A(1 + \gamma)^t K_t^\theta N_t^{1-\theta} - w_t N_t - r_t K_t \quad (6)$$

The wage rate, w_t , and the real rental price of capital, r_t , are equal to the marginal productivity of the production factors in equilibrium. Capital depreciates at a constant rate, $\delta \in (0, 1)$. There exist perfect capital markets: i.e., individuals are able to borrow and lend without any restriction at the market interest rate.

2.1 The consumer's problem

In this subsection we specify the modified model economy we are going to study throughout this paper. This economy exhibits a balanced growth path, along which the rate of

growth of the per capita variables is $(1+g)$. We detrain all the variables to eliminate the long run growth. Once done this, the consumer's problem can be written as

$$\begin{aligned} \max_{c_t^i, k_{t+1}^i} \quad & \sum_{t=0}^{\infty} \phi^t \frac{(c_t^i - \alpha)^{1-\sigma}}{1-\sigma} \\ \text{s.t.} \quad & c_t^i + (1+g)(1+\eta)k_{t+1}^i \leq w_t + (1+r_t - \delta)k_t^i \\ & k_0^i \text{ given} \end{aligned} \tag{7}$$

where $\phi = \beta(1+\eta)(1+g)^{1-\sigma}$. The factor $(1+\eta)$ is due to population growth and $(1+g)^{1-\sigma}$ is due to technological progress.¹ This economy converges to a steady state without long run growth. The evolution of prices and wealth inequality here is identical to that of the original economy.

3 The transition path and the distribution of wealth

In this section we obtain the individuals' demand functions and investment and the law of motion of capital. Once done this, we turn to analyze the evolution of the wealth distribution.

3.1 The aggregate dynamics of the model

Here we estate some properties of the individuals' demand functions. Let us consider the problem (7).

Proposition 1 *The evolution of aggregate capital, consumption, and investment does not depend on the wealth distribution.*

This result is due to the specific utility function used, the existence of perfect capital markets and the assumption that individuals do not obtain utility from leisure. Thus, in this economy, growth affects the evolution of wealth inequality, but inequality does not affect growth. The dynamics of the aggregate variables does not depend on the initial level of inequality and is identical to the dynamics of the representative agent version of this economy. Nevertheless, the dynamics of the wealth distribution and the final distribution in the steady state will depend on the initial level of inequality. Next subsection discusses this point.

¹Notice that c_t^i in problem (7) is not the same that those considered before in expression (2). If we renamed consumption in expresion (2) as $\tilde{c}_t^i$ then the relation is $\tilde{c}_t^i = c_t^i \cdot (1+g)^t$.

3.2 The evolution of the wealth distribution

To study the evolution of wealth inequality, let us define X_t^i as the ratio of individual i 's wealth to aggregate capital, $X_t^i = \frac{k_t^i}{k_t}$. Replacing the demand function in the individual budget constraint, we can obtain that the law of motion of the individual wealth follows

$$k_{t+1}^i = B_t + D_t k_t^i, \quad (8)$$

where

$$D_t = \frac{1}{(1+g)(1+\eta)}(1+r_t-\delta) \left(1 - \frac{1}{M_t}\right), \quad (9)$$

$$B_t = \frac{1}{(1+g)(1+\eta)} \left[\left(1 - \frac{1}{M_t}\right) (w_t - \alpha) - \frac{1}{M_t} \sum_{s=t+1}^{\infty} \frac{p_s}{p_t} (w_s - \alpha) \right] \quad (10)$$

We can also express the aggregate capital in terms of factors B_t and D_t ,

$$k_{t+1} = B_t + D_t \cdot k_t. \quad (11)$$

Thus, the evolution of the ratio X_t^i respect to the average is given by

$$X_{t+1}^i - 1 = \frac{D_t \cdot k_t}{B_t + D_t \cdot k_t} (X_t^i - 1) \quad (12)$$

Expression (12) shows that the share X_{t+1}^i gets closer to (further away from) the average when the factor $\frac{D_t \cdot k_t}{B_t + D_t \cdot k_t}$ is smaller (greater) than one. The value of that ratio depends on the signs of the factors B_t and D_t . It is easy to check that, since M_t is always greater than one, the factor D_t is always positive. The factor B_t , however, can have either sign. D_t is the part of the current wealth after paying interest that is not consumed today. The factor B_t is the difference of two elements: The first element, $\left(1 - \frac{1}{M_t}\right) (w_t - \alpha)$ is the part of current labor income above the minimum consumption which is not consumed in the current period. This is the factor that increases wealth. The second element, $\frac{1}{M_t} \sum_{s=t+1}^{\infty} \frac{p_s}{p_t} (w_s - \alpha)$, is the part of future earnings above α that is spent today to finance consumption. This factor decreases the current level of wealth.

Thus, we can already advance that the evolution of B_t governs the evolution of the wealth distribution in this economy. To analyze this evolution we first introduce the notion of inequality. We give the definition of Lorenz-dominance in terms of our notation.

Definition 1 *Let all agents be ordered according to their initial level of wealth. Let I be the number of types of individuals according to their level of wealth. $\frac{1}{I} X_t^i$ is the share of wealth held by group i . Then, the distribution of capital at period $t+1$ is more egalitarian than the distribution at period t if and only if it is satisfied that for $1 \leq J \leq I$*

$$\sum_{i=1}^J \frac{1}{I} X_{t+1}^i \geq \sum_{i=1}^J \frac{1}{I} X_t^i. \quad (13)$$

The following Proposition relates the level of inequality, measured using the concept of Lorenz-dominance with the aggregate dynamics of our model.

Proposition 2 *The distribution of capital at period $t + 1$ is more egalitarian than the distribution of wealth at period t if and only if B_t is non negative.*

This Proposition states that in order to know the evolution of the wealth distribution over time we just need to study the evolution of B_t . Therefore, we will be able to check whether the wealth inequality of this model economy exhibits an inverted U shape along the transition path just looking at the evolution of B_t . Nevertheless, the level of inequality also depends on initial conditions. Next Corollary states that the level of inequality at each period, measured with the Gini coefficient, is a function of the initial distribution.

Corollary 3 *The Gini coefficient at any period t depends on the initial distribution and the evolution of the aggregate variables,*

$$G_{t+1} = \prod_{r=0}^t \left(\frac{D_r k_r}{B_r + D_r k_r} \right) \cdot G_0.$$

Proof. It follows from the definition of the Gini coefficient, Proposition 2 and expression (12),

$$G_t = \frac{\sum_{j=1}^I \left(\sum_{i=1}^j \frac{1}{I} (1 - X_t^i) \right)}{\sum_{j=1}^I \sum_{i=1}^j \frac{1}{I}}.$$

■

This Corollary states that the evolution of the level of inequality is invariant with respect to the initial level of inequality.

3.3 Savings rates and the evolution of wealth distribution

We have characterized the evolution of the level of inequality as a function of the initial wealth distribution and the factor B_t . Another way of looking at the evolution of wealth

is analyzing the evolution of the savings rates. The rate of growth of individual assets holdings can be written as

$$\frac{k_{t+1}^i}{k_t^i} = \frac{1}{(1+g) \cdot (1+\eta)} \left[s_t^i \cdot \frac{y_t^i}{k_t^i} + (1-\delta) \right],$$

where s_t^i denotes individual i 's savings rate and y_t^i denotes the sum of labor earnings and return to capital. If $\frac{k_{t+1}^i}{k_t^i} > \frac{k_{t+1}^j}{k_t^j}$ and $k_t^i > k_t^j$ it must be that $s_t^i > s_t^j$, provided that $k_t^j > 0$. In other words, for the level of inequality to increase, wealthier individuals have to save a sufficiently large fraction of their income, compared to the savings rate of the poorer individuals in the society. Now, we can write the savings rate as

$$s_t^i = \left[\frac{(1+g)(1+\eta)B_t}{k_t^i} + ((1+g)(1+\eta)D_t - (1-\delta)) \right] \cdot \frac{k_t^i}{w_t + r_t k_t^i},$$

Notice that, clearly, if B_t is negative, the savings rate is higher the higher the level of wealth which implies a widening in the level of inequality. Thus, B_t determines the savings rates.

3.4 Comparative dynamics

Our framework allows comparisons of wealth dynamics across economies that differ with respect to the initial distribution of capital.

Proposition 4 *Consider two economies which are identical in all respects in period t except that $\{X_{1t}^i\}_{i=1}^I$ Lorenz-dominates $\{X_{2t}^i\}_{i=1}^I$. Then $\{X_{1\tau}^i\}_{i=1}^I$ Lorenz-dominates $\{X_{2\tau}^i\}_{i=1}^I$ for all $\tau > t$.*

Thus, initial differences in inequality will persist over time. This issue will be discussed again in section 4.

4 Quantitative implications on the evolution of inequality

Now we turn to analyze the quantitative predictions of the model about the level of inequality along the transition path. To do so, we calibrate our model economy so it matches some key features of the US economy. The linearity of the Engel curves will allow us to study this economy in two steps: We will analyze the evolution of prices and aggregate variables in the representative agent version of this model and we will turn afterwards to the full model to study the evolution of the wealth distribution.

4.1 Calibration issues

The capital share, θ is set equal to 0.4. This estimate is a bit higher than what appears elsewhere in the literature because it includes the imputed income from government capital. The rate of population growth, η , is 1 percent per year and the rate of growth of real per capita output, g , is 3 percent. The capital output ratio is chosen so the steady state value of the real interest rate, r_s , is 6.9 per annum, which corresponds to the annual average real return for the United States over the sample period 1960-1992. Given a broad definition of consumption and output, appropriate for this model, the steady state ratio output to consumption is 1.33. Furthermore, we set the initial level of technology A equal to 1. All these values are taken from Cooley and Prescott (1995).

We follow King and Rebelo (1993) to choose the initial per capita capital. We choose k_0 so that capital accumulation explains one half of the sevenfold per capita output growth observed over the period 1870-1970. Then, k_0 satisfies

$$\frac{F(k_s)}{F(k_0)} = \sqrt{7}. \quad (14)$$

We need to be careful so the initial level of capital per capita is greater than α . Otherwise, there will never exist growth.

Finally, we have left the parameters σ and α . Atkinson and Ogaki (1996) estimate that the value of the intertemporal elasticity of substitution for total consumption expenditures in US to be equal to 0.4 for the period 1968-1988. Thus we take 0.4 as the value of the intertemporal elasticity of substitution (IES, hereafter) in the steady state. In the steady state we have

$$IES = \frac{c_{ss} - \alpha}{\sigma \cdot c_{ss}} = 0.4. \quad (15)$$

Since we have assumed α to be non negative, the equality (15) imposes an upper bound to the possible values for σ : It has to be less than or equal to 2.5. We also believe reasonable to impose that α/c_{ss} should not be greater than 0.4. We have the notion that the minimum consumption α should be below 40 percent of the per capita consumption in the steady state. These considerations restrict the region of values of σ to $[1.5, 2.5]$. We try four different values for σ , 1.5, 2.0, 2.1, and 2.4. We study the evolution of the model economy for these four cases. We summarize the calibrated parameters in the following Tables,

Preferences		Technology			
β	η	A	θ	δ	g
0.997	0.01	1	0.4	0.0025	0.03

Table 1

σ	1.5	2.0	2.1	2.4
α	0.97	0.49	0.39	0.1
α/c_{ss}	0.40	0.20	0.16	0.04
α/c_0	0.92	0.53	0.44	0.12

Table 2

4.2 Predictions on aggregate dynamics

We analyze in this subsection the properties of the dynamics of this model. Figures 1a and 1b show the evolution of the aggregate variables for the different cases considered. Figure 1a shows the evolution of output, consumption, capital and investment as a fraction of their steady state value. The transition path is longest when $\sigma = 1.5$ since the value of α used implies the highest level of minimum consumption: 40 percent of the steady state consumption. Nevertheless, Table 3 shows that after 100 periods in all cases the value of output is above 92 percent of its steady state value. The main differences appear in Figure 1b, that shows the evolution of the savings rate, the real interest rate, the growth rate and the factor B_t .

σ	1.5	2	2.1	2.4
$\frac{y_{100}}{y_{ss}}$	0.922	0.946	0.947	0.952

Table 3: GDP after 100 periods

Notice that at the beginning of the transition, the growth rate of output is highest for $\sigma = 2.4$ and lowest for $\sigma = 1.5$. This is so because the minimum consumption requirement as a percentage of average consumption is much higher in the economy with $\sigma = 1.5$, which generates a much lower IES along the transition. (See Tables 2a, 3a, 4a). The economy with the highest saving rates is the one in which $\sigma = 2.4$ and the lowest when $\sigma = 1.5$. The other two cases lie in between. Williamson (1991) reports that the gross savings rate was 23 percent in 1870 and rose to 28 percent at the turn of the century. Thus, the cases in which $\sigma = 2$, and $\sigma = 2.1$ do not perform too badly in this account. He also reports that the return to conventional reproducible assets was 6.6 percent at the turn of the century. The evolution of the real interest rate is very similar in the four cases considered. It starts around 30 percent and drops to 10 percent after 30 periods.

Thus, all the cases do not differ much in the aggregate evolution of all variables, but in one: The evolution of the factor B_t . Figure 2b shows that if $\sigma = 1.5$ the factor B_t is always negative and approaches zero as the economy gets closer to the steady state. This implies that in this economy the distribution of wealth becomes more unequal along the transition path. For $\sigma = 2.4$ the factor is always positive, therefore, inequality always decreases. For $\sigma = 2$ B_t is very close to zero. In this case, inequality is invariant with respect to growth. When $\sigma = 2.1$ we get that B_t first increases and afterwards decreases. In this case the evolution of the wealth distribution has an inverted U shape. Thus, the

model is able to generate a Kuznets curve. The next point is how much variation in inequality this model generates along the transition path.

4.3 The evolution of inequality

In this subsection we analyze the size of the variation in inequality generated in the transition path and confront our results to the —some authors would say scant— evidence we have about the evolution of the US wealth distribution. To do this, we divide the individuals in ten groups and we fix an initial distribution of wealth across households. The evolution of consumption, investment and capital across households are obtained using expressions (16), (8) and the constraint of the individual problem (7). The Gini coefficient is the index used to measure the inequality of wealth and income.

The initial distribution is chosen to match the data of wealth distribution for USA in 1870. The available information on the distribution at that time is contained in Table 4. The Gini coefficient of wealth was 83.3 percent, the share of wealth held by the top 10 percent was 70 percent of the total amount of wealth and the share of the top 1 percent was 27 percent. For the rest of the deciles, there is no available information. Since we have divided the households in deciles we ignore the information about the share of the wealthiest 1 percent of the population. Many distributions of capital match the remaining two statistics. We have conducted a number of experiments with different initial distributions, and the results we are about to present are not changed in a substantial way. Thus, we consider as the initial distribution that shown in Table 5.

Year	Share top 1%	Share top 10%	Gini Coefficient
1774	13.2%	55.1%	0.64
1870	27.0	70.0	0.83
1962	15.1	35.7	n.a.
1992	29.5	n.a.	0.78

Table 4: Wealth Inequality in the US, 1774-1992

Source: Williamson (1991), pg. 18 and last row from Diaz-Giménez *et al.* (1997).

1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
0.10	0.53	0.59	0.90	1.01	3.17	5.40	8.09	10.1	70.09

Table 5: Initial distribution of wealth across deciles (in percent).

The first picture in Figures 2a, 3a, and 4a show the evolution of the Gini coefficient of wealth for $\sigma = 2.1$, $\sigma = 1.5$, and $\sigma = 2.4$. As we can see, only in the first case the Gini coefficient has an inverted U shape. The case $\sigma = 1.5$ is simulated using a much lower initial level of inequality; otherwise the majority of the population's income would fall below α . The case in which $\sigma = 2$ shows no variation in the level of inequality and it is not shown in the Figures. Thus, we think of the case in which $\sigma = 2.1$ as the one that

more closely resembles the evolution of the level of inequality in the US economy and we will refer to this case hereafter. Notice also the evolution of the aggregate intertemporal elasticity of substitution in each case (see Figures 2a, 3a, 4a). Atkeson and Ogaki (1996) estimate that the IES increased from 0.38 in 1929 to 0.40 in 1970. Notice that after 60 periods our model predicts a level of IES around that value 0.38.

Inequality across agents. Figure 2b shows the evolution of the shares of capital for 3rd, 5th, 7th, and 10th deciles, respectively. The evolution of the shares mirrors that of the Gini coefficient of wealth: Until period 13 the shares of 3rd, 5th, and 7th decrease, and that of the top decile increases. After that period, the behavior is reversed. Figure 2c shows the savings rates for the aforementioned deciles. The first thing that calls our attention is the enormous difference of savings rates across deciles: The households in the 10th decile save in the steady state more than 50 percent of their income, whereas the saving rate of those in the 7th decile is around 16.5 percent. Most of the aggregate investment is comprised by the savings of the top decile: 68.42 percent of the investment in the steady state is due to savings of the wealthiest individuals, being this fraction 74.57 percent at the peak of the wealth inequality.

The evolution of wealth and income inequality. The Gini coefficient of wealth is 83 percent in the first period, reaches 89 percent after 13 periods, which amounts to a 7.41 percent cumulative increase, and decreases to be 85.7, a 5.13 percent cumulative decrease. The Gini coefficient of income is 33.3 percent in the first period, reaches 36 percent after 17 periods, a 8 percent increase, and decreases to be 34.5, a 4.63 percent decrease. The relative size of the variation in the Gini coefficients is independent on the initial level of inequality, as it is stated in the Corollary shown in the previous subsection. The experiment also shows that after 60 periods there is little variation in the level of inequality. This accounts for the remarkable stability of wealth inequality in the post-war US economy.

Differences in wealth and income concentration. Díaz-Giménez *et al.* (1997) report that in 1992 the Gini coefficient for income was 0.57 and that for wealth 0.78. Atkinson (1997) finds that the Gini coefficient for income in 1970 was around 0.40. This model, therefore, underestimates the level of income inequality and overestimates that of wealth inequality. Two features of the model may account for this: Agents are identical in their labor earnings and they do not value leisure. The first assumption implies that the associated income distribution is always going to be more egalitarian in our artificial economy than the US economy. The second assumption implies that labor supply does not vary with wealth. If labor supply were endogenous, wealthier agents would supply fewer hours in the market than the wealth poorer agents and, consequently, wealth inequality would be lower. Thus, our model points out that in order to account for the concentration of wealth relative to the concentration of income, the wealth effect on the labor supply should be small.

Confronting the facts. Kuznets (1955) conjectured that income inequality in US increased particularly after 1870 and started declining with the first world war. He argues that industrialization and the intersectoral migration of the labor force are the key factors to understand the evolution of income inequality. More forcefully, Williamson (1995) reports that inequality in the US economy kept rising until 1929, and started decreasing afterwards. Same thing occurred to wealth inequality. He argues that rapid capital accumulation in the XIX century, migration and a widening skill premium are the responsible for the surge in inequality until 1929. In our model the only source of inequality is capital accumulation and the simulation shows that wealth inequality decreases after 13 periods. Since we have set the initial level of capital so the model explains one-half of the growth in the period 1870-1970, our model implies that inequality should have started declining before the end of the XIX century.

4.4 Discussion of the results

In this subsection we want to discuss the key features of the model that deliver the result on the evolution of the level of inequality. We all know that the evolution of the savings rates govern the evolution of the wealth distribution. The savings rates, in its turn, depend on the intertemporal elasticity of substitution (IES). In this model, the IES varies with the level of wealth because of the existence of a minimum consumption requirement. How much the IES varies along the transition path depends on the region of the parameter space we are.

Let us think of the case in which $\sigma = 2.1$. Poorer individuals have a lower IES than the wealthier individuals do (see Figure 2a). At low levels of income the level of minimum consumption is a larger fraction of the income of poorer individuals; therefore, wealthier individuals have larger savings rates. For wealthier individuals the IES is close to $\frac{1}{\sigma}$, a sufficiently high IES so they save a large fraction of their income that decreases over time. This implies a high growth in the economy. High growth implies that the minimum consumption as a fraction of the income of poorer individuals decreases, which implies an increase in the poorer individuals' savings rates. Thus, in this economy the savings rates of poorer individuals grow over time, whereas those of the wealthier individuals decrease over time. It is only after some periods that inequality starts decreasing, however. The reason is that only after the savings rates of poorer individuals are sufficiently high the rate of growth of the capital shares of the poorer individuals are higher than those of the wealthier individuals.

Let us turn to the case in which $\sigma = 1.5$. Here, the minimum consumption is a much larger fraction of the income of individuals. Thus, individuals have much lower savings rates, which implies lower growth. The higher the level of income the lower the minimum consumption, which allows rich individuals to increase their savings rates. At the same time poor individuals are running down their assets to finance consumption. Therefore, inequality increases forever.

The opposite occurs when $\sigma = 2.4$. In this case the minimum consumption relative to the level of income is small and the model predicts an ever decreasing inequality.

5 Is the Kuznets curve a feature of development?

Deininger and Squire (1998) use a very comprehensive panel data set and conclude that, of the sample of countries they have, in less than the 25% of them inequality does have the inverted U shape. Therefore, they bring into question whether the Kuznets curve is a feature of the development process. Clearly, our model shows that the Kuznets curve is not an inherent feature of growing countries. Initial conditions plus the individuals' intertemporal elasticity of substitution are key factors to determine whether the evolution of inequality will show the inverted U shape or not. Next, we review two of the questions analyzed in the literature.

The Kuznets curve and cross-country data. In the last years many authors have been debating whether the Kuznets curve is a feature of development. Due to the lack of time series data, some authors have relied on cross-country data. The underlying assumption is that poor countries should look very much like current rich countries a century ago, and rich countries are a good prediction of the future well-being of currently poor countries. Then, using cross-section data should be very informative about the evolution of inequality for a country. Many authors have then rejected the Kuznets hypothesis as a feature of development using cross-country data.²

We argue that observing a Kuznets curve for a country does not say anything about the pattern of inequality across countries. Therefore, it is not possible to infer the existence or the nonexistence of the Kuznets curve for a given country testing whether that curve exists across countries.

To illustrate our argument, we have conducted the following experiment: We have used our benchmark economy and generated a sample of 100 countries. In this sample, country $i + 1$ at period t has the per capita income of country i at period $t + 1$. We run four different simulations. Figure 5 shows the result of our experiment. In the first simulation the initial cross-section distribution of wealth is such that we observe a cross-country Kuznets curve, which persists after 20 periods. In the second simulation all countries have the same wealth distribution at period zero. After 20 periods, poorer countries have become more unequal, whereas the rest have experienced a decrease in their level of inequality. Thus, we conclude that using cross-country observations to test a hypothesis which was formulated for time series data is misleading, to say the least.

Can this model explain cross-country differences in wealth inequality? One lesson we extract from the previous experiment is that the initial level of inequality is very persistent in a country. Thus, any policy oriented to redistribute wealth would have persistent effects in this economy. This is to say, countries that start their process of sustained growth with high levels of inequality will have high inequality forever (recall Proposition 4 in Subsection 2.3). Thus, we conclude that this model does not offer explanations for cross-country differences.

²See Deininger and Squire (1998) for a review of those authors that have tested the Kuznets curve using cross-country data.

The model predicts that poor countries that start the process of sustained growth will experience a widening in their levels of inequality and a posterior narrowing of them. This is not to say that inequality levels are comparable across countries. Kuznets seems to think this in his 1955 paper. He argues that we can accept that underdeveloped countries have higher inequality than developed countries but that it is true that many of them have not started a process of industrialization and urbanization yet at that time. Therefore, the differences in inequality cannot be due to the development process (see page 23, last paragraph).

Our model says that almost all the observed cross-country wealth inequality cannot be explained by economic forces. It is true that our model is highly stylized and only has one source of inequality, but a look at the next table suggests that the cross-country wealth inequality differences are remarkably constant over time.³

Regions	1960s	1970s	1980s	1990s
Eastern Europe	22.76	21.77	24.93	28.60
South Asia	31.67	32.32	32.22	31.59
OECD and high income	32.86	33.04	32.20	33.20
East Asia and Pacific	34.57	34.40	34.42	34.80
Middle East and North Africa	41.88	43.63	40.80	39.72
Sub-Saharan Africa	49.90	48.50	39.63	42.30
Latin American	53.00	49.86	51.00	50.00

Table 6: Decadal medians of Gini coeff. of income by regions

Source: Deininger and Squire (1996).

6 Final comments

This paper has shown that a modified version of the neoclassical growth model can match the observed evolution of wealth inequality in the US economy. The only source of inequality in this model is capital accumulation. The model predicts that the level of wealth inequality first increases and decreases afterwards, remaining constant in the steady state. In other words, the evolution of wealth inequality shows a Kuznets curve. Nevertheless, we argue that the Kuznets curve is not a feature of the development process and that cross-country data is not suited to investigate this inherently time series phenomenon.

We have made two key assumptions: There are perfect capital markets and individuals do not value leisure. If there were any borrowing restriction inequality along the transition path would be lower and the level of growth would be higher. We think that perfect capital markets is not a bad assumption. Poor individuals always have available informal markets or the family network to borrow or to get insurance (see Townsend 1994, Rosenzweig and Stark 1989). As for the second assumption, our guess is that allowing for leisure decisions would result in lower levels of wealth inequality. Nevertheless, exogenous labor supply

³The table shows data on income inequality. Since wealth depends on income, if we do not observe any change in income inequality we can hardly expect any change in wealth inequality.

buys us the difference between wealth and income concentration. This suggests that we should think more deeply about the determinants of the individual's labor supply.

This model delivers some implications on convergence. If we thought of the households in the model as different countries in a world without barriers to capital flows this model would predict that countries do not converge to the same level of per capita gross national product. Thus, restrictions to capital flows would help to narrow the gap between poor and rich countries. Chatterjee (1994) obtains the same result.

In the last two decades many developed countries have experienced an upsurge in their levels of income inequality (see Atkinson 1997, Gottschalk and Smeeding 1997). These observations, more than any study, refute the existence of a Kuznets curve, in the sense that developed countries may see, as any other poorer country, a rise in inequality. To account for that increase, however, we should develop a theory of earnings distribution: The rise in the skill premium is the most likely responsible of the increase in inequality. Steps in this direction will help us to understand better the evolution of the income and wealth distribution.

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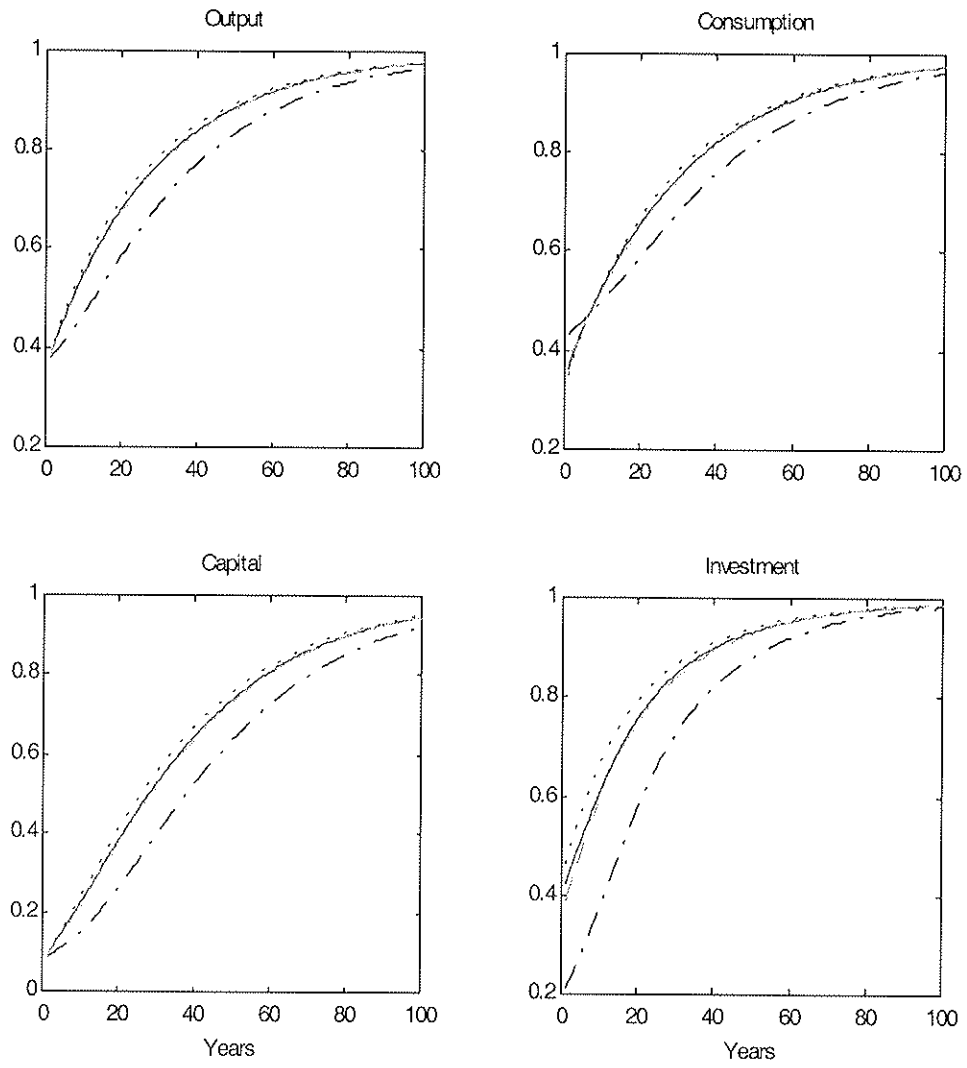


Figure 1a. Transitional Dynamics

Key: Solid line, $\sigma = 2.1$; dashed dotted line, $\sigma = 1.5$; dotted line, $\sigma = 2.4$ and dashed line, $\sigma = 2$.

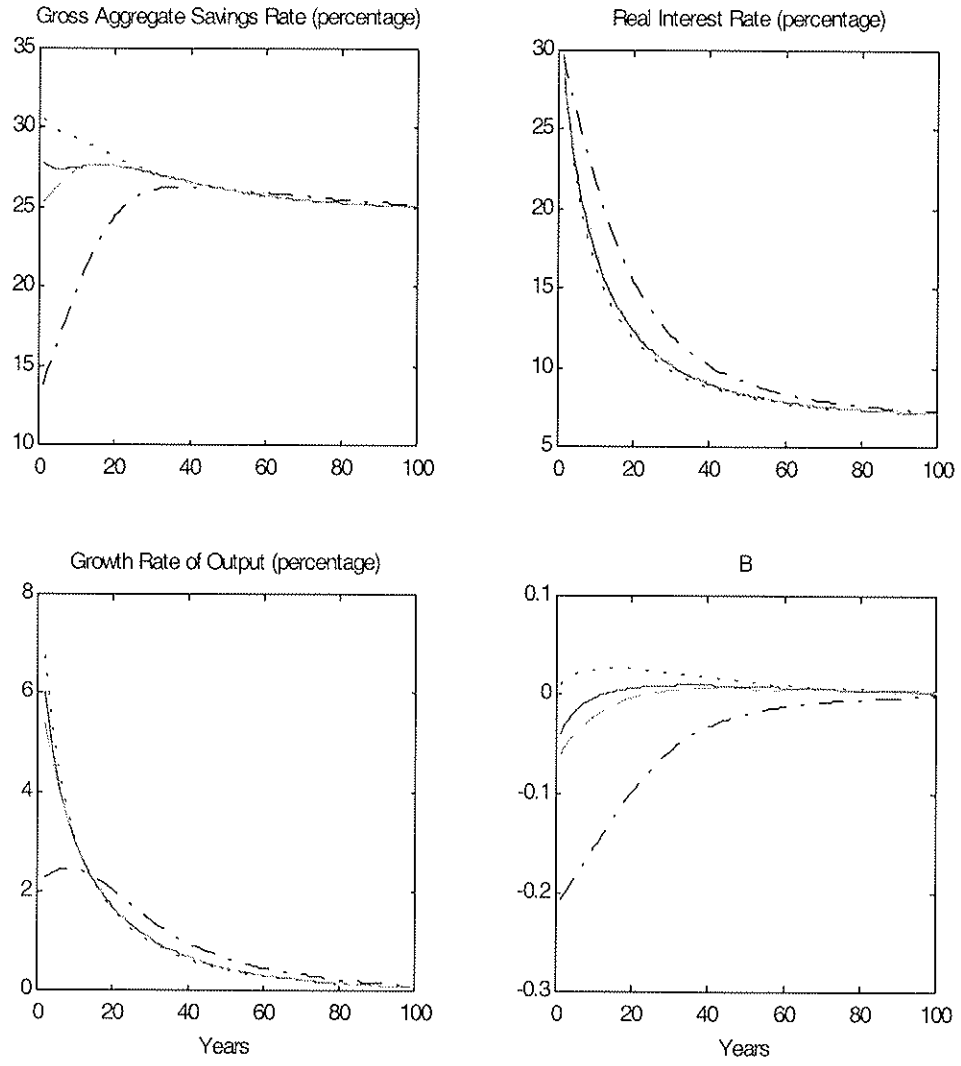


Figure 1b. Transitional Dynamics

Key: Solid line, $\sigma = 2.1$; dashed dotted line, $\sigma = 1.5$; dotted line, $\sigma = 2.4$ and dashed line, $\sigma = 2$.

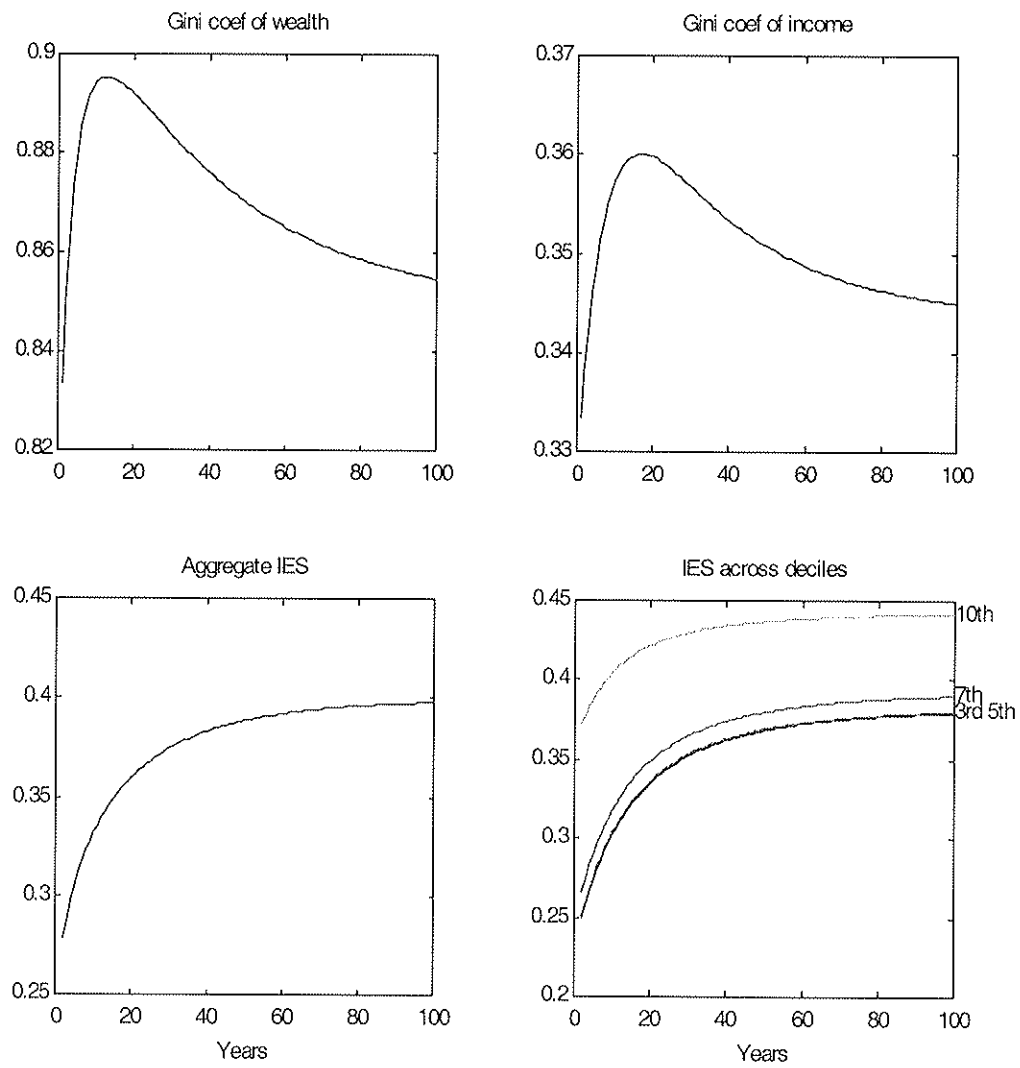


Figure 2a. Gini coefficient of wealth, income and intertemporal elasticity of substitution

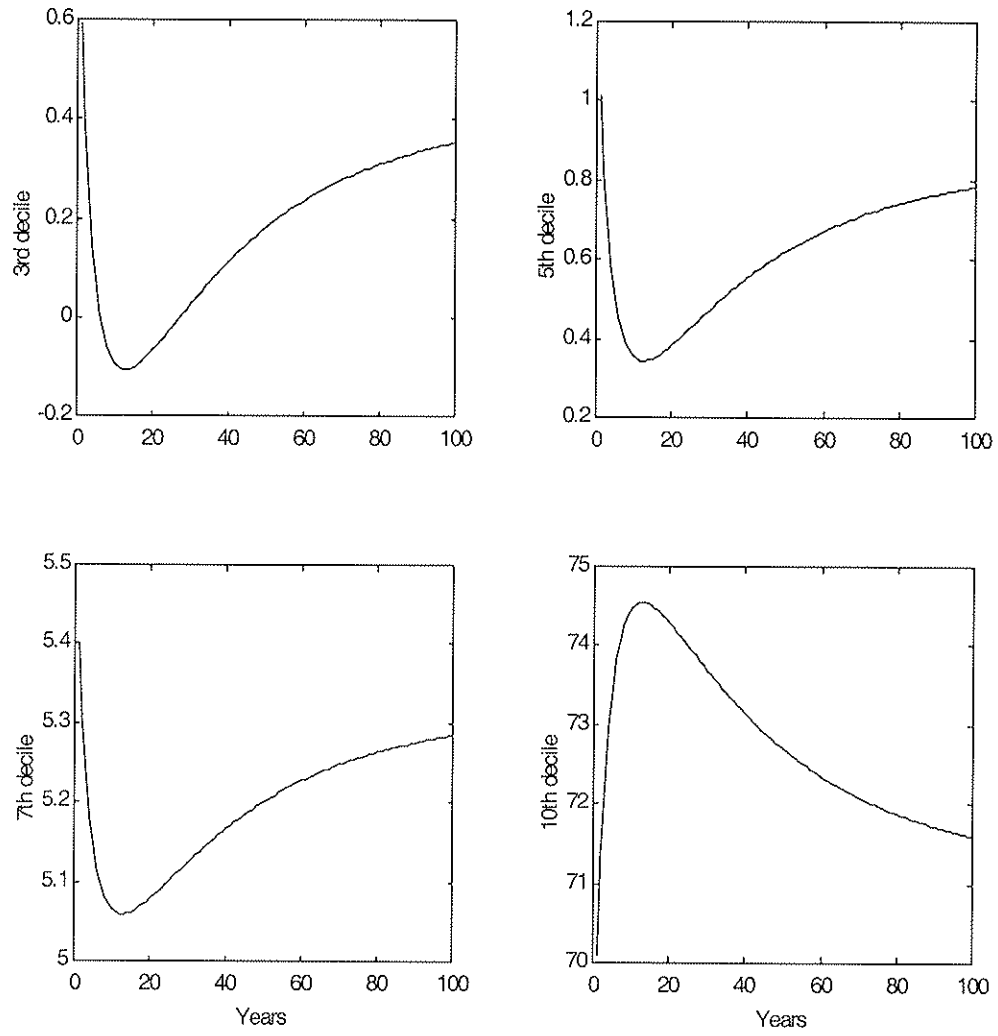


Figure 2b. Shares of wealth across deciles

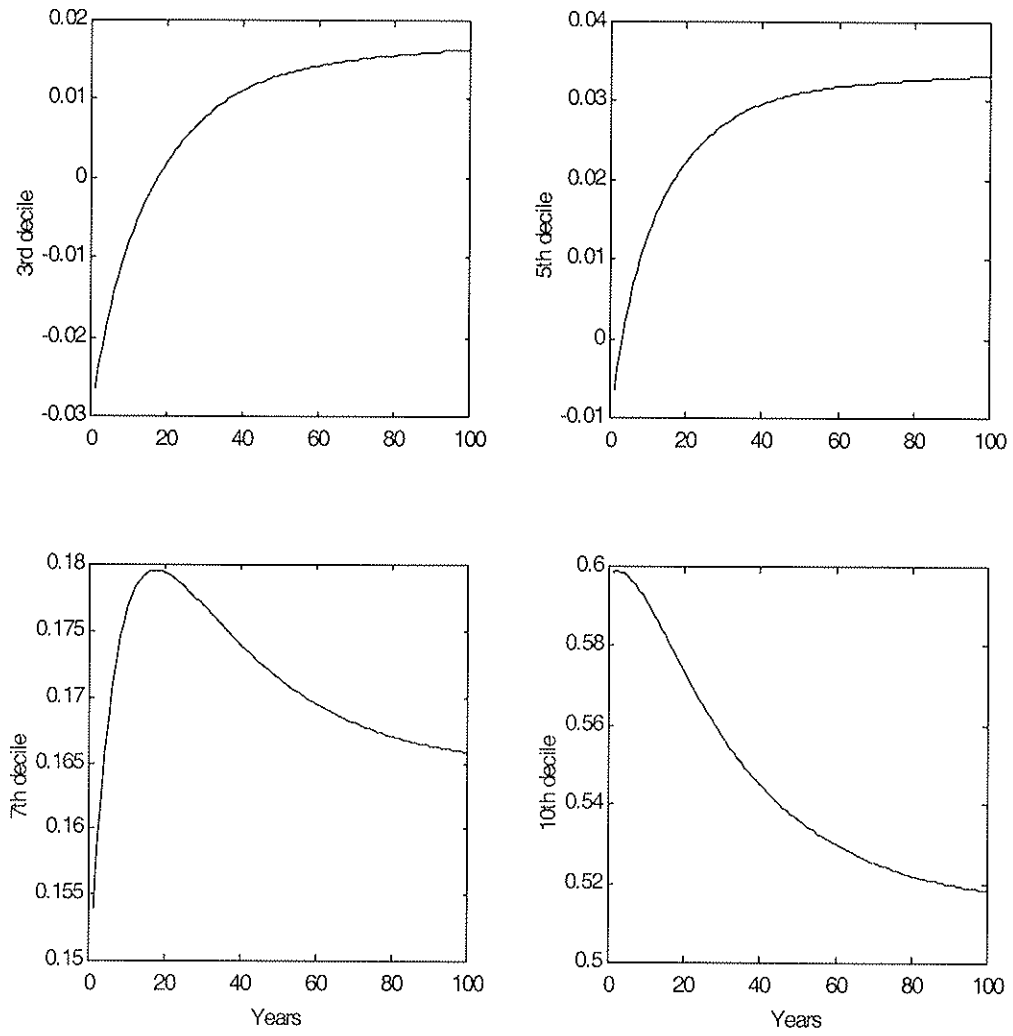


Figure 2c. Savings rates across deciles

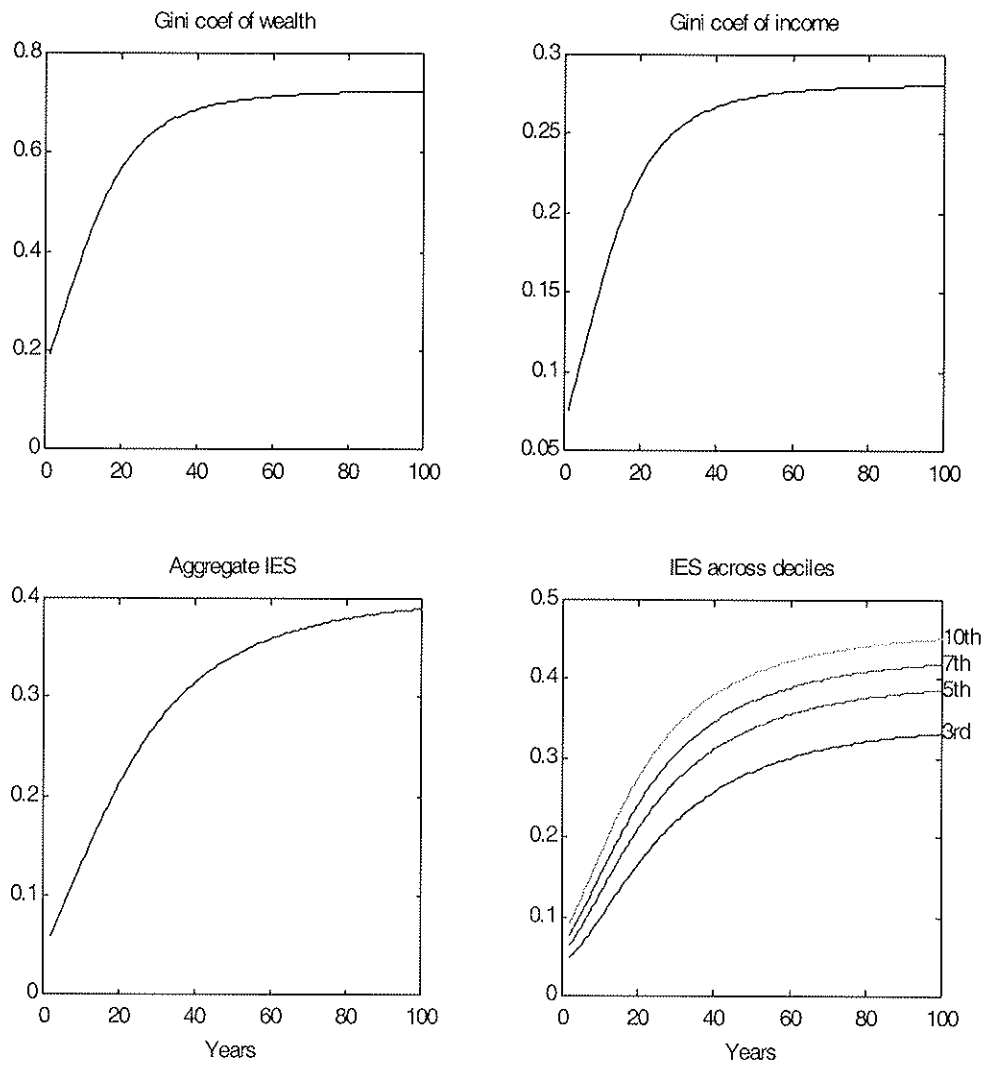


Figure3a. Gini coefficients of wealth, income and intertemporal elasticity of substitution.

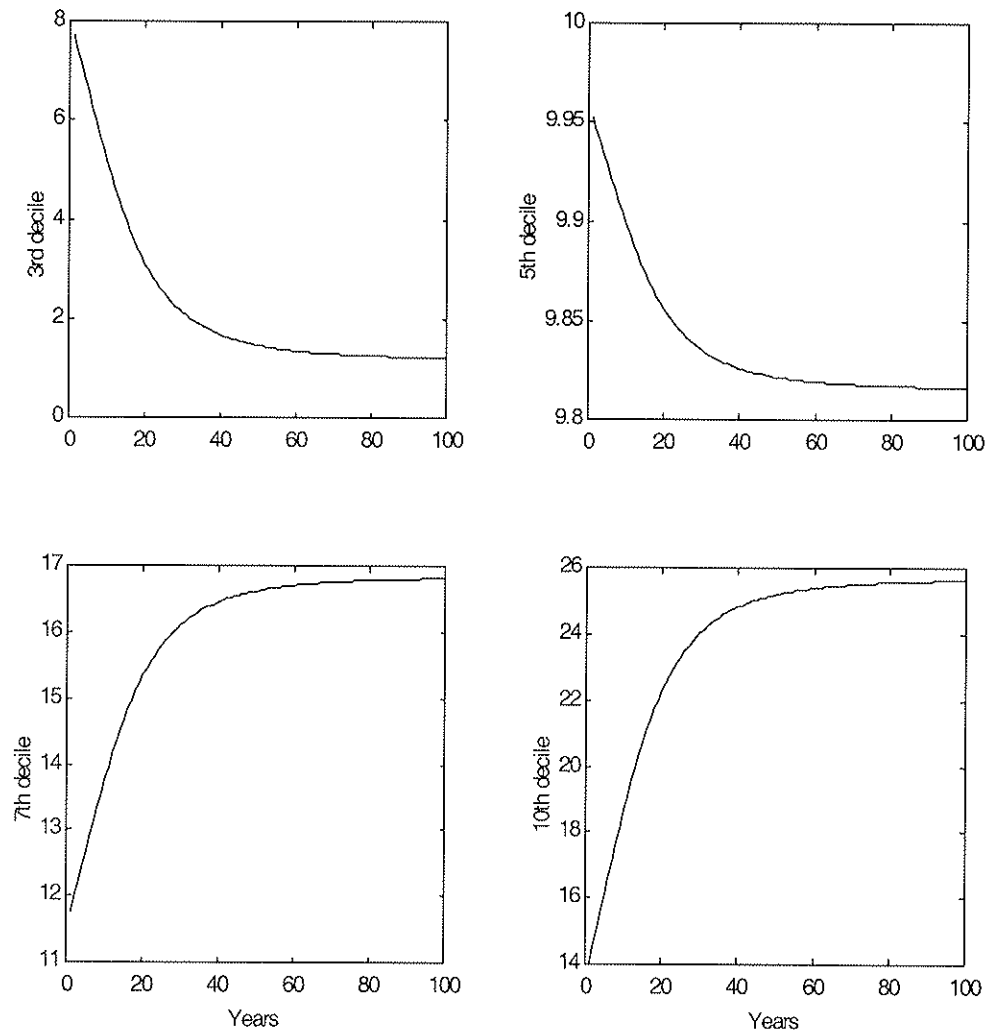


Figure 3b. Shares of wealth across deciles

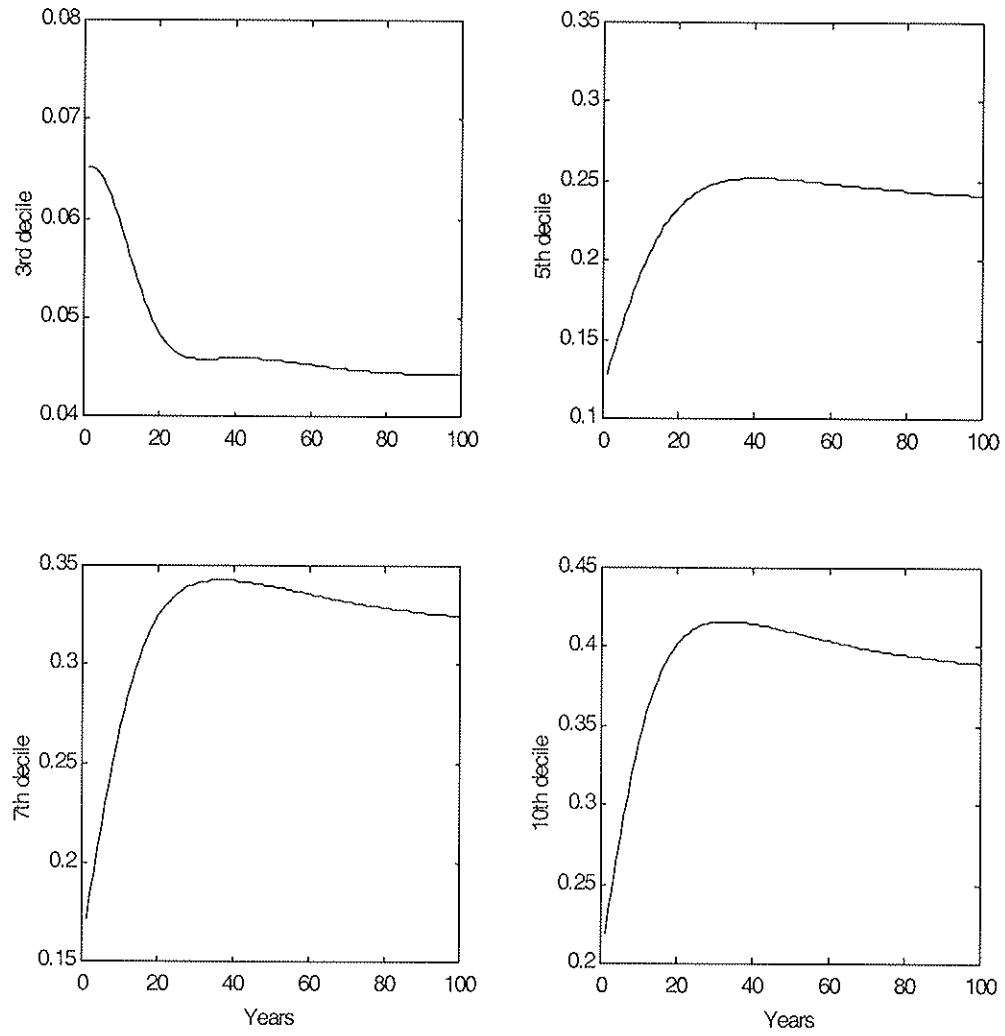


Figure 3c. Savings rates across deciles

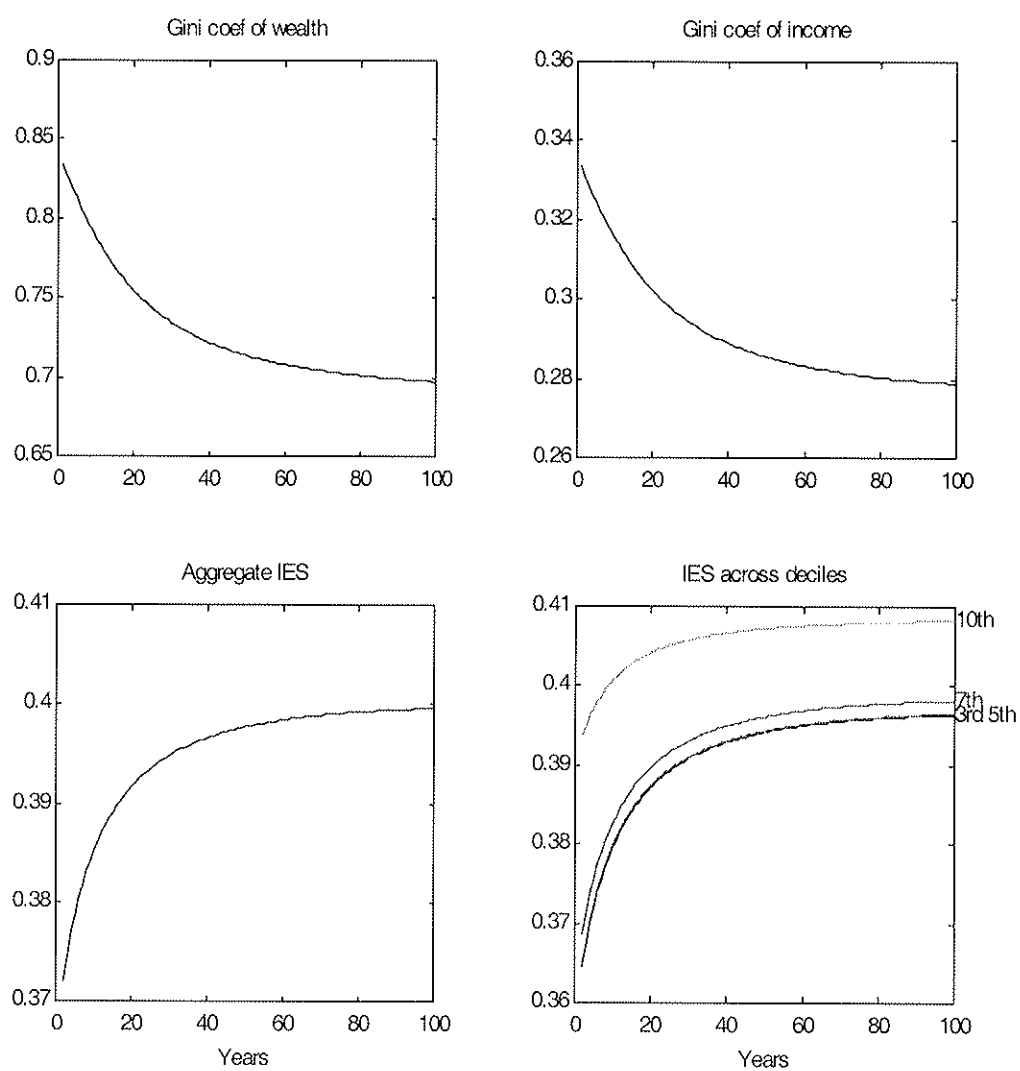


Figure 4a. Gini coefficient for wealth, income and intertemporal elasticity of substitution

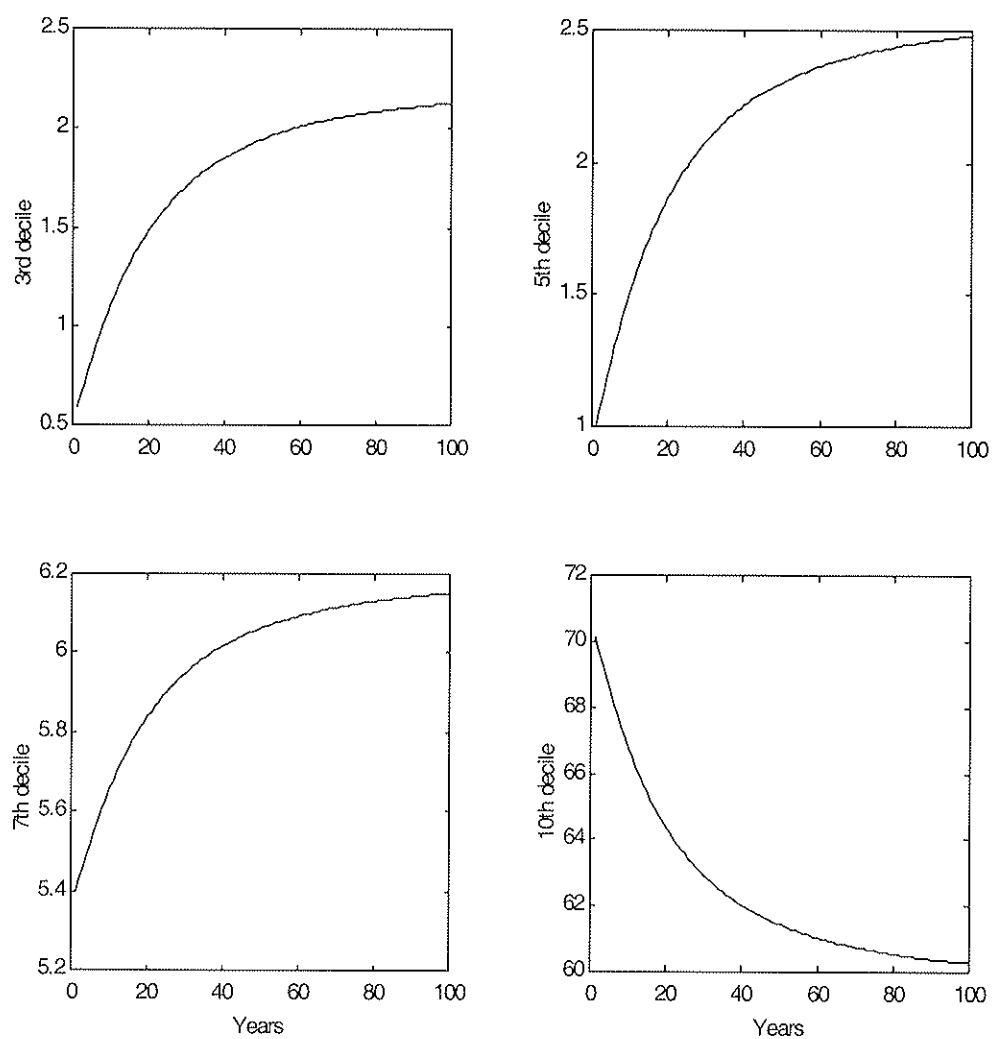


Figure 4b. Shares of wealth across deciles

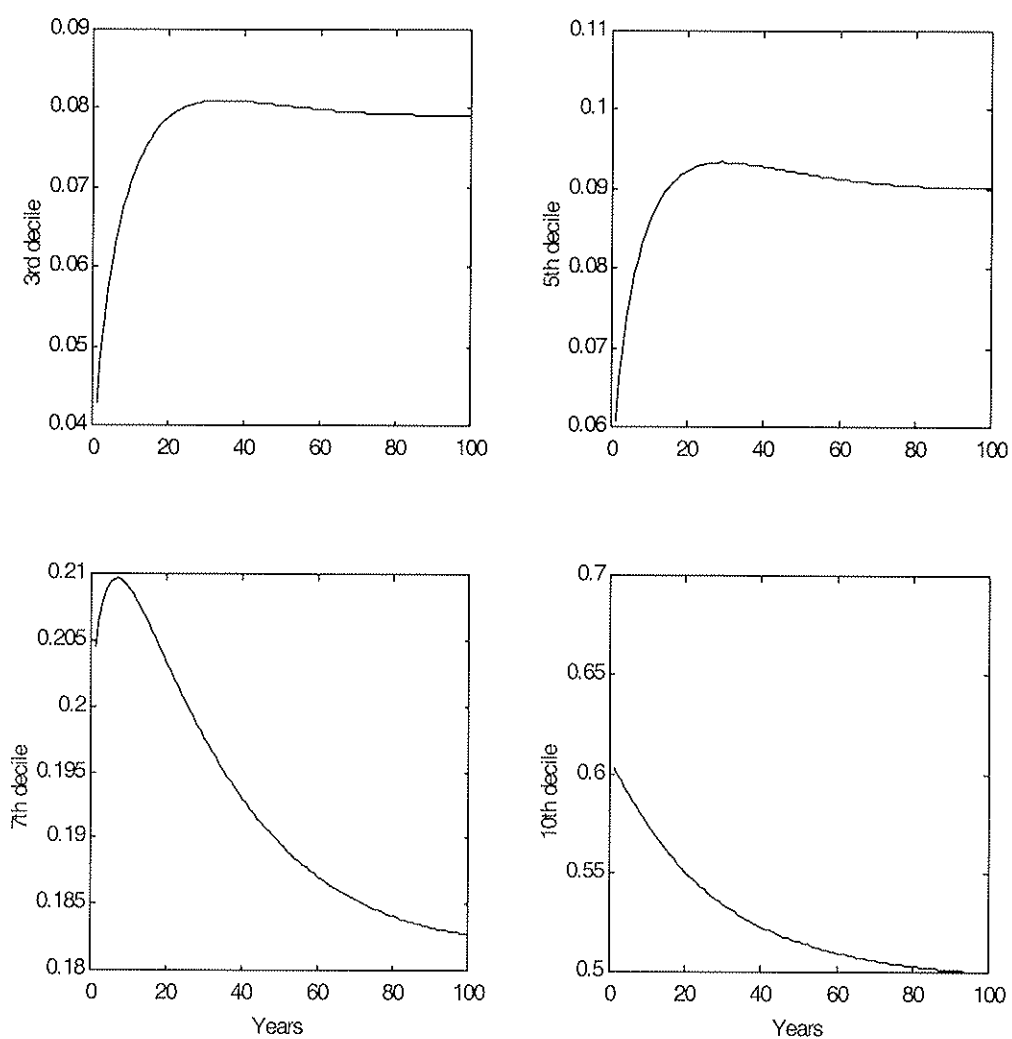


Figure 4c. Savings rates across deciles

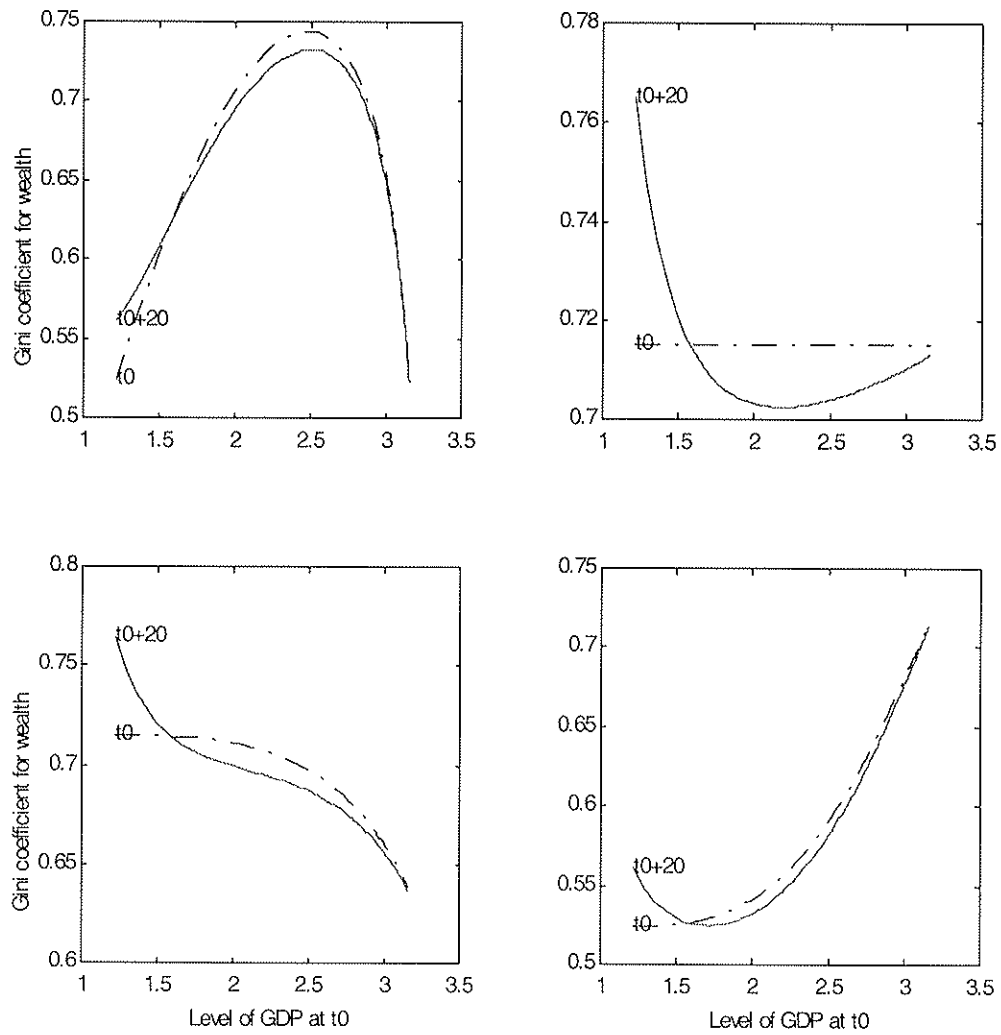


Figure 5. Wealth inequality across countries

A The transtion path and the distribution of wealth

A.1 The aggregate dynamics of the model

Proposition 1. Proof. Solving the consumer's problem (7), we have the consumer demand function

$$c_t^i = \alpha + \frac{1}{M_t} \left(\sum_{s=t}^{\infty} \frac{p_s}{p_t} (w_s - \alpha) + (1 + r_t - \delta) k_t^i \right), \quad (16)$$

$$M_t = \sum_{s=t}^{\infty} \phi^{\frac{s-t}{\sigma}} \left(\frac{p_s}{p_t} \right)^{\frac{\sigma-1}{\sigma}}. \quad (17)$$

where p_t is the price of consumption good in period t in terms of consumption good in period 0. Linearity of the Engel curves ensure that the aggregate stock of capital next period does not depend on the distribution of wealth. ■

A.2 The evolution of the wealth distribution

Proposition 2. Proof. we can write expression (13) as

$$\sum_{i=1}^J \frac{1}{I} (1 - X_{t+1}^i) \leq \sum_{i=1}^J \frac{1}{I} (1 - X_t^i), \quad (18)$$

then, substituting (12) in (18) we obtain

$$\frac{D_t \cdot k_t}{B_t + D_t \cdot k_t} \cdot \sum_{i=1}^J \frac{1}{I} (1 - X_t^i) \leq \sum_{i=1}^J \frac{1}{I} (1 - X_t^i),$$

which holds true for B_t non negative. Recall that $\sum_{i=1}^J \frac{1}{I} X_t^i$ should be less than or equal to $\frac{J}{I}$, which is the fraction of aggregate wealth held by groups 1 to J assuming capital is evenly distributed across agents. ■

A.3 Comparative dynamics

Proposition 4. Proof. Let us assume that for some $\tau > t$ $\{X_{1\tau}^i\}_{i=1}^I$ does not Lorenz-dominate $\{X_{2\tau}^i\}_{i=1}^I$. It implies that for some J ,

$$\sum_{i=1}^J \frac{1}{I} X_{1\tau}^i < \sum_{i=1}^J \frac{1}{I} X_{2\tau}^i.$$

This expression can be written as

$$\sum_{i=1}^J \frac{1}{I} (1 - X_{1\tau}^i) > \sum_{i=1}^J \frac{1}{I} (1 - X_{2\tau}^i)$$

Now, proposition 1 ensures that the aggregate dynamics of both economies are identical. Using expression (12) we can write the previous inequality as

$$\prod_{r=l}^{\tau-1} \left(\frac{D_r k_r}{B_r + D_r k_r} \right) \cdot \sum_{i=1}^J \frac{1}{I} (1 - X_{1t}^i) > \prod_{r=l}^{\tau-1} \left(\frac{D_r k_r}{B_r + D_r k_r} \right) \cdot \sum_{i=1}^J \frac{1}{I} (1 - X_{2t}^i).$$

Since the factor $\prod_{r=l}^{\tau-1} \left(\frac{D_r k_r}{B_r + D_r k_r} \right)$ is positive, it follows that

$$\sum_{i=1}^J \frac{1}{I} X_{1t}^i < \sum_{i=1}^J \frac{1}{I} X_{2t}^i,$$

which contradicts that $\{X_{1t}^i\}_{i=1}^I$ Lorenz-dominates $\{X_{2t}^i\}_{i=1}^I$. Therefore, the proposition follows. ■

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