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Competitive Markets**

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A Closer Look at the Comparative Statics in Competitive Markets*

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Abstract

In this paper we revisit the dual approach to comparative statics in competitive markets, allowing for the essential results to arise from a comprehensive and unified framework. We study, for both the long-run and the short-run, the response of all the endogenous variables to price factor changes in a way that captures the output-price effects arising from market-firm interactions. We show that it is necessary a richer characterization of the nature of factors with respect to output, connected with marginal cost and output demand elasticities, for completely determining such responses.

Keywords: Competitive Firm, Industry, Comparative Statics, Elasticity, Factor Demand.

JEL classification: D21, D41.

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1 Introduction

Most academic economists would accept that the perfect competition model was already consistently established in the works of Alfred Marshall and Frank Knight. However, the bulk of rigorous comparative statics analyses were developed more recently, and they consisted in very partial approaches to specific topics like the influence of factor price changes on different endogenous variables of the model, or the discussion about the slope of the competitive factor demand for the individual firm and, particularly, for the industry as a whole. Consequently, it does not exist a unified and comprehensive study of comparative statics allowing for the comparison of results.

Following Samuelson's (1947) pioneering contribution, the review of firms behavior in competitive markets was mainly developed taking output-prices as parameters determined by the interaction of individual agents' decisions in the market. However, this lead to disregard the feedback between the market and firms' outcomes by ignoring the induced effects of changes in the industry output-price on the decision variables at the firm level. This feature of the older studies is at the origin of the more recent controversy about the exact comparative effects on the endogenous variables caused by parameter changes.

In the context of the above debate, a second issue focused attention and enlarged the discussion because the comparative statics analysis of a competitive industry may be based upon two different views of the industry composition. The first one is, by far, the most usual in the related literature, but it is narrow because all firms in the industry are assumed identical in the sense of having an equal well-defined minimum average cost for the same strictly positive optimum size of the firm.¹ In this case, it is allowed to use the artifact of a representative firm for analysis.² Most of the results derived under this assumption, particularly such corresponding to the long-run, were already known at an early date as the middle seventies after the seminal works of Ferguson and Saving (1969)

¹Silberberg (1974) shows that a positive definite matrix of cross-partials of the average cost is a sufficient condition for a local minimum.

²Notice that this approach leads to a long-run equilibrium in which all firms in the industry are marginal.

and Silberberg (1974). However, they are based on a primal approach, building up on the properties of the production function, and some of the results were obtained under too restrictive technological conditions. Some years later, Heiner (1982) came to characterize the short-run factor demand for the entire competitive industry, showing that the law of demand is satisfied and extending this result to the long-run. He proceed by summing up individual demands which had been obtained taking into account the output-price adjustment, i.e. factor demands for firms which do not act in isolation from each other.³ The second view of the industry composition is wide and more businesslike, allowing for heterogeneous firms and hence for the existence of firms which have the same production function except for a scale factor or, even, for firms with different technologies.

The more recent works about comparative statics in competitive markets have been in fact oriented to provide the model with more realistic assumptions like multiproduct industries, less than perfectly elastic supplies and demands, and heterogeneous firms. In particular, Braulke (1984) generalized the above-mentioned analysis by Heiner to the case in which the competitive industry faces a less than infinitely elastic factor supply in some input markets.⁴ Moreover, Braulke (1987) extended the previous analyses allowing for the existence of non-identical firms.⁵ However, their results only apply to the industry at the aggregate level, showing the industry's total response to a price change without inspecting individual behaviors. The strongest difficulty associated with the heterogeneity assumption is that it makes hard to understand the true consequences of the entry and exit

³The downward slope of the industry factor demand can also be established according to Bassett and Borchering (1970b), which uses cost-minimizing input vectors, and Silberberg (1990), which uses the input vectors that maximizes profits, by making use of simple algebra without recourse to calculus. However, these two exercises are not satisfactory because they consider the output-price as an exogenous parameter.

⁴Braulke and Paech (2001) even generalizes to the case in which all supply and demand functions for the industry are less than perfectly elastic.

⁵Silberberg (1974) and Braulke (1987) show the difficulties associated with this approach, which may lead to the existence of intramarginal firms in the long-run equilibrium. Even though Silberberg seems to be quite a lot pessimistic on this point, Braulke takes a new methodological path without recourse to differentiability assumptions, which allows him to overcome such difficulties.

process, harming the predictability of the competitive industry behavior at the individual level. Consequently, these results represent an interesting extension, though of limited scope, of the pre-existing ones.

Our study relies on the standard theory of the neoclassical competitive firm, which produces and sells a single output at the industry output-price facing a vector of input prices. Each individual firm in the industry sells output to, and buys factors services from, competitive markets facing perfectly elastic output demand and factors supply respectively. The industry, assumed as composed of identical firms, operates as a whole under the usual conditions of a non-increasing market demand and a non-decreasing aggregate supply. To avoid pecuniary externalities it is also assumed that every factor supply to the industry is perfectly elastic, i.e. the whole industry is small with respect to the size of each input market. Technological externalities are removed from the analysis by assumption.

In this paper we shall adopt a partial equilibrium approach in the sense that a single market is considered without taking into account the strong economic effects on this market from interactions with other markets. The partial equilibrium behavior of competitive firms when output-price is treated as exogenous is well established in the literature, which has mainly developed comparative statics results for the isolated competitive firm.⁶ In order to overcome this limitation our analysis will take into account, explicit and systematically, the interaction between the market and the firm by allowing the industry output-price to change for market clearing and, then, studying the reaction in the decision variables at the firm level.

We propose here a comprehensive and self-consistent analysis of the comparative statics with a double perspective that, on the one hand, distinguishes between long-run and short-run and, on the other, clearly differentiates the output supply side from the factor demand side. Our paper revises the response of individual firms and the industry as a whole to an autonomous input price change, studying the effects of this change on the

⁶See into the standard microeconomic textbooks, as for example Silberberg (1990) and Gravelle and Rees (1992).

endogenous variables of the model which also include the firm and the industry derived factor demands. In section 2 we study the long-run comparative statics while section 3 is devoted to the short-run analysis. These exercises have been performed on the basis of the standard dual approach developed for profit maximizing and constrained cost minimization models.⁷ Consequently, our results are local and hold in the neighborhood of the equilibrium point. They are summarized making use of some fundamental and very common elasticity concepts and input shares.

2 Long-run analysis

We start by considering the study of comparative statics in the long-run, a period of time large enough for all inputs to be considered fully variable, and for firms to freely join or leave the industry. We assume that there are no entry (exit) costs to (from) the industry and potential entrants can reproduce the technology available for existing firms. This technology does not impose any additional adjustment cost to firms beyond the usual production costs.

Definition 1 *A long-run competitive industry equilibrium is a state of the industry in which: (i) entry or exit have lead the market output-price to the minimum average cost of the marginal firm in the industry, (ii) every price-taking firm maximizes profits, and (iii) output market clears. The state of long-run equilibrium is represented by a vector of values for the endogenous variables industry price and output, firm's output, and number of firms in the industry, $(p(\omega), Q(\omega), q(\omega), n(\omega))$, which for any exogenously given vector of factor prices, $\omega = (\omega_1, \dots, \omega_j, \dots, \omega_m)$, simultaneously solves the system*

$$p = \frac{C(q, \omega)}{q}, \quad (1)$$

$$p = C_q(q, \omega), \quad (2)$$

$$Q = D(p), \quad (3)$$

⁷Silberberg (1990) considers duality theory as opposed to the traditional methodology, which is based on the properties of the production function.

$$n = \frac{Q}{q}, \quad (4)$$

$$x_j = x_j(q(p, \omega), \omega) \quad \forall j = \{1, \dots, m\}. \quad (5)$$

Equation (1) says that, because of free entry and exit, in a long-run equilibrium quasi-rents are zero at the firm level, i.e. total revenue equals total cost, $C(q, \omega)$. Equation (2) shows that, in order to achieve profit maximization, firm's marginal cost, $C_q(q, \omega)$, has to be equal to the industry output-price. From now on, marginal cost will be assumed strictly increasing with respect to firm's output, $C_{qq} > 0$. The two previous equations taken together imply that firm's output at the long-run equilibrium is given by the optimum size of the firm, i.e. the level of production associated with the minimum point on the average cost curve. The necessary condition for market-clearing (3) requires equality between aggregate industry supply and market demand, $D(p)$, which is assumed strictly decreasing, $D'(p) < 0$. For this, we use the aggregation condition (4), which determines the number of firms in the industry through the well-known relationship between market demand and the optimum size of the firm. Finally, equation (5) represents both the firm cost-minimizing demand for factor x_j and its corresponding firm profit-maximizing demand evaluated at equilibrium. The profit-maximizing demand for factors are deduced, given the strictly quasi-concave and twice continuously differentiable production function $q = F(x_1, \dots, x_j, \dots, x_m)$, from the first order conditions $pF_j(x_1, \dots, x_j, \dots, x_m) = \omega_j \quad \forall j = \{1, \dots, m\}$. Moreover, corresponding to these factor demands there is a profit-maximizing output, $q(p, \omega)$, which substituted into the output variable of the cost-minimizing demands establishes the previous correspondence. This closes the set of equations characterizing the long-run competitive industry equilibrium in which price-taking firms are simultaneously maximizing profits and minimizing cost.

Related to the functions involved in previous definition, three main elasticities will play an important role in the discussion below. These are: (i) the elasticity of long-run marginal cost with respect to firm's output, $\xi_{MC} \equiv C_{qq} \frac{q}{C_q} > 0$; (ii) the absolute value of price elasticity of the market demand, $|\varepsilon_p| \equiv -D'(p) \frac{p}{Q} > 0$; and (iii) the elasticity of firm's long-run demand for factor x_j with respect to firm's output, $\eta_j^q \equiv \frac{\partial x_j}{\partial q} \frac{q}{x_j}$. According

to the latter, inputs may be classified as inferior when $\eta_j^q < 0$ or as superior (normal) when $\eta_j^q > 0$; in case of having $\eta_j^q > 1$ the input is strongly superior.

Proposition 1 *In the long-run equilibrium, output-price always changes in the same direction of the input-price change.*

Proof. Total differentiation of (1) under $d\omega_i = 0 \ \forall i \neq j$ and (2) give us $q dp = C_{\omega_j} d\omega_j$. Then, applying Shephard's lemma: $C_{\omega_j} \equiv \partial C(q, \omega) / \partial \omega_j = x_j(q, \omega)$, where $x_j(q, \omega)$ is the cost-minimizing demand for x_j , it follows

$$\frac{dp}{d\omega_j} = \frac{x_j}{q} > 0. \quad \blacksquare \quad (6)$$

The dimension of the change in output-price depends positively on the relative demand for factor x_j , which also represents the inverse of the average product for this input. This result may be rewritten in terms of the factor share in total cost as $\frac{dp}{d\omega_j} = \alpha_j \frac{p}{\omega_j}$, showing that the dimension of the change in output-price depends positively on the share of x_j .⁸

Proposition 2 *In the long-run equilibrium, an increase (decrease) in the input-price leads the optimum size of the firm to (i) increase (decrease) when $\eta_j^q < 1$, (ii) not to change when $\eta_j^q = 1$, and (iii) decrease (increase) when $\eta_j^q > 1$.*

Proof. From (1) and (2) we have $C(q, \omega) = q C_q(q, \omega)$. Total differentiation under $d\omega_i = 0 \ \forall i \neq j$ gives $q C_{qq} dq = (C_{\omega_j} - q C_{q\omega_j}) d\omega_j$. Moreover, Young's theorem and Shephard's lemma imply $C_{q\omega_j} = C_{\omega_j q} = \partial x_j / \partial q$. Then, substituting for the elasticity of long-run marginal cost with respect to firm's output and the elasticity of firm's long-run demand for factor x_j with respect to firm's output, it follows

$$\frac{dq}{d\omega_j} = \frac{1 - \eta_j^q}{\xi_{MC}} \frac{x_j}{p} = \alpha_j \frac{1 - \eta_j^q}{\xi_{MC}} \frac{q}{\omega_j}. \quad (7)$$

Therefore, the sign of $dq/d\omega_j$ clearly depends on the value of η_j^q with respect to unity at the long-run equilibrium. $\blacksquare$

⁸Notice that multiplying both sides of (6) by $\frac{\omega_j}{p}$, we get the elasticity of output-price with respect to input-price: $\frac{dp}{d\omega_j} \frac{\omega_j}{p} = \frac{\omega_j x_j}{p q}$, which given the zero profits condition becomes $\frac{dp}{d\omega_j} \frac{\omega_j}{p} = \frac{\omega_j x_j}{C(q, \omega)} = \alpha_j$, being α_j the share of factor x_j in total cost.

As we can see, the point here is not whether the factor is inferior or superior but whether it is strongly superior or not.⁹ Moreover, the dimension of the change in the optimum size of the firm depends negatively on the elasticity of long-run marginal cost with respect to firm's output. Given that higher ξ_{MC} implies a more vertical representation of the marginal cost, the higher the slope of the marginal cost function the lower the variability of the firm production level at equilibrium. The dimension of the change also depends positively on the share of factor x_j in total cost.

Proposition 3 *In the long-run equilibrium, the output of the industry always changes in opposite direction to the input-price change.*

Proof. Total differentiation of (3) under $d\omega_i = 0 \ \forall i \neq j$ and (6) give us $dQ = D'(p) \frac{x_j}{q} d\omega_j$. Then, substituting for the price elasticity of the market demand, it follows

$$\frac{dQ}{d\omega_j} = -|\varepsilon_p| \frac{n x_j}{p} = -\alpha_j |\varepsilon_p| \frac{Q}{\omega_j} < 0. \quad \blacksquare \quad (8)$$

The dimension of the change depends positively on the market demand elasticity. Given that higher $|\varepsilon_p|$ implies a more horizontal representation of the market demand, the higher the absolute value of price elasticity of the market demand the higher the variability of the industry production level at equilibrium. It also depends positively on the factor share in total cost.

Proposition 4 *In the long-run equilibrium, an increase (decrease) in the input-price leads the number of firms to (i) decrease (increase) when $\eta_j^q < 1 + |\varepsilon_p| \xi_{MC}$, (ii) not to change when $\eta_j^q = 1 + |\varepsilon_p| \xi_{MC}$, and (iii) increase (decrease) when $\eta_j^q > 1 + |\varepsilon_p| \xi_{MC}$.*

Proof. Total differentiation of (4) under $d\omega_i = 0 \ \forall i \neq j$ and the previous results (7) and (8), lead us to the result

$$\frac{dn}{d\omega_j} = - \left(|\varepsilon_p| + \frac{1 - \eta_j^q}{\xi_{MC}} \right) \frac{n x_j}{p q} = -\alpha_j \left(|\varepsilon_p| + \frac{1 - \eta_j^q}{\xi_{MC}} \right) \frac{n}{\omega_j}. \quad (9)$$

⁹Ferguson and Saving (1969) uses a different input classification (inferior, normal, and superior) based on the expenditure elasticity instead of the output elasticity. Although their conclusions about the impact of a factor price change on the endogenous variables in the long-run are similar to the ones developed in this paper, we share the objections remarked in Silberberg (1974) concerning their method of derivation.

The sign of $dn/d\omega_j$ depends on the relationship between η_j^q , $|\varepsilon_p|$, and ξ_{MC} evaluated at the long-run equilibrium. ■

Again, the point here is whether the input is strongly superior or not. According to the previous propositions, when $\eta_j^q < 1$ an increase in the input-price will lead to a reduction in the number of firms accompanied of an increase in the optimum size of the firm. Instead, when $\eta_j^q > 1 + |\varepsilon_p| \xi_{MC}$ an increase in the input-price will lead to an increase in the number of firms accompanied of a reduction in the optimum size of the firm. In this case, the size of the firm will be reduced in such a strong way that it allows for new entrants to join the industry in spite of the output reduction experienced at the industry level. For the interval $1 < \eta_j^q < 1 + |\varepsilon_p| \xi_{MC}$ an increase in the input-price will lead to a simultaneous decrease in the output of the industry, the optimum size of the firm, and the number of firms. The dimension of the change in the number of firms depends positively on the share of factor x_j in total cost.

Corollary 1 *If the market demand is completely inelastic an increase (decrease) in the input-price, which has no effect on the output of the industry, leads the number of firms to (i) decrease (increase) when $\eta_j^q < 1$, (ii) not to change when $\eta_j^q = 1$, and (iii) increase (decrease) when $\eta_j^q > 1$.*

In this particular case, face to a factor price change the number of firms and the optimum size of the firm always move in opposite direction, irrespective of the input classification.

Proposition 5 *The slope of the firm's long-run profit-maximizing factor demand is not unambiguously predetermined, but it follows from the relationship between the substitution effect (always negative) and a mixed effect, which sign depends on the elasticity of such a factor demand with respect to firm's output, η_j^q . According to the value of the latter, we can identify the following cases:*

i) if $\eta_j^q = 0$ or $\eta_j^q = 1$, there is no mixed effect and the firm demand for factor x_j at equilibrium is negatively sloped,

ii) if $\eta_j^q < 0$ or $\eta_j^q > 1$, mixed effect reinforces substitution effect and the firm demand for factor x_j at equilibrium is negatively sloped,

iii) if $0 < \eta_j^q < 1$, mixed effect plays against substitution effect and, for this reason, the firm demand for factor x_j at equilibrium may be positively sloped.

Proof. Total differentiation of (5) under $d\omega_i = 0 \ \forall i \neq j$ gives $dx_j = \frac{\partial x_j}{\partial q} dq + \frac{\partial x_j}{\partial \omega_j} d\omega_j$. Then, using the definition of η_j^q and the result (7), we get

$$\frac{dx_j}{d\omega_j} = \frac{\partial x_j}{\partial \omega_j} + \frac{\eta_j^q (1 - \eta_j^q)}{\xi_{MC}} \frac{(x_j)^2}{p q}. \quad (10)$$

This expression shows that the long-run response of firm's factor demand to a change in its own input-price is composed of a direct *substitution effect* and a mixed effect which combines both an *output effect* and an *output-price effect*. This may be more clearly inspected into the following expansion of the previous differential expression

$$\frac{dx_j}{d\omega_j} = \frac{\partial x_j}{\partial \omega_j} + \frac{\partial x_j}{\partial q} \frac{\partial q}{\partial \omega_j} + \frac{\partial x_j}{\partial q} \frac{\partial q}{\partial p} \frac{\partial p}{\partial \omega_j}. \quad (11)$$

The first term on the right-hand side represents the rate of change of x_j when output is held constant. The second term on the right-hand side represents the indirect rate at which x_j changes because of the effect of the change in ω_j on the optimal output level of the firm. Finally, the third term on the right-hand side represents the indirect rate at which x_j varies because of the effect of changes in ω_j on the industry output-price. Except for the sign of the substitution effect, the sign of the two last effects are not predetermined. Consequently, although the direct substitution effect is always negative under strictly convex isoquants, the sign of $dx_j/d\omega_j$ is ambiguous and depends on the sign and magnitude of the second compounded term on the right-hand side of (10), which is contingent on the input classification in terms of η_j^q . ■

This proposition says that the usual long-run factor demand curve for the firm is unambiguously downward-sloping only for inferior or strongly superior inputs.¹⁰ Moreover, according to Bassett and Borchering (1970a), we also conclude that surprisingly the

¹⁰The traditional analysis, such as is found in Samuelson (1947) and mainly reflected in standard microeconomic handbooks, ignores the indirect output-price effect and offers a partial and distorted

long-run response of the individual firm may entail, although not necessarily, an upward-sloping factor demand curve for the case of normal (not strongly superior) factors of production.

Proposition 6 *In the long-run equilibrium, the aggregate factor demand for the entire competitive industry is always negatively sloped.*

Proof. Whatever the number of firms in the industry, at the equilibrium the aggregate demand for factor x_j is $X_j = n x_j (q(p, \omega), \omega)$. Total differentiation under $d\omega_i = 0 \forall i \neq j$ leads to $\frac{dX_j}{d\omega_j} = x_j \frac{dn}{d\omega_j} + n \frac{dx_j}{d\omega_j}$. Then, substituting the results (9) and (10), we get

$$\frac{dX_j}{d\omega_j} = n \left(\frac{\partial x_j}{\partial \omega_j} - \left(|\varepsilon_p| + \frac{(1 - \eta_j^q)^2}{\xi_{MC}} \right) \frac{(x_j)^2}{p q} \right) < 0. \quad \blacksquare \quad (12)$$

This proposition reveals that ambiguities in the long-run response of the firm demand for factor x_j to a change of its own input-price are resolved at the aggregate level for the entire industry factor demand.¹¹ It is well-known that the factor demand function for picture of the long-run factor demand responses adopted by an individual price-taking firm, in which the negative slope is out of question. The reason is that under the fixed price assumption for the output of the industry there is no output-price effect of a change in the input-price and, also, the output effect always reinforces the substitution effect irrespective of the input classification. However, as we have shown, things are more complex and the results richer than these ones when output-price is not exogenously determined.

¹¹Bassett and Borchering (1970a) concludes that it is not possible to rule out positively sloping demands for normal factors at the industry level. They argue that whether for the ambiguity about the slope of the individual factor demand or because the change in the number of firms cannot be predicted, the slope of the industry factor demand is indeterminate. However, as we have shown, this is not true. We have perfectly identified the cases in which the size of the firm and the number of firms increase or decrease, and our results show that face to an increase in the input-price, even if factor demand is positively sloped, when $0 < \eta_j^q < 1$ the exit of firms effect dominates. But it is also shown that when $\eta_j^q > 1 + |\varepsilon_p| \xi_{MC}$, even if there is entry of firms, the decline of factor demand at the firm level will dominate. In any case, Bassett and Borchering propose as a solution for indeterminacy to assume an aggregate production function homogeneous of the first degree. This implies homotheticity and, according to Silberberg (1974), this one implies $\eta_j^q = 1$. Consequently, this strong technological assumption leads to remove both the output effect of the firm's factor demand and the case in which there is entry of firms.

the aggregate competitive industry does not correspond to the horizontal summation of the individual firms' factor demand functions when the output-price is allowed to adjust, unless the individual demands should have been obtained taking into account such an output-price adjustment. Although this is the case in our framework,¹² it must be pointed out that we are not telling a story about factor demand functions but developing a local analysis around the equilibrium point, for which we aggregate quantities alone and, then, we study the partial derivatives evaluated at such a point.

3 Short-run analysis

The short-run competitive equilibrium has been frequently represented as an intermediate situation between two polar cases in which, first, the industry output supply is totally inelastic so that the only adjustment mechanism is through the output-price and, second, the industry supply is perfectly elastic so that the only adjustment mechanism is through the output of the industry. In the benchmark model the short-run is the period over which the industry has a fixed capacity and, because of the study of isolated firms behavior, most of the traditional literature considers the output-price as a parameter exogenously given. Instead, our general goal can be better achieved assuming that the output-price is endogenously determined by the market clearing condition and, consequently, as responding to input-price changes. Moreover, we adopt as essential features of the short-run analysis both the existence of at least one fixed input in the technology of the individual firm, and a constant number of firms in the industry.¹³ Finally, it is still assumed to be no

¹²It is also the case of the monopolistic industry where the sole firm faces a decreasing output demand, and the output-price varies as the firm adjusts to an input-price change.

¹³Although in the short-run the firm does not necessarily produces at the minimum of its average cost curve the reader may assume, for the sake of simplicity, that the short-run cost structure for the representative firm is such that the fixed inputs are at the long-run cost minimizing level for the particular output level corresponding to the optimum size of the firm. That is, the minimum of the short-run average cost occurs simultaneously at the minimum of the long-run average cost. This assumption, however, is completely innocuous for our results.

adjustment costs originated in the change of input levels.

Definition 2 *A short-run competitive industry equilibrium is a state of the industry in which: (i) there is a fixed number of firms $\bar{n}$, (ii) every price-taking firm maximizes profits, (iii) output market clears, and (iv) inputs are partitioned in two sets: the vector of variable inputs, $\mathbf{x} = (x_1, \dots, x_j, \dots, x_m)$, and the vector of fixed inputs, $\bar{\mathbf{x}} = (\bar{x}_{m+1}, \dots, \bar{x}_{m+l}, \dots, \bar{x}_{m+k})$. The state of short-run equilibrium is represented by a vector of values for the endogenous variables industry price and output, firm's output, and firm's profits, $(p(\boldsymbol{\omega}, \bar{\mathbf{x}}), Q(\boldsymbol{\omega}, \bar{\mathbf{x}}, \bar{n}), q(\boldsymbol{\omega}, \bar{\mathbf{x}}), \pi(\boldsymbol{\omega}, \bar{\mathbf{x}}))$, which for any exogenously given vector of factor prices, $\boldsymbol{\omega} = (\omega_1, \dots, \omega_j, \dots, \omega_m; \omega_{m+1}, \dots, \omega_{m+l}, \dots, \omega_{m+k})$, simultaneously solves the system*

$$\pi = p q - C^s(q, \boldsymbol{\omega}, \bar{\mathbf{x}}), \quad (13)$$

$$p = C_q^s(q, \boldsymbol{\omega}, \bar{\mathbf{x}}), \quad (14)$$

$$Q = D(p), \quad (15)$$

$$\bar{n} q = Q, \quad (16)$$

$$x_j^s = x_j^s(q(p, \boldsymbol{\omega}, \bar{\mathbf{x}}), \boldsymbol{\omega}, \bar{\mathbf{x}}) \quad \forall j = \{1, \dots, m\}. \quad (17)$$

Equation (13) shows the profit level of the firm, π , as the difference between total revenue and total short-run cost, $C^s(q, \boldsymbol{\omega}, \bar{\mathbf{x}})$, which includes additively two components: variable and fixed cost. Equation (14) represents the short-run profit maximization which needs that firm's marginal cost, $C_q^s(q, \boldsymbol{\omega}, \bar{\mathbf{x}})$, be equal to the industry output-price. From now on, short-run marginal cost will be assumed strictly increasing with respect to firm's output, $C_{qq}^s > 0$. The necessary condition for market-clearing (15) requires equality between aggregate industry supply and market demand, $D(p)$, which is assumed strictly decreasing, $D'(p) < 0$. Because of the assumed symmetry for firms, the aggregation condition (16) determines industry's output as $\bar{n}$ times firm's output. Finally, equation (17) represents both the firm cost-minimizing short-run demand for a variable factor x_j and its corresponding firm profit-maximizing short-run demand evaluated at equilibrium.

The short-run demands for fixed factors are $x_{m+l}^s = \bar{x}_{m+l} \quad \forall l = \{1, \dots, k\}$. The profit-maximizing demand for variable factors are deduced, given the strictly quasi-concave and twice continuously differentiable production function $q = F(\mathbf{x}, \bar{\mathbf{x}})$, from the first order conditions $pF_j(x_1, \dots, x_j, \dots, x_m; \bar{\mathbf{x}}) = \omega_j \quad \forall j = \{1, \dots, m\}$. Moreover, corresponding to these factor demands there is a profit-maximizing output, $q(p, \boldsymbol{\omega}, \bar{\mathbf{x}})$, which substituted into the output variable of the cost-minimizing demands establishes the previous correspondence. This closes the set of equations characterizing the short-run competitive industry equilibrium in which price-taking firms are simultaneously maximizing profits and minimizing cost.

Related to the functions involved in previous definition, three main elasticities will play an important role in the discussion below. These are: (i) the elasticity of short-run marginal cost with respect to firm's output, $\xi_{MC}^s \equiv C_{qq}^s \frac{q}{C_q^s} > 0$; (ii) the absolute value of price elasticity of the market demand, $|\varepsilon_p| \equiv -D'(p) \frac{p}{Q} > 0$; and (iii) the elasticity of firm's short-run demand for factor x_j with respect to firm's output, $\eta_j^{qs} \equiv \frac{\partial x_j^s}{\partial q} \frac{q}{x_j^s}$. According to the latter, inputs may be classified as inferior when $\eta_j^{qs} < 0$ or as superior (normal) when $\eta_j^{qs} > 0$; in case of having $\eta_j^{qs} > 1$ the input is strongly superior.

Next, we shall study how the endogenous variables of the model will move at equilibrium face to a change in the price of a variable input.

Proposition 7 *In the short-run equilibrium, an increase (decrease) in the input-price leads the output-price to (i) decrease (increase) when the input is inferior, $\eta_j^{qs} < 0$, (ii) not to change when $\eta_j^{qs} = 0$, and (iii) increase (decrease) when the input is superior, $\eta_j^{qs} > 0$.*

Proof. From (14), (15), and (16) we get

$$p = C_q^s \left(\frac{D(p)}{\bar{n}}, \boldsymbol{\omega}, \bar{\mathbf{x}} \right). \quad (18)$$

Total differentiation under $d\omega_i = 0 \quad \forall i \neq j$ gives $dp = C_{qq}^s \frac{1}{\bar{n}} D'(p) dp + C_{q\omega_j}^s d\omega_j$. Then, applying Shephard's lemma: $C_{\omega_j}^s \equiv \partial C^s(q, \boldsymbol{\omega}, \bar{\mathbf{x}}) / \partial \omega_j = x_j^s(q, \boldsymbol{\omega}, \bar{\mathbf{x}})$, i.e. the cost-minimizing short-run demand for x_j , Young's theorem: $C_{q\omega_j}^s = C_{\omega_j q}^s = \partial x_j^s(q, \boldsymbol{\omega}, \bar{\mathbf{x}}) / \partial q$,

and substituting for the elasticity of short-run marginal cost with respect to firm's output, the absolute value of price elasticity of the market demand, and the elasticity of firm's short-run demand for factor x_j with respect to firm's output, it follows

$$\frac{dp}{d\omega_j} = \frac{\eta_j^{qs}}{1 + |\varepsilon_p| \xi_{MC}^s} \frac{x_j^s}{q}. \quad (19)$$

Therefore, the sign of $dp/d\omega_j$ depends on the sign of η_j^{qs} at the short-run equilibrium, which is negative for an inferior input and positive for a superior input. ■

Beyond the question of the sign, for which it is crucial whether the factor is inferior or superior, there is also the issue of the dimension of the change in the equilibrium output-price. This one depends positively on the relative demand for factor x_j , which also represents the inverse of the average product for this input. Moreover, it depends negatively on both the absolute value of price elasticity of the market demand and the elasticity of short-run marginal cost with respect to firm's output. In fact, higher ξ_{MC}^s implies a more vertical representation of the short-run marginal cost function and, hence, a less elastic output supply curve. Therefore, the higher the slope of the marginal cost and the lower the slope of the inverse market demand, the lower the variability of the equilibrium output-price.¹⁴

Proposition 8 *In the short-run equilibrium, an increase (decrease) in the input-price leads the firm's output to (i) increase (decrease) when the input is inferior, $\eta_j^{qs} < 0$, (ii) not to change when $\eta_j^{qs} = 0$, and (iii) decrease (increase) when the input is superior, $\eta_j^{qs} > 0$.*

Proof. Total differentiation of (14) under $d\omega_i = 0 \ \forall i \neq j$ gives $dp = C_{qq}^s dq + C_{q\omega_j}^s d\omega_j$. Then, applying Young's theorem and Shephard's lemma, and using the profit-maximizing condition and the previous result (19) for substitutions, we get

$$\frac{dq}{d\omega_j} = - \frac{\eta_j^{qs} |\varepsilon_p|}{1 + |\varepsilon_p| \xi_{MC}^s} \frac{x_j^s}{p}. \quad (20)$$

¹⁴In the short-run, total revenue is not necessarily equal to total cost because of profits, $pq = \pi + C^s(q, \omega, \bar{x})$. However, we can still define the share of factor x_j in total revenue as $\beta_j = \frac{\omega_j x_j^s}{pq}$. Then, we get the following expressions which may be useful for substitution in the previous and the next two propositions: $\frac{x_j^s}{q} = \beta_j \frac{p}{\omega_j}$, $\frac{x_j^s}{p} = \beta_j \frac{q}{\omega_j}$, and $\frac{\bar{\pi} x_j^s}{p} = \beta_j \frac{Q}{\omega_j}$.

Therefore, the sign of $dq/d\omega_j$ depends inversely on the sign of η_j^{qs} at the short-run equilibrium, which is negative for an inferior input and positive for a superior input. ■

Once again, it is important whether the factor is inferior or superior for the matter of the sign, but the dimension of the change in the equilibrium output of the firm depends negatively on the elasticity of short-run marginal cost with respect to firm's output (positively on the elasticity of the output supply curve) and positively on the absolute value of price elasticity of the market demand. In the short-run, face to a factor price change, the equilibrium output-price and firm's output always move in opposite direction, irrespective of the input classification.

Proposition 9 *In the short-run equilibrium, an increase (decrease) in the input-price leads the output of the industry to (i) increase (decrease) when the input is inferior, $\eta_j^{qs} < 0$, (ii) not to change when $\eta_j^{qs} = 0$, and (iii) decrease (increase) when the input is superior, $\eta_j^{qs} > 0$.*

Proof. Total differentiation of (16) gives $dQ = \bar{n} dq$. Then, using the previous result (20), we get

$$\frac{dQ}{d\omega_j} = - \frac{\eta_j^{qs} |\varepsilon_p|}{1 + |\varepsilon_p| \xi_{MC}^s} \frac{\bar{n} x_j^s}{p}. \quad (21)$$

Given that the number of firms is fixed at the short-run equilibrium, the sign of $dQ/d\omega_j$ depends inversely on the sign of η_j^{qs} , which is negative for an inferior input and positive for a superior input. ■

The dimension of the change in the equilibrium output of the industry depends positively on the elasticity of the market demand and negatively on the elasticity of the short-run marginal cost (positively on the elasticity of the output supply curve). In the short-run, face to a factor price change, the equilibrium output of the industry always change in the same direction as the output of the firm, irrespective of the input classification.

Proposition 10 *In the short-run equilibrium, an increase (decrease) in the input-price leads the profits level of the firm to (i) decrease (increase) when $\eta_j^{qs} < 1 + |\varepsilon_p| \xi_{MC}^s$, (ii) not to change when $\eta_j^{qs} = 1 + |\varepsilon_p| \xi_{MC}^s$, and (iii) increase (decrease) when $\eta_j^{qs} > 1 + |\varepsilon_p| \xi_{MC}^s$.*

Proof. By total differentiation of (13) under $d\omega_i = 0 \ \forall i \neq j$ and noticing that $p = C_q^s$, we have $d\pi = q dp - C_{\omega_j}^s d\omega_j$. Then, applying Shephard's lemma, $C_{\omega_j}^s = x_j^s$, and using for substitutions the usual elasticity definitions and the result (19), we get

$$\frac{d\pi}{d\omega_j} = \frac{\eta_j^{qs} - 1 - |\varepsilon_p| \xi_{MC}^s}{1 + |\varepsilon_p| \xi_{MC}^s} x_j^s. \quad (22)$$

According to this, except for the case in which the input is very strongly superior, $\eta_j^{qs} \geq 1 + |\varepsilon_p| \xi_{MC}^s$, the input-price and the profits level of the firm will move in opposite direction, including the particular case of inferior inputs. ■

The dimension of the change in the equilibrium level of profits of the firm depends positively on both the elasticity of the market demand and the elasticity of the short-run marginal cost (negatively on the elasticity of the output supply curve). It also depends positively on the absolute demand for factor x_j .¹⁵

Corollary 2 *If the market demand is completely inelastic an increase (decrease) in the input-price, which has no effect on the equilibrium levels of the output of the firm and the output of the industry, leads the profits level of the firm to (i) decrease (increase) when $\eta_j^{qs} < 1$, (ii) not to change when $\eta_j^{qs} = 1$, and (iii) increase (decrease) when $\eta_j^{qs} > 1$.*

In this particular case, face to a factor price change the equilibrium output-price and the equilibrium level of profits move in the same direction as long as the input is inferior or strongly superior. However, they move in opposite direction when the input is only superior.¹⁶

¹⁵Bear (1965) studies some short-run results of the theory of the competitive firm but he does not accept the feasibility of $\frac{d\pi}{d\omega_j} > 0$. Thus, he uses this premise to refuse the possibility of factor inferiority, at least for all levels of output. However, as it is shown here, Bear's logic is wrong because he associates the result $\frac{d\pi}{d\omega_j} > 0$ with inferior factors, but also because his premise is false.

¹⁶Meyer (1967, 1968) analyzes the *paradoxical* case in which $\frac{d\pi}{d\omega_j} > 0$, concluding that conditions for this result are met with relatively high frequency, as suggested by casual empiricism. He also finds that $|\varepsilon_p| = 0$ or $\xi_{MC}^s = 0$ are sufficient conditions for a higher factor price to result in larger profits, but this is a conclusion which is not supported by our results. Finally, as a corollary of the above paradox, Meyer infers that profits are no longer a good signalling device for the entry/exit process. We find, however, that

Proposition 11 *The slope of the firm's short-run profit-maximizing factor demand is always negative, irrespective of the input classification.*

Proof. Total differentiation of (17) under $d\omega_i = 0 \ \forall i \neq j$ gives $dx_j^s = \frac{\partial x_j^s}{\partial q} dq + \frac{\partial x_j^s}{\partial \omega_j} d\omega_j$. Then, using the definition of η_j^{qs} and the result (20), we get

$$\frac{dx_j^s}{d\omega_j} = \frac{\partial x_j^s}{\partial \omega_j} - \frac{|\varepsilon_p| (\eta_j^{qs})^2}{1 + |\varepsilon_p| \xi_{MC}^s} \frac{(x_j^s)^2}{p q} < 0. \quad (23)$$

The sign of the slope is unambiguous because the direct substitution effect (the first term on the right-hand side) is non-positive and the second compounded term reinforces the previous one. ■

Another view of total differentiation of (17) under $d\omega_i = 0 \ \forall i \neq j$ is $dx_j^s = \frac{\partial x_j^s}{\partial q} \frac{\partial q}{\partial p} dp + \frac{\partial x_j^s}{\partial q} \frac{\partial q}{\partial \omega_j} d\omega_j + \frac{\partial x_j^s}{\partial \omega_j} d\omega_j$. But, as we have previously shown, (18) implicitly defines a function $p = p(\omega, \bar{n}, \bar{\mathbf{x}})$, which after substituting into (17) implies that at the short-run equilibrium

$$\frac{dx_j^s}{d\omega_j} = \frac{\partial x_j^s}{\partial \omega_j} + \frac{\partial x_j^s}{\partial q} \frac{\partial q}{\partial \omega_j} + \frac{\partial x_j^s}{\partial q} \frac{\partial q}{\partial p} \frac{\partial p}{\partial \omega_j}. \quad (24)$$

This expression is equivalent to that provided by Heiner (1982). As the author points out, the full short-run factor demand response to a change of its own price, at the firm level, can be divided into three components: the output constant effect, the output response effect, and the output-price response effect. The first one is the *substitution effect* (i.e. the factor demand response for constant firm's output). The second one is the *output effect* (i.e. the factor demand response of the induced change in firm's profit-maximizing output for fixed output-price). Finally, the third component is the *output-price effect*, which captures the additional influence of the induced change in the market output-price. Coming back to the context of the paper, the short-run equilibrium conditions (14), (15), and (16) allow us to write $D(p) = \bar{n} q(p, \omega, \bar{\mathbf{x}})$. Then, total differentiation under $d\omega_i = 0$

this is not true because when conditions for the paradox hold, in the corresponding long-run equilibrium there is margin for the encouraged firms to join the industry. Actually, Meyer's argument is based on the assumption that isoquants are homothetic. But, as Silberberg (1974) shows, if the production function is homothetic the required conditions for Meyer's paradox are not feasible because the output elasticity of demands for variable inputs are all unity.

$\forall i \neq j$ leads to $D'(p) dp = \bar{n} \frac{\partial q}{\partial \omega_j} d\omega_j + \bar{n} \frac{\partial q}{\partial p} dp$. Taking this one and the result shown in (19), we can substitute both into (24) getting exactly the result (23).

Proposition 12 *In the short-run equilibrium, the aggregate factor demand for the entire competitive industry is always negatively sloped.*

Proof. Quantities summation at the equilibrium over a fixed number of firms leads to the aggregate demand for factor x_j , $X_j^s = \bar{n} x_j^s(q(p, \omega, \bar{x}), \omega, \bar{x})$. Total differentiation under $d\omega_i = 0 \ \forall i \neq j$ gives $\frac{dX_j^s}{d\omega_j} = \bar{n} \left[\frac{\partial x_j^s}{\partial \omega_j} + \frac{\partial x_j^s}{\partial q} \frac{\partial q}{\partial \omega_j} + \frac{\partial x_j^s}{\partial q} \frac{\partial q}{\partial p} \frac{\partial p}{\partial \omega_j} \right] = \bar{n} \frac{dx_j^s}{d\omega_j}$, and substituting the result (23) we get

$$\frac{dX_j^s}{d\omega_j} = \bar{n} \left[\frac{\partial x_j^s}{\partial \omega_j} - \frac{|\varepsilon_p| (\eta_j^{qs})^2}{1 + |\varepsilon_p| \xi_{MC}^s} \frac{(x_j^s)^2}{pq} \right] < 0. \quad \blacksquare \quad (25)$$

4 Conclusions

In this paper we have shown how the comparative statics results of the competitive model may be obtained in a comprehensive framework, integrating simultaneously the short and long-run analyses and the output-price feedbacks between firms and the industry. Our results pivot on the concepts of elasticity of the marginal cost with respect to firm's output, the absolute value of price elasticity of the market demand, and the elasticity of firm's factor demand with respect to firm's output. Connected with the latter, factors may be classified as inferior, superior or strongly superior, and we have shown that otherwise we could not report the richness and variability of the results just obtained.

In the case of the slope of the firm's factor demand, it could be normal to ask whether the LeChâtelier principle holds for the short-run equilibrium result compared to the long-run one. However, it seems not to exist an evident answer to this question because from the comparison it emerges the ambiguity. We find that the nature of the factor with respect to output as well as its definition in terms of gross substitutability/complementarity do play a central role, which may or may not be conclusive and prevents us from getting a clear verdict.

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