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Modeling the Immigration Shock

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1. Introduction

We are interested in the following question: what are the intergenerational effects of a large and unexpected immigration flow? How does it affect the welfare of the different generations living in the country that receives the immigration flow and, in particular, how does it impact on intergenerational arrangements such as public education and pensions? To begin answering these questions we develop a simple theoretical framework with overlapping generations that live for three periods, accumulate human capital in the first, work in the second and retire in the third living off the return from their investments. The latter include both physical capital and the interest from the money they lent to the young people to invest in human capital.

We model the immigration shock as a sudden increase in the supply of middle-age labor accompanied by a reduction in the average human capital. The immigrants are, in other words, new middle age workers somewhat less skilled than the native ones. The shock lasts one period, after which the economy moves along its new growth path with a larger number of, now heterogeneous, middle age workers. We assume that the children of the immigrants perfectly integrate, hence after one

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period they accumulate as much per capita human capital as the offsprings of the native workers.

In the baseline model we assume financial markets are sequentially complete. This means that - because there are always two possible states of the world next period, one with and one without immigration shock - there are two financial assets agents buy from and sell to each other in every period. One that pays one unit of consumption only when there is an immigration shock and one that pays a unit of consumption only when there is no immigration shock. Through these two assets - accessible to all individuals that are alive during the period - young people can insure their human capital investment from the negative impact of an immigration shock, which they do by purchasing insurance from the middle age people. The middle age people - who are earning labor income and saving part of it for retirement - can use the extra payoff they would receive from their capital investment if, next period, the immigration shock were realized to provide the young people with such insurance. Old people do not trade in assets as we assume that they must die without debt and the bequest motive is absent. Notice that the buying and selling of insurance takes place at the same time and through the same instruments that middle age and young use to lend/borrow to/from each other.

More precisely: middle age individuals invest in physical capital (by purchasing assets issued by a one period firm, that will carry out production next period) and in human capital (by purchasing assets issued by the young agents that are carrying out such investment). Because the capital investment in the firm pays off more when there is immigration (more, and cheaper workers are available) this extra return is used to compensate for the payoff from the human capital investment, which will be lower in that case. This assures that both young and middle age people implement as much consumption smoothing as it is feasible (consumption taking place, respectively, when middle age and old).

This does not imply that there is perfect consumption smoothing, nor that some ex-ante notion of efficiency is satisfied at the equilibrium of our model. This is because agents cannot insure beforehand against the risk of being born right after a period of high immigration. Young agents born in the period after a positive immigration shock are worse off than they would be otherwise, as they must compete with a larger number of people (the offsprings of the immigrants) in borrowing funds to invest in human capital this period and in the labor market next period. This type of risk cannot be insured away with the market structure we have assumed. It would be possible if parents were altruistic and internalized,

via bequests, the future welfare of their children. We assume, instead, that parents are selfish and do not leave anything to their children, hence the latter must bear the cost of being born in the "wrong" period. The extension to the case in which a bequest motive leads parents to purchase insurance for the future generations is an interesting venue for future research.

The key channels through which immigration affects welfare in this economy is that it increases labor supply in the face of a predetermined stock of capital. This lowers wages and increases the return on capital, shifting income from one generation to another. In this sense, factor prices move around because we have assumed zero capital mobility. If there were perfect capital mobility, capital would flow into the country from outside on the footsteps of immigrant labor, and the capital intensity ratio would remain unchanged. In this case, factor prices are unaffected by immigration, which amounts to nothing more than an increase in the size of the economy. Under constant returns to scale in production, which we assume, this does not affect the welfare of the native agents. Nevertheless, if there are frictions in international financial markets and capital adjustment is not instantaneous, i.e. it takes time for the capital stock of the country to be built up to restore the initial K/L ratio, then immigration causes a redistribution between generations, as outlined above. Notice that, the latter observation suggests that the larger is the trade deficit following an immigration shock the quicker will be the adjustment toward a the old K/L ratio hence the smaller the redistribution away from native workers and toward native owners of capital.

We ask next whether government policies can be used to substitute for the credit and insurance markets of the base model when these, as it is often the case in reality, are either absent or largely incomplete. To do this we build on previous results presented in Boldrin and Montes [REStud, 200?], and which answered the question in the affirmative for the case of no immigration shock, adapting that framework to the particular circumstances at hand. In the present case we show that pension payments and social security contributions must be negatively indexed to the size of the immigration flow, while educational expenditures and the issuance of public debt that finances it should be positively correlated. Intuitively, this is because social security contributions play the role that the repayment of debt plus interest, by the currently middle age generation to those that lend them money to invest in human capital, plays in the model with sequentially complete financial markets. The pension payments are nothing but these contributions being paid to the old people: they correspond to the payoff from the securities that were traded to finance the human capital investment of the young generation

in the previous period. Likewise, the educational investment (financed via the issuance of bonds) corresponds to the issuance of the same securities in this period, hence it should increase as the size of the young generation is larger than expected.

The rest of the paper proceeds as follows. In Section 2 we describe the basic model; in Section 3 we replicate the results of Boldrin and Montes [REStud 200?] for this particular case; in Section 4 we look at an open economy with complete markets; in Section 5 we look at an open economy without any financial market but the one in which physical capital is traded but with public education and pensions. Section 6 concludes.

2. The basic model

We study an OLG model with a representative agent in each generation, who lives for three periods - youth, middle age and old. There is aggregate uncertainty due to an immigration flow that may unexpectedly increase the size of the middle age group, thereby affecting the total supply of labor, the wage rate, the return on capital and aggregate output.

The structure of the population in period t is (N_t^y, N_t^m, N_t^o) , with $N_t^m = (1 + z_t)N_{t-1}^y$ and $N_t^y = (1 + n)N_t^m$, where $-1 < n$ and z_t is the realization of the immigration shock in period t . We use superscripts, y , m and o to denote, respectively, young, middle-age and old people. For simplicity, we assume that the shock z follows a two-state Markov process, with state space $Z = \{\bar{z}, 0\}$, $\bar{z} > 0$. The notation $\pi(z_{t+1}|z_t)$ denotes the probability of $z_{t+1} \in Z$, given z_t .

In each period $t = 0, 1, \dots$ a new generation $N_t^y = (1 + n)N_t^m$ is born, with a per-capita endowment h_t^y of basic knowledge, which is an input in the production of future human capital according to $h_{t+1}^m = h(d_t, h_t^y)$. With d_t we denote the physical resources, invested in education, that the young must purchase on the market, as specified below. The function $h(d, h^y)$ is a constant returns to scale neoclassical production function. During the second period of life, individuals work and decide how much of their income to consume, how much to save, and how to allocate the latter among different financial instruments. When old, they have no decisions to make: they consume all their income, and then die. We assume agents draw utility from consumption when middle age and old. We assume immigrants enter the country with a fraction $0 \leq \gamma \leq 1$ of the human capital level of the natives and with zero capital or financial assets. Neither consumption when young, nor leisure, nor the welfare of descendants affects lifetime utility.

Initial conditions are: K_0 , for the capital stock, (N_0^y, N_0^m, N_0^o) for the popula-

tion, h_0^m for the level of per-capita human capital of the middle age and $A_{-1}^y(0)$, $A_{-1}^m(0)$ for the portfolios of middle age and old people, respectively, and $A_{-1}^f(0)$ for the representative firm carrying out aggregate production, and which owns K_0 . Finally, we assume that there are no immigrants in the first period, hence the initial N_0^m middle age people all have the same human capital level and the same portfolio of financial assets, as do the initial N_0^o old people.

The preferences of an individual born in period $t - 1$ are represented by the following expected lifetime utility

$$E_{t-1} [u(c_t^m(z_t)) + \delta E_t [u(c_{t+1}^o(z_{t+1}))]] ,$$

where δ is the period discount factor and E the expectation operator. The function $u : \mathbb{R}_{++} \rightarrow \mathbb{R}$ is assumed to be strictly increasing, strictly concave and C^2 .

2.1. Market structure

Normalize to one the price of output in the initial period, in which the state is $z = 0$, and write $p_t(z)$ for the price of consumption in period t and state $z \in Z$ in all subsequent periods. We assume sequentially complete financial markets, *i.e.* that - given the current state z_t and the set Z of possible future states - for all $z \in Z$ there exists a competitive market in which contingent claims $A_t(z)$ are traded, with payoff, in units of next period consumption, $b[A_t(z), z_{t+1}] = 1$ if $z_{t+1} = z$, and zero otherwise. We assume agents cannot die in debt, *i.e.* we impose $A_t^m(z) \geq 0$ for all t and z . Let $q(z, z_t)$ be the price, in units of consumption at t , of asset $A(z)$ in period t and state z_t . Notice that here, to save notation, the symbol $A_t(z)$ indicates also the number of units of that asset traded in a given period.

2.2. Firms

There is a representative firm, which in each period uses human and physical capital to produce the consumption good according to $Y_t = F(K_t, H_t)$, where aggregate human capital is $H_t = (1 + z_t\gamma) h_t^m N_{t-1}^y$, $0 < \gamma < 1$ and $F(K, H)$ is a constant returns to scale neoclassical production function. Firms last one period and own the physical capital, which they finance by issuing state-contingent securities. More specifically, in each period t the representative firm issues securities $A_t^f(z)$ at a price $q(z, z_t)$, for $z \in \{\bar{z}, 0\}$, with the proceedings of which they purchase K_{t+1} , used for production next period. In period $t + 1$, after the realization

of the shock, the firm hires workers, carries out production, pays off wages, honors its financial liabilities and then dissolves.

Let $w_t(z)$ be the nominal wage in period t and state $z \in Z$. Write $w(z_t)/p(z_t) = \omega(z_t)$ and $\varphi(z_t) = p(z_t)F_K(K_t, H(z_t))$. The problem of the firm is

$$\max_{A_t^f(z), H_{t+1}} E_t \left\{ p_{t+1}(z) \left[F(K_{t+1}, H_{t+1}(z)) - \omega_{t+1}(z)H_{t+1}(z) - A_t^f(z) \right] \right\}$$

subject to,

$$K_{t+1} = \sum_{z \in Z} q(z, z_t) A_t^f(z).$$

The first order conditions for H and for $A^f(z)$ are

$$\omega_{t+1}(z) = F_H(K_{t+1}, H_{t+1}(z)), \text{ and} \quad (1.a)$$

$$q(z, z_t) = \frac{\pi(z|z_t)p_{t+1}(z)}{\sum_{z \in Z} \pi(z|z_t)\varphi_{t+1}(z)} \text{ for each } z \in Z. \quad (1.b)$$

2.3. Consumers

For a native agent born in period $t - 1$ when the state is z_{t-1} , the life-time optimization problem is

$$\max_{d(z_{t-1}), A_{t-1}^y(z), A_t^m(z)} E_{t-1} \left\{ u(c_t^m(z)) + \delta E_t [u(c_{t+1}^o(z))] \right\}$$

subject to,

$$d(z_{t-1}) + \sum_{z \in Z} q(z, z_{t-1}) A_{t-1}^y(z) \leq 0 \quad (2.a)$$

$$c^m(z_t) + \sum_{z \in Z} q(z, z_t) A_t^m(z) = \omega(z_t)h_t^m + A_{t-1}^y(z_t) \quad \forall z_t \in Z \quad (2.b)$$

$$c^o(z_{t+1}) = A_t^m(z_{t+1}) \quad \forall z_{t+1} \in Z \quad (2.c)$$

$$h_t^m = h[d(z_{t-1}), h_{t-1}^y] \quad (2.d)$$

The first order conditions for the choice of $\mathcal{A}^y(z_{t-1}) = \{A_{t-1}^y(z), \text{ for all } z \in Z\}$ and $d(z_{t-1})$ boil down to

$$q(z, z_{t-1}) = \frac{\pi(z|z_{t-1})u'(c_t^m(z))}{\sum_{z \in Z} \pi(z|z_{t-1})u'(c_t^m(z))\omega_t(z)h_d[d(z_{t-1}), h_{t-1}^y]} \quad \forall z \in Z \quad (3.a)$$

$$1 = \sum_{z \in Z} q(z, z_{t-1})\omega_t(z)h_d[d(z_{t-1}), h_{t-1}^y]. \quad (3.b)$$

The first order condition for each of the $A_t^m(z)$ reads

$$q(z, z_t) = \frac{\pi(z|z_t)\delta u'(c_{t+1}^o(z))}{u'(c_t^m(z))} \quad \forall z \in Z. \quad (3.c)$$

For a middle age immigrant, arriving in the state of the world z_t with human capital $\bar{h}_t^m = \gamma h_t^m$ and $A_{t-1}^y(z_t) = 0$, the maximization problem is:

$$\max_{A_t^m(z)} u(\bar{c}^m(z_t)) + E_t \delta u(\bar{c}^o(z_{t+1}))$$

subject to,

$$\bar{c}^m(z_t) + \sum_{z \in Z} q(z, z_t) \bar{A}_t^m(z) = \omega(z_t) \bar{h}_t^m \quad \forall z_t \in Z \quad (2.b')$$

$$\bar{c}^o(z_{t+1}) = \bar{A}_t^m(z_{t+1}) \quad \forall z_{t+1} \in Z. \quad (2.c')$$

The first order conditions determining $\bar{A}^m(z_t)$ are the same as in (3.c);

$$q(z, z_t) = \frac{\pi(z|z_t)\delta u'(\bar{c}_{t+1}^o(z))}{u'(\bar{c}_t^m(z))} \quad \forall z \in Z. \quad (3.c')$$

2.4. Financial markets

It is clear from the budget constraints that the net financial position of young agents will be non-positive (i.e. $\sum_{z \in Z} q(z, z_{t-1})A_{t-1}^y(z) \leq 0$) while that of middle age agents are non-negative (i.e. $\sum_{z \in Z} q(z, z_t)A_t^m(z) \geq 0$). When the latter is positive it corresponds to aggregate national saving, which is invested in the physical capital of firms and in the education of the young agents. The first order conditions for profit maximization of the firm imply

$$q(z, z_t) = \frac{\pi(z|z_t)p_{t+1}(z)}{\sum_{z \in Z} \pi(z|z_t)\varphi_{t+1}(z)} \quad \text{for each } z \in Z.$$

Multiplying (4.a) by $F_K(K_{t+1}, H_{t+1}(z))$ and aggregating in $z \in Z$ we get

$$\sum_{z \in Z} q_t(z, z_t) F_K(K_{t+1}, H_{t+1}(z)) = 1. \quad (4)$$

2.5. Competitive equilibrium

A *competitive equilibrium* is a mapping from the current state of the world into a distribution of future quantities and prices at all times t . For a given initial conditions $(K_0, H_0, z_0, N_0^y, N_0^m, N_0^o, A_{-1}^y(z_0), A_{-1}^m(z_0), A_{-1}^f(z_0))$ a competitive equilibrium is a collection of choices: (1) for native, $\{d(z_t), c_t^m(z), c_t^o(z), A_t^y(z), A_t^m(z)\}_{t=0}^\infty$, and immigrant, $\{\bar{c}_t^m(z), \bar{c}_t^o(z), \bar{A}_t^m(z)\}_{t=0}^\infty$, households, and, (2) for the representative firm, $\{K_t(z), H_t(z), A_t^f(z)\}_{t=0}^\infty$, as well as prices, $\{p_t(z), q(z, z_t), \omega_t(z), \varphi_t(z)\}_{t=0}^\infty$; such that for all t and $z \in Z$, the consumers and the firm maximize their payoffs and the markets clear. [Need to provide the appropriate definition for a Markovian equilibrium in this setting.]

In each period t and state z there are three sets of markets to clear:

i) Output market:

$$C_t^m(z) + C_t^o(z) + d_t(z) N_t^y(z) + K_{t+1}(z) = F(K_t, H_t(z)). \quad (5.a)$$

where $C_t^m(z) = [c_t^m(z) + \bar{c}_t^m(z) z] N_{t-1}^y$ and $C_t^o(z) = [c_t^o(z) + \bar{c}_t^o(z) z_{t-1}] N_{t-2}^y$.

ii) Labor market:

$$(1 + z\gamma) h_t^m N_{t-1}^y = H_t(z). \quad (5.b)$$

iii) Capital market:

$$\begin{aligned} \sum_{z \in Z} q(z, z_t) A_t^f(z) &= K_{t+1}, \\ A_t^f(z) &= N_{t-1}^y A_t^m(z) + z_t N_{t-1}^y \bar{A}_t^m(z) + N_t^y A_t^y(z), \\ \sum_{z \in Z} q(z, z_t) A_t^y(z) &= d(z_t). \end{aligned} \quad (5.c)$$

For each state $z \in Z$, the payoff from security $A_t^f(z)$ is:

$$b[A_t^f(z), z_{t+1} = z]A_t^f(z) = [F(K_{t+1}, H_{t+1}(z)) - \omega_{t+1}(z)H_{t+1}(z)] = F_K(K_{t+1}, H_{t+1}(z))K_{t+1}. \quad (5.d)$$

2.6. Example 1:

Consider an economy with logarithmic utility function and Cobb Douglas production functions: $u(c) = \log c$, $F(K, H) = AK^\alpha H^{1-\alpha}$ and $h(d, h^y) = Bd^\beta (h^y)^{1-\beta}$.

Write, $\tilde{\omega}(z_t) = \omega(z_t)h(d_{t-1}, h_{t-1}^y)$. From (3.c) we have, for a native middle age person

$$q(z, z_t) = \frac{\pi(z|z_t)\delta}{A_t^m(z)} \left[\tilde{\omega}(z_t) + A_{t-1}^y(z_t) - \tilde{A}^m(z_t) \right] \quad \forall z \in Z,$$

where $\tilde{A}^m(z_t) = \sum_{z \in Z} q(z, z_t)A_t^m(z)$. Multiplying by $A_t^m(z)$ and aggregating in $z \in Z$ we arrive at the total demand for contingent securities of a native middle age individual

$$\tilde{A}^m(z_t) = \frac{\delta}{1+\delta} \left[\tilde{\omega}(z_t) + A_{t-1}^y(z_t) \right].$$

The demand for consumption in middle-age, and the demand for each component $A_t^m(z)$ of $\mathcal{A}^m(z_t)$ are

$$\begin{aligned} c^m(z_t) &= \frac{1}{1+\delta} \left[\tilde{\omega}(z_t) + A_{t-1}^y(z_t) \right], \\ A_t^m(z) &= \frac{\delta}{1+\delta} \left[\tilde{\omega}(z_t) + A_{t-1}^y(z_t) \right] \frac{\pi(z|z_t)}{q(z, z_t)}. \end{aligned}$$

For an immigrant in t we get

$$\begin{aligned} \bar{c}^m(z_t) &= \frac{1}{1+\delta} \gamma \tilde{\omega}(z_t), \\ \tilde{\bar{A}}^m(z_t) &= \frac{\delta}{1+\delta} \gamma \tilde{\omega}(z_t), \\ \bar{A}_t^m(z) &= \frac{\delta}{1+\delta} \gamma \tilde{\omega}(z_t) \frac{\pi(z|z_t)}{q(z, z_t)}. \end{aligned}$$

Now, substituting $A_{t-1}^m(z)$ and $\bar{A}_{t-1}^m(z)$ in (5.d) and using (5.c) we have the demand for each component $A_{t-1}^y(z)$ of $\mathcal{A}^y(z)$:

$$A_{t-1}^y(z) = -d(z_{t-1}) \frac{\pi(z|z_{t-1})}{q(z, z_{t-1})} + \frac{K_t}{N_{t-1}^y} \left[\frac{\varphi_t(z)}{p_t(z)} - \frac{\pi(z|z_{t-1})}{q(z, z_{t-1})} \right].$$

Furthermore, the expected return on human and physical capital must be equalized in equilibrium. Using this condition we have:

$$-\tilde{A}^y(z_{t-1}) N_{t-1}^y = d(z_{t-1}) N_{t-1}^y = \eta K_t \Psi(z_{t-1}),$$

where $\Psi(z_{t-1}) = E_{t-1} \{p_t(z) (1 + \gamma z)^{-\alpha}\} / E_{t-1} \{p_t(z) (1 + \gamma z)^{1-\alpha}\}$ and $\eta = \beta(1 - \alpha) / \alpha$. Therefore

$$A_{t-1}^y(z) = -d(z_{t-1}) \frac{\pi(z|z_{t-1})}{q(z, z_{t-1})} + \frac{d(z_{t-1})}{\eta \Psi(z_{t-1})} \left[\frac{\varphi_t(z)}{p_t(z)} - \frac{\pi(z|z_{t-1})}{q(z, z_{t-1})} \right].$$

Finally, from (1) we obtain the equilibrium prices for each period t and state $z \in Z$:

$$\begin{aligned} \omega_t(z) &= (1 - \alpha) A K_t^\alpha H_t(z)^{-\alpha} \\ \varphi_t(z) &= p(z_t) \alpha A K_t^{\alpha-1} H_t(z)^{1-\alpha} \\ q(z, z_t) &= \frac{\pi(z|z_t) p_{t+1}(z)}{E_t \{p_{t+1}(z) \alpha A K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha}(z)\}}. \end{aligned}$$

Note also that in equilibrium $p_t(\bar{z}) c_t^m(\bar{z}) = p_t(0) c_t^m(0)$. Substituting the values of $c_t^m(\bar{z})$ and $c_t^m(0)$ we arrive to $p_t(\bar{z}) = p_t(0) \frac{(1+\gamma\bar{z})^\alpha}{1+\alpha\gamma\bar{z}}$, where $(1 + \gamma\bar{z})^\alpha < 1 + \alpha\gamma\bar{z}$ and we normalize $p_t(0) = 1$.

Set $h_t^y = H_t / N_t^y$ so that an autonomous system can be derived. Given initial conditions for $(K_0, H_0, d_{-1}, A_{-1}^y(z_0), A_{-1}^m(z_0), N_0^o)$, the following system describes the dynamic of the economy for a given sequence of shocks $(z_0, z_1, \dots)$,

$$\begin{aligned} K_{t+1} &= \Omega(z_t, z_{t-1}) A K_t^\alpha (H(z_t))^{1-\alpha} \\ H(z_{t+1}) &= (1 + \gamma z_{t+1}) B (\eta A \Omega(z_t, z_{t-1}) \Psi(z_t))^\beta K_t^{\alpha\beta} (H(z_t))^{1-\alpha\beta}. \end{aligned} \tag{6}$$

where

$$\Omega(z_t, z_{t-1}) = \frac{\delta}{1 + \delta} \frac{1}{(1 + \eta \Psi(z_t))} \left(1 - \alpha \frac{\left[\pi(\bar{z}|z_t) \frac{(1+\gamma\bar{z})^\alpha}{1+\alpha\gamma\bar{z}} + \pi(0|z_t) \right]}{(1 + z_t \gamma)^{1-\alpha}} (\eta \Psi(z_{t-1}) + 1) \right)$$

and

$$\Psi(z_t) = \frac{\pi(\bar{z}|z_t) \frac{1}{1+\alpha\gamma\bar{z}} + \pi(0|z_t)}{\pi(\bar{z}|z_t) \frac{(1+\gamma\bar{z})}{1+\alpha\gamma\bar{z}} + \pi(0|z_t)}.$$

Given a sequence of shocks $(z_0, z_1, \dots)$ the evolution of the factor intensity ratio $X = K/H$ is given by

$$X_{t+1} = \frac{(\Omega(z_t, z_{t-1}) A)^{1-\beta}}{(1 + \gamma z_{t+1}) B(\eta \Psi(z_t))^\beta} X_t^{\alpha(1-\beta)}.$$

Set $(z_t, z_{t+1}, z_{t+2}, \dots) = (0, 0, 0, \dots)$. The ray

$$X^* = \left[\frac{(\Omega(0, 0) A)^{1-\beta}}{B(\eta \Psi(0))^\beta} \right]^{\frac{1}{1-\alpha(1-\beta)}}$$

defines a balanced growth path. For all initial conditions $(H_0, K_0) \in \mathfrak{R}_+^2$, iteration of (6) leads (H_t, K_t) to the ray X^* .

Along the balanced growth path, the two stocks of capital expand (or contract) at the factor

$$1 + g^* = \left[\frac{(\Omega(0, 0) A)^\beta}{B(\eta \Psi(0))^\beta} \right]^{\frac{\alpha-1}{1-\alpha(1-\beta)}}.$$

2.6.1. Numerical example

Let us now consider the practical implications of an immigration shock using a numerical example. We assign reasonable values to preferences and technological parameters, but make no claim to be using empirically "certified" values. We compare two economies: an economy with no immigration $(z_0, z_1, z_2, \dots) = (0, 0, 0, \dots)$ and another with only one immigration shock $(z_0, z_1, z_2, \dots) = (0, \bar{z}, 0, \dots)$. We assume $\bar{z} = 0.3$, $\gamma = 0.7$, $\pi(\bar{z}|z_t) = \pi(0|z_t) = 0.5$. We normalize $N_0^m = 1$ and assume an annual growth rate of the population equal to 0. Recall that a period in this model is about 30 years. With respect to the production technology, α is fixed at 0.3 and the scale parameter A is fixed at 1. In the human capital technology we set $\beta = 0.13$, which corresponds to an elasticity of output with respect of education of 0.09. The discount factor δ is set to match the ratio of investment

over output $I/Y = 22\%$. This yields a value of $\delta = 0.904$, which corresponds to an annual discount rate of 0,996641. We set the scale parameter B equal to 4 to get an annual rate of aggregate output growth equal to 3% along the balance growth path. With these parameter values we have an annual interest rate of 4,07% (and a capital-output ratio equal to 1.69 in annual terms, which is certainly small when compared to the often used values). The fraction of production dedicated to education is equal to 6%, again a number on the low side for the US but not for many European countries such as Spain.

Assume the same initial conditions for both economies and assume they are already in their balance growth path from the start. Denote with hat symbols the variables in the economy with an immigration shock in $t = 1$. In Table 1 we

show the change (relative to the case of no shock) in utility and consumption, of middle age and old, caused by an immigration shock in period $t = 1$. Notice, first, that the generations alive when the shock hits consume more than in the economy with $(z_0, z_1, z_2, \dots) = (0, 0, 0, \dots)$, because output is much higher during that period and the insurance mechanism redistributes this extra income to both middle age people and old. The future generations, nevertheless, are worse off in the economy with $(z_0, z_1, z_2, \dots) = (0, \bar{z}, 0, \dots)$ because, as pointed out before, they could not purchase insurance against the immigration shock before being born. The immigration shock affects future generations negatively through two channels. First, because the increase in labor supply reduces per-capita wages in a way that is not compensated by the increase in the marginal productivity of capital that the abundance of labor in future periods entails. Second, because the new immigrant workers need to be endowed with productive capital, and this requires additional investment, which reduces consumption for at least one period. More precisely, the mechanism at work is the following.

Our is a model of endogenous growth driven by constant returns to scale in the two reproducible factors, physical and human capital. The long run growth rate does not depend on the size of the economy, which increases after the immigration shock, but only on the technology and preferences parameters. Once the labor supply increases, equilibrium dictates that an extra amount of output must be allocated to endow these workers, and their offsprings, with physical and human capital in all periods following the one of the shock. Hence, saving rates must temporarily increase until the factor proportions reach again the balanced growth levels, after which growth resumes at the same rate as in the economy without shock. Because the initial extra-investment reduces consumption for two periods, after which consumption grows at the same rate as in the original equilibrium

path, the new path will always stay below the original one. Hence, consumption levels, while growing at the same rate, will be lower forever. This explains the lower steady state utility for the representative agent in the economy with the immigration shock. Put it differently: because the immigrants come without physical capital (and less human capital) this must be provided by the natives, who give up some consumption during the adjustment periods. Once the adjustment is completed (which takes three periods, counting the one with the shock) in our calibration, growth resumes at the same rate but at a consumption level that is lower forever.

In table 2 we show the effect of the immigration shock on the annual interest rate, wage and annual growth rate of aggregate output. As just argued, the immigration shock has a positive effect on the investment rate in the period in which the immigrants arrive and a temporarily negative (positive) effect on labor (capital) productivity. After the adjustment is completed, the growth rate and the marginal productivities resume their original long-run levels.

Table 1: Change in life-cycle utility and consumption

t	$\hat{U}_{t-1} - U_{t-1}$	$\hat{c}_t^m(z)/c_t^m(0)$	$\hat{c}_t^o(z)/c_t^o(0)$
0	0,0035	1	1
1	-0,006	1,0039	1,0039
2	-0,121	0,9401	0,9883
3	-0,126	0,9363	0,9363
4	-0,127	0,9353	0,9353
5	-0,127	0,9350	0,9350
6	-0,127	0,9350	0,9350

Table 2 Changes caused by an immigration shock in $t = 1$.

t	$\hat{r}_t$ annual	$\hat{\omega}_t$	$\hat{g}$ Y annual
0	0,0407	0,2500	0,03004
1	0,0453	0,2361	0,03660
2	0,0402	0,2517	0,03015
3	0,0406	0,2504	0,03007
4	0,0407	0,2501	0,03004
5	0,0407	0,2500	0,03004
6	0,0407	0,2500	0,03004

3. Equilibrium when credit and insurance markets are missing

This will be added, but it is substantially similar to what we find in Boldrin and Montes [REStud 200?]. The only variation is that now we have uncertainty, hence one wants to analyze separately what happens when insurance markets are not available from what happens when also the lending/borrowing markets are not available and the only possible investment is to purchase the capital stock. This gives rise to the following two cases.

1. Young people cannot trade in $A^y(z)$, still they can borrow to invest d in human capital, but must repay at the same rate with or without immigration shock next period. In this case, even if the middle age people could try to trade in state contingent $A^m(z)$ assets, it would make no sense. The only entity they can trade those assets with is the firm, which cannot insure them against anything as it has no compensating source of resources in bad states. The income of old age people, therefore, becomes random and is equal to the fixed return on d plus the random return on capital investment. Young people bear quite a bit of risk because, once they become middle age, they must pay a fixed amount $d(1+r)$ no matter what the state of the world is. This implies that, when there is an immigration flow, middle age natives have less income than in the complete markets model to allow for consumption and savings. As in the world with complete markets, their wage bill is lower (marginal productivity of labor decreases due to arrival of immigrants), but their debt payment is now higher, which leaves less for $c^m(z_t) + \sum_{z \in Z} q(z, z_t) A_t^m(z)$. Total investment, then decreases.
2. Young people cannot borrow at all, hence middle age people can only invest in the physical capital. Obviously this implies that there is a much lower level of human capital in the economy, and there is no growth. Leaving this aside, this is the case in which "workers" bear the downside of the risk (i.e. they either do "normal" or do "worse") whereas the owners of capital bear all the upside (i.e. they either do "normal" or do "better").

4. The Welfare State of a Closed Economy

Let us start with the case 2. of the previous section, in which all credit and insurance market other than the market for physical capital have been shut down. In this case both, $F_H(K_t, H_t)$ and $F_K(K_t, H_t)$, are affected by the shock and neither one of the two factor owners can manage to insure against it. We want to derive policies that are able to implement the complete market allocation (CMA) of Section 2. They turn out not to be very different from those derived in Boldrin and Montes [REStud 2005], apart from the fact that contributions and benefits are now state contingent. With uncertainty we need to use the welfare state to also allocate risk efficiently between generations and not just, as in the deterministic case, to allow for intergenerational trade. Think of what happens when there is an unexpected flow of immigrants ($z = \bar{z}$): the marginal productivity of labor decreases and the marginal productivity of capital increases. If we simply levy a social security contribution in the amount $t_t^p = d^* R^*$ and nothing else, the per capita income of the middle age individuals decreases compared to the CMA. In this class of models, immigration causes a redistribution from working to retired people; in general it redistributes from labor to capital. Furthermore, the decrease in net labor income will tend to reduce workers' saving, implying an under-investment in physical capital compared to the CMA.

There can be gains from allowing the middle-age and old generations to share the immigration risk. We should stress here a relatively delicate point: an immigration shock causes aggregate uncertainty (it increases aggregate output) but, because in a competitive world it affects the two factors of productions differently, part of this uncertainty is insurable. In particular, the native workers face the risk of a reduced per capita income, while the native capital owners face the risk of an increased per-capita income. On the other hand, if there is no immigration, the native workers earn a higher per capita salary, while the capital owners earn a smaller share of output. Insurance, then, must work the following way: when there is immigration the old people (owning capital) pay something to the native middle age people, viceversa in those periods in which there is no immigration. We, therefore need to add a policy emulating the way in which intergenerational insurance markets would work.

Assume a period-by-period balanced budget and introduce two tax and transfer schemes similar to those studied in our earlier work; we call the first a "pension scheme" and the second an "education scheme". Write

$$t_t^p(z) N_{t-1}^y + \bar{t}_t^p(z) z N_{t-1}^y = b_t(z) N_{t-2}^y + \bar{b}_t(z) z_{t-1} N_{t-2}^y,$$

for the pension scheme, and

$$t_t^e(z) N_{t-1}^y + \bar{t}_t^e(z) z N_{t-1}^y = e_t(z) N_t^y,$$

for the education scheme. Let us start from the last equation. Here $e_t(z)$ denotes the educational transfer received from each member of the currently young generation. On the other side of the budget constraint, we find the contributions provided, respectively, by the middle age natives ($t_t^e(z)$) and by the middle age immigrants ($\bar{t}_t^e(z)$). In the optimal policy, we treat working immigrants differently from working natives, as they receive different wages in light of their different human capital. On the other hand, the optimal policy dictates treating all young people alike, those born to immigrants and those born to natives.

The budget constraint for the pension scheme can be interpreted similarly, but here we need treating natives and immigrants differently on either side. They pay different contributions ($t_t^p(z)$ and $\bar{t}_t^p(z)$, respectively) and receive different benefits when retired, $b_t(z)$ and $\bar{b}_t(z)$. Again, this mimics what would have happened in an economy like that of section 2, where markets were dynamically complete. The important point to note here is that, in both schemes, the contribution and benefit rates are state contingent, i.e. change depending on the immigration flow. The latter is an aggregate variable, hence the state contingent policy does not depend on any private information but on a state variable that should, at least in principle, be observable by the policy maker.

Under these policies, the budget constraints for the representative member of the generation born in period $t - 1$ become

$$\begin{aligned} d(z_{t-1}) &\leq e(z_{t-1}) \\ c^m(z_t) + s(z_t) &= \omega(z_t) h(d(z_{t-1}), h_{t-1}^y) - t^e(z_t) - t^p(z_t) \quad \forall z_t \in Z \\ c^o(z_{t+1}) &= s(z_t) R(z_{t+1}) + b(z_{t+1}) \quad \forall z_{t+1} \in Z \end{aligned}$$

For an immigrant arriving in period t , the budget constraints read

$$\begin{aligned} \bar{c}^m(z_t) + \bar{s}(z_t) &= \omega(z_t) \gamma h(d(z_{t-1}), h_{t-1}^y) - \bar{t}^e(z_t) - \bar{t}^p(z_t) \quad \forall z \in Z \\ \bar{c}^o(z_{t+1}) &= \bar{s}_t R(z_{t+1}) + \bar{b}(z_{t+1}) \quad \forall z \in Z, \end{aligned}$$

The symbol $s(z_t)$ is the investment in physical capital an individual makes in period t and state z , and $R(z_{t+1}) = \varphi(z_{t+1})/p(z_{t+1})$. If we set $e(z_{t-1}) = d^*(z_{t-1})$ (starred symbols for now on refer to the CMA), *i.e.* we transfer educational

resources to the young generation up to the point at which the expected return on education is equal to the expected return on physical capital,

$$\sum_{z \in Z} \pi(z|z_{t-1}) F_K(K_t, H_t(z)) = \sum_{z \in Z} \pi(z|z_{t-1}) \omega_t(z) h_d(d(z_{t-1}), h_{t-1}^y)$$

we reach the efficient level of human capital in period t . In Boldrin and Montes (2005) we show that in a deterministic world this policy, together with $t_t^p = d^*(z_{t-1})R^*(z_t)$, $\bar{t}^p(z_t) = 0$, $b_t = t_{t-1}^e R^*(z_t)$ and $\bar{b}_t = \bar{t}_{t-1}^e R^*(z_t)$, implements the efficient CMA overall. Pension benefits received (social security contributions) must correspond to the capitalized value of the lifetime contributions to aggregate human capital accumulation (educational services received).

Comparison the last budget restrictions with the budget restrictions in the CMA of section 2 (2.a) – (2.d) shows that, if the lump-sum tax-transfer amounts satisfy

$$t^p(z_t) = A_{t-1}^{y*}(z_t); \bar{t}^p(z_t) = 0,$$

$$b(z_{t+1}) = A_t^{m*}(z_{t+1}) - \frac{\lambda K_{t+1}^*}{N_{t-1}^y} F_k^*(z_{t+1}); \bar{b}(z_{t+1}) = \bar{A}_t^{m*}(z_{t+1}) - \frac{(1-\lambda) K_{t+1}^*}{z_t N_{t-1}^y} F_k^*(z_{t+1}),$$

and

$$t^e(z_t) = \tilde{A}_t^{m*}(z_t) - \frac{\lambda K_{t+1}^*}{N_{t-1}^y}; \bar{t}^e(z_t) = \tilde{\bar{A}}^m(z_t) - \frac{(1-\lambda) K_{t+1}^*}{z_t N_{t-1}^y},$$

where λ is the portion of aggregate investment in physical capital that it is made by a native people in the CMA, the competitive equilibrium under the new policy achieves the CMA. Important to note that our efficient pension system achieve the efficient investment in physical and human capital related to the CMA and therefore the "efficient" crowding-out of private saving.

A benevolent planner can restore efficiency, improve long-run growth rates, and preserve intergenerational fairness by establishing a correct design of public financing of education *and* pay-as-you-go pensions system. In this sense, the combination of both system can substitute for some missing or imperfect private credit and insurance markets.

4.1. Example 1 (continue)

People carry-out their saving decision during middle age. In the CMA the return on the saving of the middle age generation is $E_t \varphi_{t+1}^*(z) / p_{t+1}(z)$.

TO BE COMPLETED

5. The Welfare State of an Open Economy.

How should the previous analysis be altered for the case of an open economy? It is easy to realize that it may change completely if capital mobility is both instantaneous and perfect, as capital will flow into the country at the same time at which immigrants flow, so as to restore equality between the internal rate of return on capital and the one established on the international capital markets. When this is the case, the immigration shock has no economic relevance as neither the wage of the native workers nor the return on capital of the native capital owners will be affected by the arrival of new workers. Efficient and perfectly frictionless capital markets may act, in this context, as insurance devices rendering the state contingent assets essentially redundant. This is a somewhat interesting result as it suggests that, in the light of the simulations presented earlier, the Spanish trade deficit was beneficial, in terms of consumption and overall utility, to both the native households and the immigrant ones.

This observation helps explaining, at least in part, what we have observed happening in Spain during the last twelve years or so: as the flow of immigration into the country continued and even increased, Spain external trade deficit ballooned while productivity showed little signs of increasing, if at all. Capital flew into Spain, if not at the same rate at which labor was entering, certainly at a very high rate, thereby preventing the K/H ratio from falling and the real wage rate with it. Analysis of the actual data is obviously difficult, not to say impossible, by means of a model as simplified as this, in which there is no distinction between durable and non durable goods and one period lasts roughly thirty years of which, since the immigration shock first hit, we have observed at most half.

Nevertheless, a simple back of the envelope calculation may help us estimate how much the immigration shock helps understanding the Spanish trade deficit of the second half of the 1990s and, especially, of the eight first years of this century. Assume, therefore, that capital flew into Spain at roughly the rate needed, year after year, to keep the internal K/H ratio constant.

TO BE COMPLETED

The bottom line of the calculation is the following. In Spain the K/Y ratio without housing is 2.8, and higher than 4.0 with housing.

Employment in 1996 was still about 13 million (a little less) today it is 20 M, of which 3 M are immigrants. The K/Y ratio has not changed much since (slightly increased). Hence, respect to the original work force the immigrants

added almost 24%, while they are about 15% of the current work force. Take a number in between to account for the fact that this took place over, roughly, a decade (in fact less).

This implies that, if (i) the immigrants come without any K, (ii) the saving rate of the natives remains constant (it roughly did); (iii) the final K/L and K/Y ratios for immigrants is similar to those for natives, there would be the need to borrow from abroad the resources needed to increase the original stock of K of about 20%. In fact, the number is larger, because we have, on top of the 3M immigrant workers, about other 4M native workers that become employed and were not such, at least officially, before. The problem with these are more complicated as part of them were clearly underground, part had accumulated savings they invested in their own K, and so on.

Bottom line, assuming that about 20% of the new stock of capital had to be imported, in the face of constant saving rates, is a reasonable lower bound. Given the K/y ratios, this implies that a (cumulated total) between 56% and 80% of the Spanish GNP had to be imported, everything else equal. The trade deficit, in percentage of GNP, adds up to 50.1% between 1995 and 2007. Hence, we are pretty much nailing it.

In other words, the trade deficit and borrowing from abroad was a substitute for the missing internal insurance markets.

6. Conclusions

TO BE ADDED.

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