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Abstract

Governments often impose choices regarding the levels of savings and the composition of the portfolio of assets in mandatory pension systems; either the share of pay-as-you-go vs. financial assets or the structure of default portfolios to which a majority of workers stick. Yet, it is well known that the optimal savings rate and the structure of the portfolio of assets depend on individual preferences and the properties of human capital. For example, workers whose labor income is very volatile or is highly correlated with the returns on risky financial assets should tilt their portfolios towards safe assets early in life. In this paper we explore the potential welfare gains derived from incorporating this basic principle into the design of the default portfolios offered by DC pension plans, based on the case of the Chilean pension system. We estimate the properties of labor earnings for several representative individuals, simulate their optimal life-cycle portfolio choices and compare with the current institutional defaults. We find very sizable welfare improvements for several of the groups of workers studied. The results suggest that policymakers should take into account education and occupation when defining portfolio defaults. These principles apply more generally to the choice between pay-as-you-go vs. financial assets – and we argue – could improve incentive for some groups to contribute.

Keywords: Pension system, optimal retirement portfolio, Chile

JEL-class: H55, G11

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1. Introduction

The majority of countries, regardless of income, have implemented mandatory pension programs. One of the common justifications for these programs is that, left to their own devices, individuals would not save enough for retirement. This is either because they have inconsistent time preferences, because it is difficult to have the discipline to commit to a given savings plan, or simply because they lack the information and skills to make good investment decisions (see, for example, Benartzi and Thaler, 2007). As a result, most governments have set up legislation that mandates individuals to join and contribute to public pension plans. Although there is a large heterogeneity in how these plans are designed and implemented, they always involve two critical choices: setting the targeted income replacement rate for a given retirement age and a given number of years of contributions; and defining how savings can be invested. In pure pay-as-you-go pension plans savings are mostly invested in pay-as-you-go assets (e.g., Brazil), in funded plans savings are invested in diversified portfolios of financial assets (e.g., Chile), whereas in “multi-pillar" systems savings are allocated to both types of assets (e.g., Sweden and Poland). In the case of financial assets, there are also restrictions regarding the types of securities that can be purchased and more and more countries are introducing default portfolios --to which most individual stick.

An important policy question is how efficient are these choices. Having a single mandatory savings rate that applies to individuals with different –unobservable-preferences already suggests that, for many, joining the public pension plan can be welfare decreasing (even after taking into account “myopia”), particularly if there are imperfections in financial markets that make it difficult for people to dissave. Making universal choices for the portfolio of pay-as-you-go and financial assets can further aggravate the problem and provide incentives for evasion and informal work. Indeed, the optimal assets portfolio depends not only on preferences, but also observable individual characteristics that are correlated with life-time labor income.

In this paper we consider these two problems in the case of Chile, paying special attention to asset allocations provided by default portfolios. . Thus, we explore how current default portfolios differ from optimal portfolios and whether there is room to improve their design. Our results suggest that policymakers should consider different default portfolios for males and females with different education attainment and/or professional occupation. These observable characteristics are good predictors of the “riskiness” of the individual human capital (and its correlation with returns on assets) which, in turn, lead to large differences in the structure of the portfolio of assets. An important corollary is that the savings rate and the share of pay-as-you-go vs. financial assets could also be set as a function of observable individuals characteristics. We argue that doing so could improve incentives to enroll and contribute among individuals who today are outside the system by choice (see Perry et al., 2007 and Ribe et al., 2012) either as informal wage employees or high-skilled self-employed.

Our analysis contributes to two strands of literature. It joins a growing body of research focusing on the impact of labor income on portfolio choice exemplified by, for example, Davis and Willen (2002), Campbell and Viceira (2002), Cocco, Gomes, and Mahenhout (2005), and Gomez, Kotlikoff, and Viceira (2008). Our analysis also links to research about the optimal design of defined contributions funded pension systems such as Bovenber et al. (2007) and Miles and Cerny (2006).

The main findings of our work, which display little sensitivity to the degree of individuals' risk aversion, can be summarized as follows:

1. The labor income of the average wage earner in Chile (male, private sector, average education) exhibits a rather low degree of volatility and is largely uncorrelated with the returns of risky assets. Consequently, the optimal default portfolio in their retirement account should have a higher exposure to risky assets. The welfare gains from this change are appreciable and the implicit risk very small, meaning that those who *ex post* suffer with the change could be easily compensated for their losses.

2. For females and public sector workers the conclusion is similar but weaker. The welfare gains are, on average, smaller and the size of the *ex post* losers is larger.

3. Highly educated workers and employers have more volatile income profiles and higher correlation with aggregate stock returns. As o result, the optimal portfolio should be more "conservative" than that provided by the current legislation. This reallocation, however, would generate a large number of *ex post* "losers" (i.e., workers who experience a favorable combination of aggregate and idiosyncratic shocks and may, consequently, regret the prudence embedded in the *ex ante* optimal portfolio). The average gains are also smaller than in the benchmark case (the average wage earner). This then emphasizes the importance of combining default options with information to plan members about the tradeoffs involved when choosing among different portfolios.

The paper is organized as follows. Our model of optimal life-cycle choices is explained in section 2. The empirical procedures followed by its calibration to the Chilean case are the subject of section 3. The simulation results are presented in section 4. We finish with some conclusions and proposals for further research in section 5. The more technical parts of the paper are confined to a set of dedicated appendices.[1]

2. A model of life cycle behavior

We model the "rational" consumption and portfolio allocation of individuals that operate in an uncertain environment. Rationality in this context means that people's preferences are represented by an additively separable utility function (taking absolute consumption as its argument) and that choices are made so as to maximize the expected discounted value of that lifetime utility. More precisely, individuals maximize:

$$U_{t0} = E_{t0}\left[\sum_{i=t0}^{T} \beta^{(i-t0)}\ S_i\ \frac{C_i^{1-\eta}}{1-\eta}\right]$$

where time is indexed by age, i, $t0$ is the age of entrance in the labor market, T is the

[1]Appendix 1 explores the link between labor income and portfolio choice, reviews the details of the life-cycle model and present a sensitivity analysis. Appendix 2 provides a brief overview of the Chilean pension system and its calibration for our numerical experiment. .

maximum lifespan, β is a pure time discount factor, S_i is the probability of surviving to age i (conditional on surviving to $t0$), C_i is age-i consumption and η is the degree of relative risk aversion (assumed constant). Note that we omit the dependence on calendar time to ease notation.

Each year before retirement (assumed to take place at the exogenous age R), individuals receive labor income in the form of an exogenous and stochastic gross salary Y_i. As is standard, we decompose Y_i into the product of a permanent, P_i, and a transitory, Θ_i, component ($Y_i = P_i\,\Theta_i$).[2] The transitory income shock is lognormally distributed, with unitary mean and independent of the other sources of uncertainty in the model. The permanent labor income component is obtained as follows:

$$P_{i+1} = \Gamma_{i+1}\,\Psi_{i+1}\,P_i$$

where Ψ_i is a mean-one, lognormal shock to permanent income and the deterministic factor Γ_i reproduces the average growth rate of labor earnings at age i.[3] The log of the permanent component, then, is the sum of a persistent shock and a deterministic factor calibrated to reproduce the average profile of life-cycle earnings in the population (which is typically humped shape). The persistent shocks can be correlated with the stochastic return of risky assets in the economy described below (see, for example, Cocco et al (2005) for a discussion of the use of this particular model of labor income in the literature of life-cycle consumption and portfolio choice).

We also take into account that the *disposable* labor income of the individual differs from the *gross* income due to the outflows derived from two compulsory government programs: the social contributions (to the individual retirement account, IRA, described in the next paragraph), and the general income taxes. For simplicity, the latter takes the form of a pair of constant tax rates, (τ and τ_R), applied, respectively to labor and capital income.

Throughout his life, our representative individual is enrolled with the Public Pension System[4], which is *fully funded* and *defined contributions*. Each individual pays social contributions into his/her account before retirement, chooses the composition of the portfolio of assets in the account and collects the resulting pension benefit after retirement.[5] The compulsory contributions can be invested in two mutual funds: a riskless one that pays R_f with certainty, and a risky one with stochastic gross real return$R$. We assume R to take the form of independent lognormal shocks with mean $E[R]$ and variance σ_R^2. We assume that the innovations to R can be correlated with the permanent component of labor income, denoting their covariance by ρ. The proportion of the portfolio allocated to risky assets at age i is represented by ω, and the value of the stock of assets accumulated in the IRA at the same age is represented by Π_i. The annual contribution rate, ς, is invariant during the life of the worker. Note that the accumulation of pension wealth depends on the particular realizations of the gross-income process of the individual *and* (whenever $\omega_i > 0$) on the sequence of aggregate returns of the risky asset:

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[2]We follow the notation in Carroll (2009).

[3]Formally, we assume $Log\,\Theta \sim \mathcal{N}(-\sigma_\theta^2/2, \sigma_\theta^2)$ and $Log\,\Psi \sim \mathcal{N}(-\sigma_\Psi^2/2, \sigma_\Psi^2)$. These assumptions guarantee that $E(\Theta) = 1$ and $E(\Psi) = 1$, implying that the average growth rate of the process is set by Γ_i.

[4]As mentioned in section 1, we do not consider the (serious) issue of non participation in the pension system in this paper.

[5] Note that the individual is not allowed to withdraw funds from his/her pension account until retirement, even in presence of unfavorable labor income shocks. For simplicity, we assume that the claim of the pension and the retirement from the labor force take place at the same time.

$$\Pi_{i+1} = (R_f + (R_{i+1} - R_f)\,\omega_i)\,\Pi_i + \varsigma\, Y_{i+1}$$

The individual has a second source of liquid income in the principal and interest obtained from the stock of previously accumulated private savings A_{i-1}. *Cash-in-hand* or the total income available for consumption and savings at age i is represented by M_i. It evolves according to the simple budget constraint:

$$M_i = (1 - \tau_R)\, R_f\, A_{i-1} + (1 - \tau)\,(1 - \varsigma)\, Y_i$$

Note that, while M_i represents the funds available at the start of the period, A_i represents the end-of-period stock of assets (i.e., those remaining after the consumption decision have been made, $A_i = M_i - C_i$). Private liquid savings are assumed to be held in deposit accounts that pay the riskless rate R_f. This strong assumption captures well the observed behaviour of a large part (but by no means all) of the targeted population of this study .

After retirement, the individual problem is substantially simplified. The pension wealth accrued in the IRA is converted into an individual pension benefit, B, according to the norms of the pension system (discussed in section 1). The pension payment takes the form of a constant annuity, computed according to the average survival probabilities of the individual. For married men, the annuity is a joint annuity that provides the surviving widow with a certain percentage of the pension of the male worker. We thus model a deterministic flow of income during the retirement stage.[6]

In this work we do not let the individual take “short” or leveraged positions in any of the two available mutual funds (i.e. we impose $0 \leq \omega_i \leq 1$) and also prevent the individual from borrowing from future income at any age (ie. the balance of the deposit account should always be positive $A_i \geq 0$). This is especially important after retirement to avoid the risk of dying with outstanding debts.

2.1. Recursive formulation of the problem

As usual in the life-cycle literature, the individual problem is too complex to be solved analytically and we must use recursive computational methods instead. Each period, the state of the individual under study is characterized by the vector $\{i, M, \Pi, P\}$. To solve the model we must express the consumption and portfolio choices as functions of that state vector: $C(i, M, \Pi, P), \omega(i, M, \Pi, P)$. This is achieved by solving the Bellman equation of the problem:[7]

$$V_i(M, \Pi, P) = \max_{\{C,\,\omega\}}\; u(C) + \beta\, S_i E[\,V_{i+1}(M', \Pi', P')] \tag{1}$$

$$M' = (M - C)\,(1 - \tau_R)\, R_f + (1 - \varsigma)\,(1 - \tau)\, Y'$$

$$Y' = P'\,\Theta' \tag{2}$$

$$P' = \Gamma_{i+1}\,\Psi'\,P \tag{3}$$

[6]Health shocks are a potential source of variability in (net) income at advanced ages, but we do not include them is this study. We also abstract from the existence of a bequest motive for saving in our representative individuals.

[7]Our formulation has the advantage that the *value function* is homogeneous with respect to the permanent income P. As a result, the Bellman equation can be reformulated with all variables expressed as ratios to the current value of the permanent shock, P. This reduces the dimension of the problem, making the transformed problem significantly easier to solve. See appendix B for more details.

$$\Pi' = (R_f + (R' - R_f)\,\omega)\,\Pi + \varsigma\,Y' \quad (4)$$

where future variables are denoted with a prime. A discussion of the first order conditions of the problem, the properties of the optimal behavioral rules and the numerical technique employed to solve the model (Carrol (2006) endogenous grid point algorithm) can be found in appendix B.

3. Calibration of the illustrative examples

In this section we apply the theoretical life-cycle model of the previous section to the Chilean economy and pension system. The Chilean system of individual pension accounts is one of the most sophisticated in the world (see Appendix 2 for a description). Among its many features, the system provides its affiliates with age-varying defaults for their asset allocation (see Berstein et al., 2011b). These defaults are followed by almost 70% of the workers enrolled and are already conditional on individual characteristics (the gender of the affiliate). As discussed above, our aim is to explore the convenience of expanding the set of observable characteristics that are explicitly considered when designing default portfolios.

We work with a stylized version of the public pension and tax systems and the set of assets available to transfer income across time and states of nature. All our representative individuals operate in the same environment and share a common set of mortality probabilities and preferences over life-cycle consumption and leisure (see Table 1 for a summary of the parameters defining institutions and preferences). They only differ in the properties of the stochastic process underlying the dynamics of their labor income. Differences in these processes are important enough to generate a large variation in the set of optimal individual decisions.

τ	& Income tax rate	0.22
ς	Contribution rate	0.1
R	Retirement age	65
B_m	Minimum Pension (% average labor income)	0.08
	Asset returns	
$R_f - 1$	Riskless rate (Multifondo E)	5.35
$E[R] - 1$	Mean return of risky assets (Multifondo A)	8.30
σ_R	Standard deviation of risky assets (Multifondo A)	0.209
	Preferences	
η	Risk aversion	5
β	Discount factor	0.98

Table 1. Parameter values used in the modeled economic environment

Survival probabilities are taken from the Human Mortality Database (2011) and correspond to the year 2000. For preferences, the parameters' values selected for our benchmark case are intended to result in standard life-cycle behaviors. The pure time discount factor, β, is set to reproduce the ratio of private assets to labor income at the age of 60, according to the 2006-wave of the EPS survey.[8] More specifically, we set β to the value that generates a median A/Y at 60 close to the median of the private wealth-to-income ratio for workers with average education.[9] As indicated in the bottom rows of Table 1, the resulting individual is remarkably patient; he/she discounts the future at a 2% annual rate. The degree of risk aversion, η, was set to a value 5 which is common in simulation analysis of individual portfolio choice.[10] Such a degree of risk aversion is also quite close to values used in the macroeconomics literature (which, typically, assume a slightly higher degree of inter-temporal substitution). In appendix 1 section C we explore the stability of our main findings to changes in this parameter value.

The characterization of the properties of the life-cycle labor income flows is a centerpiece of the calibration exercise. There is extensive evidence in the economic literature of large differences in the properties of labor income across different groups of individuals. The systematic exploration of the data has been done mainly for developed countries, but there is also a growing literature on developed economies.[11] These countries have, in general, less developed *formal* social protection networks, which can result in larger volatility of labor income and more intra-cohort heterogeneity. For Chile, there have been few systematic attempts to assess the differences in the labor income processes of different social groups. The analysis in Repetto and Huneeus (2005), also based on ESI data, may be the most outstanding example.[12] In our analysis we use a broad definition of gross individual income, which includes wages, pensions, government benefits and income from individual business. We also do not limit ourselves to currently employed workers, reflecting in this way the volatility in income induced by unemployment spells. We undertake separate estimations according to three observable characteristics: education (disaggregated in three groups), gender, and type of work defined by sector of occupation (private vs. public) and type of labor relation (wage earners vs. employers or self-employed). We thus have a total of 24 different types of workers. In this section, however, we limit ourselves to only five "representative" cases.

We take as our benchmark a male of average education working in the private sector as a wage earner (see first row in Table 2). This is the type of worker most frequently observed in our sample.[13] The second profile is that of is a female counterpart of our benchmark case (referred as "Fml" as indicated in the second row of Table 2). The other three profiles involve a highly educated male employee referred as "HighE," the public sector *alter ego* of our benchmark named "Public," and a representative of employers

[8]The EPS (Encuesta de Protección Social) is a biannual longitudinal survey conducted by Universidad de Chile.

[9]The median value of the ratio is approximately 3.7. The median private wealth (including liquid financial wealth, voluntary retirement savings and housing assets) is 12 million pesos; the average labor income is 3.2 millions.

[10]See, for example, Campbell and Viceira (2002) and Gomez, Kotlikoff, and Viceira (2008). In (Cocco, Gomes, and Mahenhout 2005) the benchmark value is 10, but a simulation with our less extreme value is also provided.

[11]See, for example, Davis and Willen (2002) for the USA. In this country, the PSID (Panel Study of Income Dynamics) has provided the foundations for a large number of life-cycle analysis like Storesletten, Telmer, and Yaron (2004) or Campbell et al. (2001) (or the references therein).

[12]ESI (Encuesta Suplementaria de Ingresos) is a supplement to the annual Labor Survey organized by INE, the Chilean national statistics institute. It is a rotatory panel, with households staying in the sample for a maximum of 6 quarters. The sample is nationwide representative, with a sample size close to fifty thousand observations per annum. We use the information corresponding to years 2001 to 2009.

[13]To guarantee an accurate estimation of the statistical properties of their labor income, we only work with type of workers that are well represented in our estimation sample.

identified as "Emp." We follow standard econometric procedures to estimate the statistical properties of labor income flows for all our representative cases.[14]

Short-name	Gender	Educ	type	Avg	Tran	Perm	Cov
				$\overline{Y}$	σ_θ	σ_Ψ	ρ
Base	Male	Avrg	Wage earner	2.9	0.035	0.040	0.0
Fml	Female	Avrg	Wage earner	2.4	0.073	0.094	0.1
Public	Male	Avrg	Public sector	4.0	0.146	0.039	0.5
HighE	Male	High	Wage earner	9.0	0.077	0.155	0.3
Emp	Male	Avrg	Employer	7.1	0.264	0.051	0.75

Table 2. Characterization of the representative agent by gender, education type of contract and properties of their life-cycle labor-income processes (average, variance of the transitory and permanent components and covariance with the returns of the risky assets).

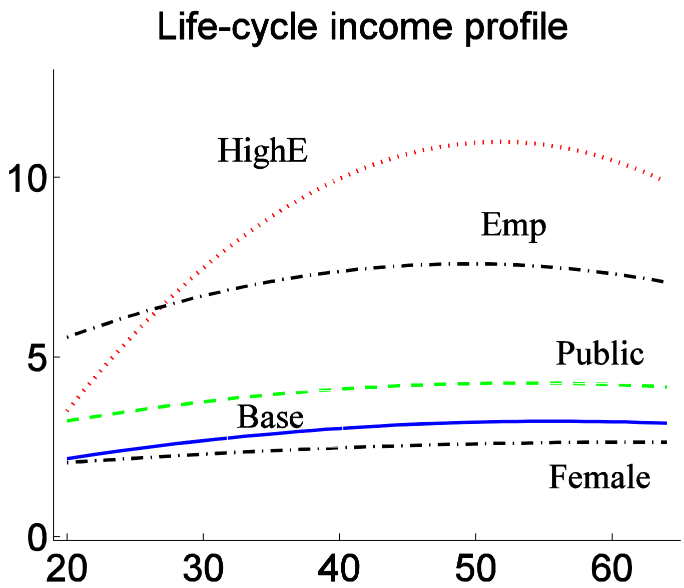

Figure 1. Average labor income by age for our five representative individuals.

Our key findings can be summarized as follows:

4. The life-cycle profiles of labor income display standard dynamics (see Figure 1). All the profiles are "hump-shaped", showing raising incomes early in life that eventually flats out and ultimately declines. There are, however, strong

[14] The predicted component of life-cycle income is estimated by running regressions of the log of gross annual income on a quadratic polynomial. We use the residuals from these regressions to estimate the variance decomposition following the methodology in Carroll and Sanwick (1997). In summary, it proceeds in two steps: First, we compute the variance of the differences in incomes residuals separated d periods, $Var(\hat{y}_{t+d} - \hat{y}_t)$. The permanent/transitory components are then subsequently recovered by using that $Var(\hat{y}_{t+d} - \hat{y}_t) = d\,\sigma_P^2 + 2\,\sigma_T^2$. The correlation of labor income shocks and stock returns is estimated following a very similar approach (see page 208 in Campbell and Viceira (2002)): we construct the excess returns of the risky asset by computing the average annual return enjoyed by "Multifondo" A in the interval 2000/2011 (obtained from the daily data provided by Superintendencia de AFP). ρ is then obtained from an OLS regression of the annual cross-section averages on the constructed excess returns.

differences in the curvature of the profiles. Highly educated workers have the steepest growing profile when entering the labor market, while all others display a more moderate growth rate. This growth rate becomes negative late in life for the highly educated (in the mid 50s) and somewhat earlier for the rest (typically in the mid forties), although the declines are very mild in most cases. Our female and public workers, in particular, have largely flat income profiles. Regarding average income levels the results are not surprising. Compensation is the highest for highly educated wage earners and employers, followed by public sector workers, male wage earners with average education and, finally, female wage earners with average education.

5. We find large differences in the estimated value of the permanent and transitory components of labor income. The permanent component, σ_Ψ, is lower in the case of the public sector workers and higher in the case of wage earners with high education (see the second and third columns in Table 2). As discussed in the previous section and in appendix A, σ_Ψ is expected to be a fundamental determinant of the individual portfolio choice. It is important to observe the much larger degree of volatility (both temporary and permanent) of females vs. males, and the fact that the large variability of the income of employers seems to be mostly transitory.[15]

6. As expected, we find the labor income of employers to be strongly correlated with asset returns, which are themselves strongly procyclical (see rightmost column of Table 2 where assets refer to “Multifondo” A). Similarly, highly educated workers show a positive correlation, smaller than that of the employers but still larger than that of our base case (both male and female). Public employees have also a positive correlation, which is not surprising, but the level found (0.5) is larger than expected. As with the variance of the permanent component, standard theoretical results (see appendix A) suggest a strong inverse relation between ρ and the proportion of risky assets in the optimal portfolio, making it another critical component of the portfolio choice.

4. Results

This section discusses the results of the calculation of the optimal life-cycle behavior of our representative individuals.[16] We start by describing the optimal portfolio of male wage earners with an average education. Then, we quantify the welfare consequences of following the portfolio default provided by the current pension system rather than the optimal schedule. Finally, we look at the optimal portfolios of our other representative

[15] Exploring these rough stylized facts in more detail is an important aspect that we leave for future research.

[16] In the simulation, we generate one particular time-series of the returns of the risky asset (following the stochastic process described in section 2. This realization covers the entire professional life of each of the individuals of our simulation sample (a cohort of 500 *ex-ante* identical individuals). We then simulate time series of permanent shocks for each of the individuals in our artificial sample, reflecting the (type-specific) correlations between permanent shocks and asset returns. Finally we generate independent life-cycle profiles of transitory income shocks for each of the individuals. We let workers react optimally to their specific shocks and store their resulting life-cycle paths of consumption, portfolio choices and stocks of private and public (IRA) wealth. This process is repeated 25 times in different aggregate scenarios (ie. with different realizations of the common time-series of the returns of the risky asset). In what follows we report medians across individuals and across aggregate scenarios of the different statistics of interest (like optimal portfolios or welfare changes induced by policy changes).

individuals.

4.1. The benchmark individual

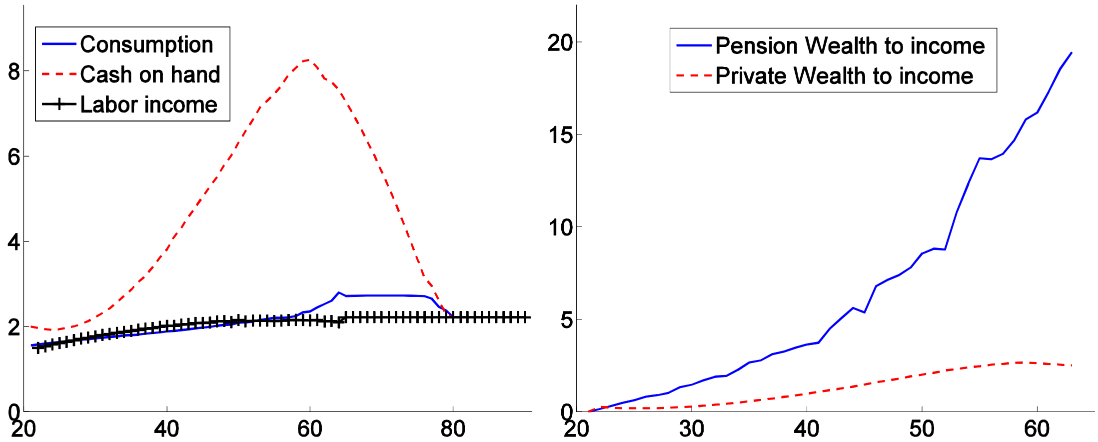

Figure 2. Simulation results for the benchmark individual (wage earner of average education): median life-cycle consumption, labor income and cash-in-hand (left panel); median ratios of accumulated private and pension (IRA) wealth to income (right panel)

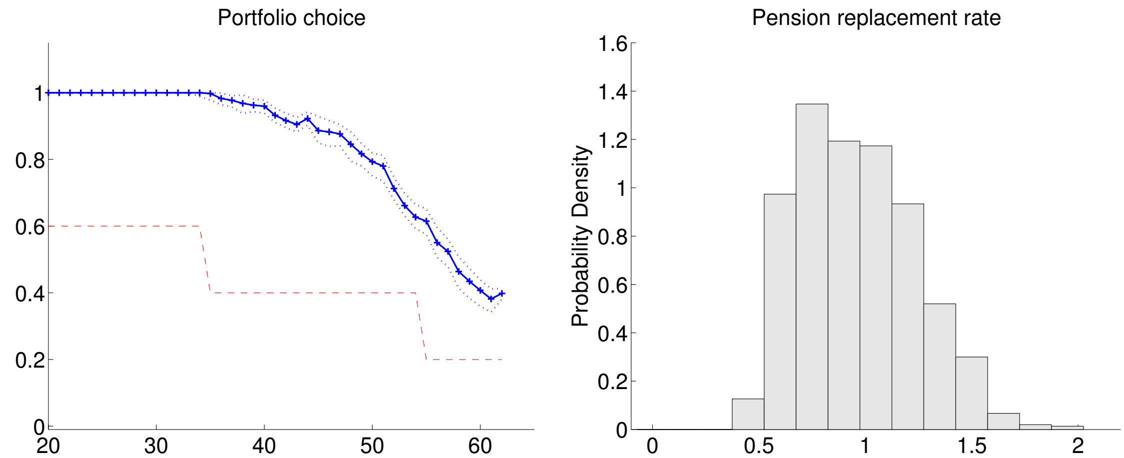

Figure 3. Left panel: median proportion of risky assets in the IRA account of the benchmark individual (blue continuous line), with bands corresponding to the 5% and 95 % percentiles of the distribution of income shocks (blue dotted lines) versus the current institutional default in Chile (red dashed line). Right panel: simulated distribution of pension replacement rates -ratio of the pension benefit to labor income at the age of 64-, under the optimal portfolio rule.

Figures 2 and 3 illustrate the simulation results obtained for the benchmark individual introduced in section 2: a wage earner with average education employed in the private sector. The panel on the left in Figure 2 displays the optimal consumption/savings behavior of the agent. We observe that consumption is close to income early in life and private assets accumulate at a very slow pace (see right panel). This is the usual response to the upward-sloping profile of net labor income: for precautionary reasons, workers build a relatively small buffer stock of assets very early in life, but their saving rates go subsequently down and stay low until the mid-forties. Serious voluntary savings only materialize in the twenty years immediately preceding retirement, with saving rates increasing progressively until they reach a maximum in the mid-fifties. By the time the worker reaches retirement age, he/she has accumulated around 4 years of net income in private liquid assets. The assets accumulated in the IRA are, at the same age, more than five times larger despite the smaller 10% mandate savings rate, which is the result of a higher compound average interest rate of return. This higher return stems from the inclusion of a large proportion of risky assets in the IRA (see left panel of Figure 3). Of course, these higher average returns only come as a compensation for the extra risk incurred, which is apparent in the simulated distribution of the level of the pension obtained upon retirement (see right panel of Figure 3). This pension ranges between less than 50% of pre-retirement earnings to 200%.[17] After retirement, individuals are assumed not to take portfolio decisions (all the liquid wealth stays in the deposit account), and run down their accumulated private assets quite rapidly. Eventually, late in their seventies or early in their eighties, they hit a borrowing constraint (they are not allowed to borrow from future pensions) and simply consume their pension benefit thereafter.

The optimal portfolio choice of our benchmark individual (see left panel of Figure 3) is characterized by a large share of risky assets early in life, followed by a progressive conversion into the safer bonds offered by Fund E. This decreasing life-cycle pattern of ω is consistent with well known principles of life-cycle portfolio choice when human capital is accounted for.[18] As workers get older, human capital constitutes a diminishing proportion of their total wealth. For our benchmark individual, human capital is an order of magnitude safer than risky assets, meaning that their implicit stock of private wealth tilts naturally towards riskier assets as the individual ages (i.e., the share of the risky asset in the total increases). It follows that, to keep a constant proportion of *total* wealth invested in risky assets, workers must progressively reduce the riskiness of their pension savings with age.

Figure 3 also displays the 5% and 95 % quantiles of the simulated optimal portfolio choices. They illustrate the range of different outcomes that result from the accumulation of different income and return shocks throughout the working career of our representative individual. The dispersion increases with time, with very little heterogeneity at the onset of the professional life and a more distinctive widening of the upper and lower bounds in the ten years before retirement. Still, the decreasing pattern for ω is evident.

We find a higher proportion of risky asset in the optimal portfolio than in the default portfolio for the average worker.[19] Here we take as the "recommended" benchmark the default proposed by the Chilean authorities for the workers enrolled in the System of "Multifondos" (see page 4 of Berstein et al., 2011a). This benchmark prescribes Fund B as the default option for (male) workers of up to 35 years of age, Fund C for workers between 35 and 55 years, and Fund D for workers older than 56. Given the legal limitations on the assets composition of the different "Multifondos", we can estimate the proportion of risky assets in the pension portfolio by age. This share is plotted as the step-wise decreasing dashed-line in Figure 3, starting with a 60% share before the age of 35 and ending with just 20% after age 55. This legal default is appreciably more conservative than our optimal portfolio rule, meaning that there can be welfare gains from moving into riskier default portfolios.

4.2. Welfare analysis

To assess the potential gains from a change in the asset allocation default, we compare the utility derived by our representative individuals when they follow the optimal portfolio

[17]To interpret these high replacement rates, recall that we are simulating workers that are permanently enrolled with the pension system.
[18]They are reviewed in section A of Appendix 1.
[19]And, also higher that the proportion currently observed in the EPS data.

rule and when they follow the default portfolio.[20] To focus exclusively on the differences stemming from the portfolio choice, we assume the same saving behavior in both cases (i.e. we apply the optimal saving rates to the simulation with the institutional default).

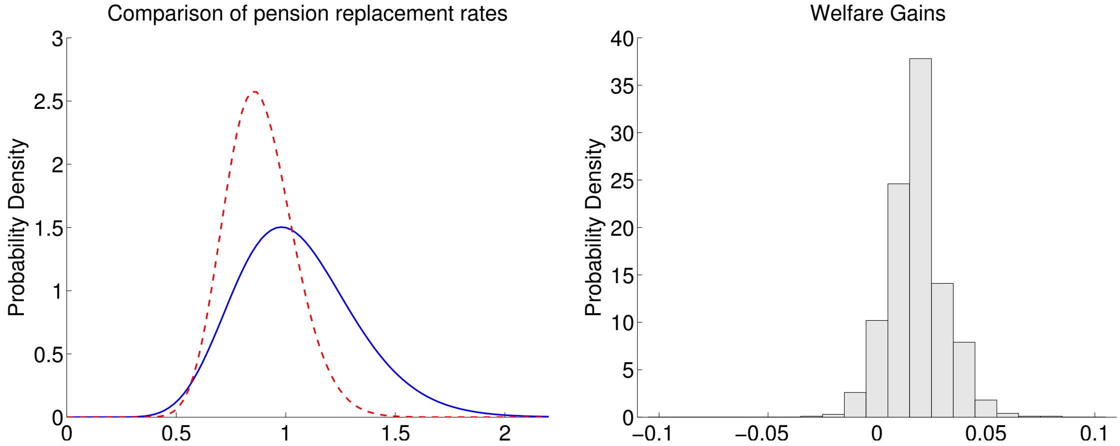

Figure 4. Left panel: distribution of the pension replacement rates obtained under the current portfolio default (red, dashed line) versus those with the optimal portfolio rule (blue, continuous line) for the benchmark individual; Right panel: Distribution of the welfare changes induced by the change in portfolio default for the benchmark individual

Name	Gender	Educ	type	Repl. Rate	Welfare			
					Q5%	Median	Q95%	% losers
Base	Male	Avrg	Wage earner	1.01/0.87	-0.63	1.73	4.45	7.17
Fml	Female	Avrg	Wage earner	1.06/0.92	-1.60	1.00	6.80	20.70
Public	Male	Avrg	Public sector	1.08/0.93	-1.68	1.56	4.42	10.15
HighE	Male	High	Wage earner	0.93/0.87	-57.6	0.06	37.3	47.19
Emp	Male	Avrg	Employer	0.99/0.98	-42.5	0.34	4.04	42.40

Table 3. Simulation results for each representative agent: pension replacement rates (under the optimal vs. current portfolio default) and selected statistics from the distribution of welfare changes induced by the substitution of the optimal for the current portfolio default.

The relevant indicators are the pension replacement rate and the equivalent variation in life-cycle consumption (the standard welfare index used in this literature). For each individual in our simulation sample, the latter is defined as the increase in life-cycle consumption needed to keep the worker indifferent between the current default and the optimal portfolio rule (expressed as a percentage of consumption under the current default, and arranged in such a way that a positive value implies a welfare gain from adopting the optimal rule).[21]

[20] Berstein et al. (2011b) show that the institutional default weights have a very strong impact of the observed real-world behavior.

[21] Formally, for each individual we compute the level of consumption that, if maintained constant throughout his/her life, results in the same expected utility level as that obtained in the simulation. These "certainty-equivalent" consumption levels are computed under the two portfolio rules considered (the current default and our optimal rule), resulting in the utility indexes $\overline{c}_d$ and $\overline{c}_o$ respectively. The welfare index is, simply, $\overline{c}_o/\overline{c}_d - 1$. See, for example, the appendix in Cocco et al. (2005) for a more complete description of this standard welfare metric.

The results for our reference individual are presented in Figure 4 and in the top row of Table 3. The optimal policy rule is more “aggressive”, which naturally leads to larger (average) stocks of pension wealth and, consequently, higher replacement rates. The optimal share of risky assets is, on average, 84% throughout life, going down to 49% after age 55. On average, the accumulated wealth at the age of 65 amounts to almost 22-times current income under the optimal rule, versus around 19-times under the current default. This translates into an average replacement rate of almost 100% under the optimal system versus 87% under the current default. But averages are not enough to assess changes in welfare, given that higher returns come at the expense of additional risk. To check for this, the left panel of Figure 4 illustrates the estimated distribution of the replacement rates.[22] The picture is clear: there is some increase in risk, but it is definitely small. The shift in the density function is very close to a “first order stochastic dominance”.

The distribution of the formal welfare index (the equivalent variation in consumption) confirms the results obtained with the replacement rate (see right panel of Figure 4). On average, the welfare gain is 1.8%, the median gain is 1.7%, and the estimated proportion of welfare losers is 7.2%. The distribution is skewed, with maximum gains (95% quantile) close to 4.5% of annual equivalent consumption and maximum losses (5% quantile) of -0.6%. The order of magnitude of these figures is very close to those found in Cocco et al. (2005) for a similar experiment in the USA. Our interpretation of these results is that, in the absence of changes in the economic environment, a shift from the current default life-cycle portfolio to our proposed optimal portfolio should lead to an appreciable increase in welfare for a vast majority of male wage earners of average education. And, it should be possible to set up a mechanism to compensate the ex-post losers and, therefore, guarantee an across the board increase in welfare.

4.3. Simulation results for alternative types of individuals

Shall we recommend the optimal portfolio found in the previous section as a new default for all affiliated workers? In this section we analyze the optimal portfolio choices of the other individual types. The results are reported in Table 3 and in Figures 5 to 8.

Female wage earners

Female wage earners differ from our base case essentially in that the life-cycle profile of their endowment of human capital is flatter, yields smaller income flows and, more importantly, the volatility of those flows is substantially higher.[23] Our simulation results (Figure 5) indicate that these differences do not add up to much in terms of changes in the optimal portfolio composition of their pension wealth. The extra income volatility results in a flatter distribution of the realized replacement rates (see left-bottom panel of Figure 5) but the optimal life-cycle portfolio choice is very similar: ω is slightly higher despite the higher volatility and the small positive correlation.[24] Overall, women expose

[22] We fit by maximum likelihood a gamma distribution to the simulated replacement rates. The plot displays the probability density functions of the resulting model.

[23] Note that differences by gender in life expectancy are not reflected in this simulation.

[24] This is slightly puzzling as the stock of human capital is, on average, also smaller and stronger precautionary savings should result in a smaller H/A ratio than their male counterparts. Note, however, that in this model the risky stock of wealth is confined to the retirement account, which is accumulated at a constant rate (leading to a smaller stock of “financial” wealth than males). Besides, note that the differences in the timing of the income flows (i.e. the slope of the life-cycle profile of income) also matters for the timing of the portfolio choice.

themselves to a little more risk than men (87 vs. 84 % on average), but follow an almost identical profile of progressive reductions in the riskiness of their IRAs with age. Individuals *ex post* profiles are, however, more disperse (the upper and lower bands in the graph are slightly more distant).

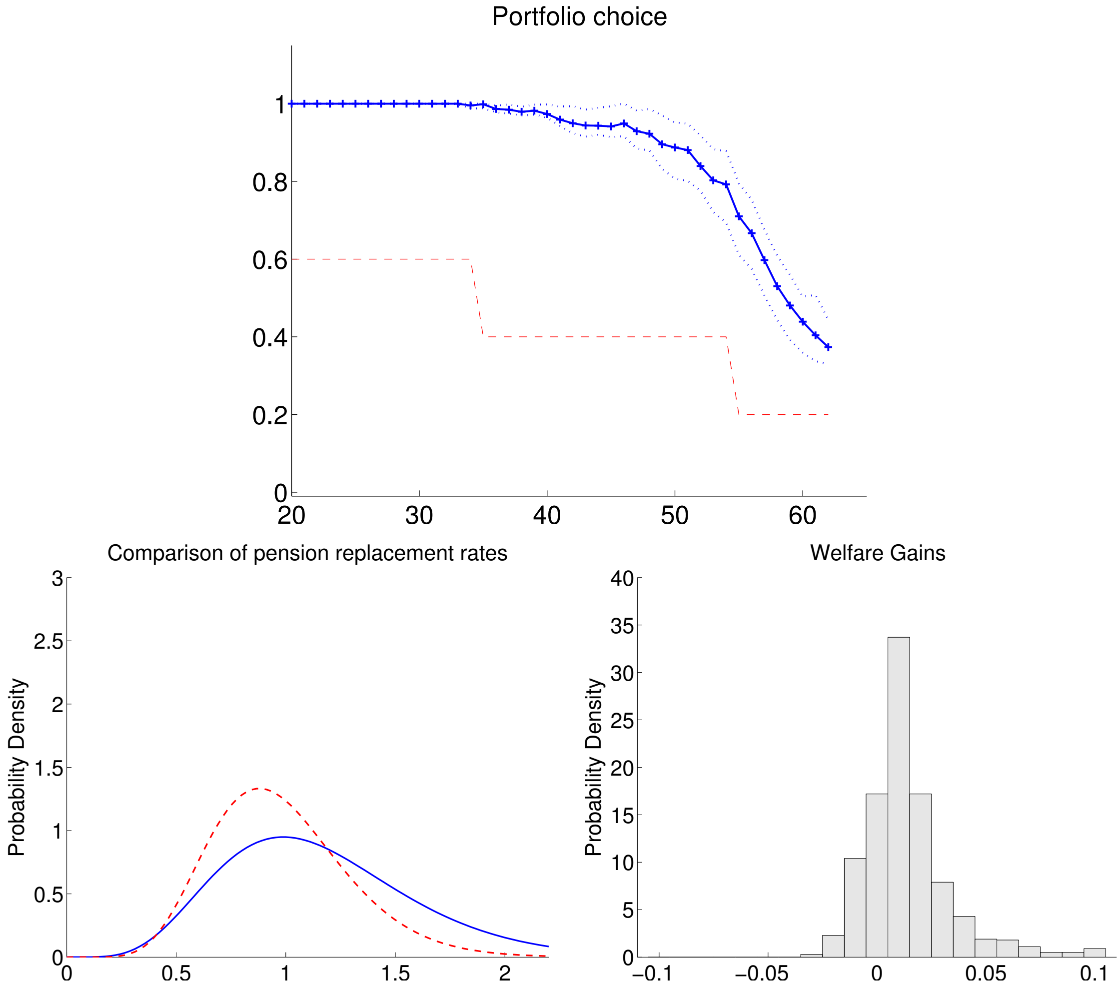

Figure 5. Simulation results for a female wage earner of average education. *Top panel*: median optimal portfolio choice (blue, continuous line) vs. current institutional default (red, dashed line); *Left-bottom panel*: distribution of the pension replacement rates obtained under the current portfolio default (red, dashed line) versus those with the optimal portfolio rule (blue, continuous line); *Right-bottom panel*: Distribution of the welfare changes induced by the change from the current portfolio default to the optimal.

For the welfare experiment we consider a slightly different benchmark than in the base case. Indeed, the Chilean legislation adjusts the default portfolio on the basis of gender – although the differences are small. Fund C is prescribed for workers between 35 and 50 years (vs. 55 for males) and Fund D for workers older than 51 years of age (vs. 56 for males). The results of the simulations are very similar (see second row of Table 3). Changing to the optimal portfolio rule is still welfare improving (1.4% gain on average and 1.0% median gain), but the extra risk of the change is bigger as 20.1% of workers, *ex post,* find themselves worse-off under the optimal rule. Their welfare losses are non-negligible with the 5% lower percentile of the distribution experiencing a 1.6% drop in life-cycle consumption. At the same time, the top 95% of the distribution experiences a 6.8% annual welfare gain. Given the size of the losses, it is no longer obvious that a compensating scheme can be constructed to guarantee uniform welfare gains.

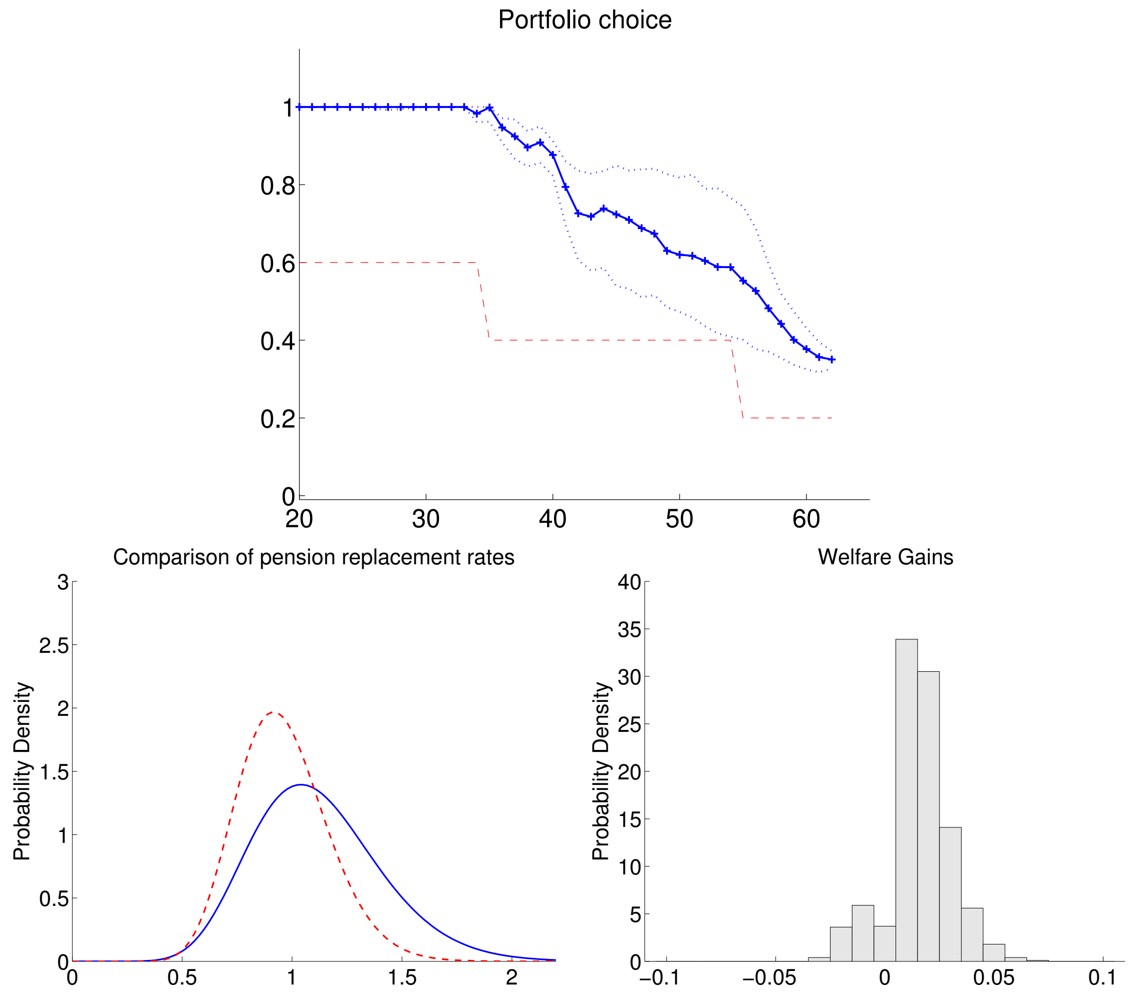

Figure 6. Simulation results for a public sector-wage earner of average education. *Top panel*: median optimal portfolio choice (blue, continuous line) vs. current institutional default (red, dashed line); *Left-bottom panel*: distribution of the pension replacement rates obtained under the current portfolio default (red, dashed line) versus those with the optimal portfolio rule (blue, continuous line); *Right-bottom panel*: Distribution of the welfare changes induced by the change from the current portfolio default to the optimal.

Public workers

Workers employed in the public sectors are also potential candidates for a different portfolio default. On the one hand, the stochastic properties of their labor income suggest the need for a more prudent portfolio composition: their income shocks display a higher permanent volatility and show an appreciable degree of synchronicity with the shocks on asset returns. But, on the other hand, they hold a substantially bigger stock of human capital, which can act as a cushion for unfavorable investment returns. In the end, our simulations (reported in the third row of Table 3 and in Figure 6) indicate that the first factor predominates, leading to a smaller average exposure to risky assets (78% versus 84% in the benchmark). Notwithstanding, the life-cycle profile of the optimal ω is very similar to that in the benchmark case. The main differences appear in later ages, as a result from a higher volatility of income (and a higher correlation with risky returns). Thus, there are important differences in the speed of risk-reduction after the age of 40.

In terms of welfare, the results fall in between the base case and the case of females studied above. The average welfare results are positive (1.6% gain both in median and average annual consumption), but there is more risk than in the benchmark (10.1% of losers and a 1.7% loss for the 5% percentile).

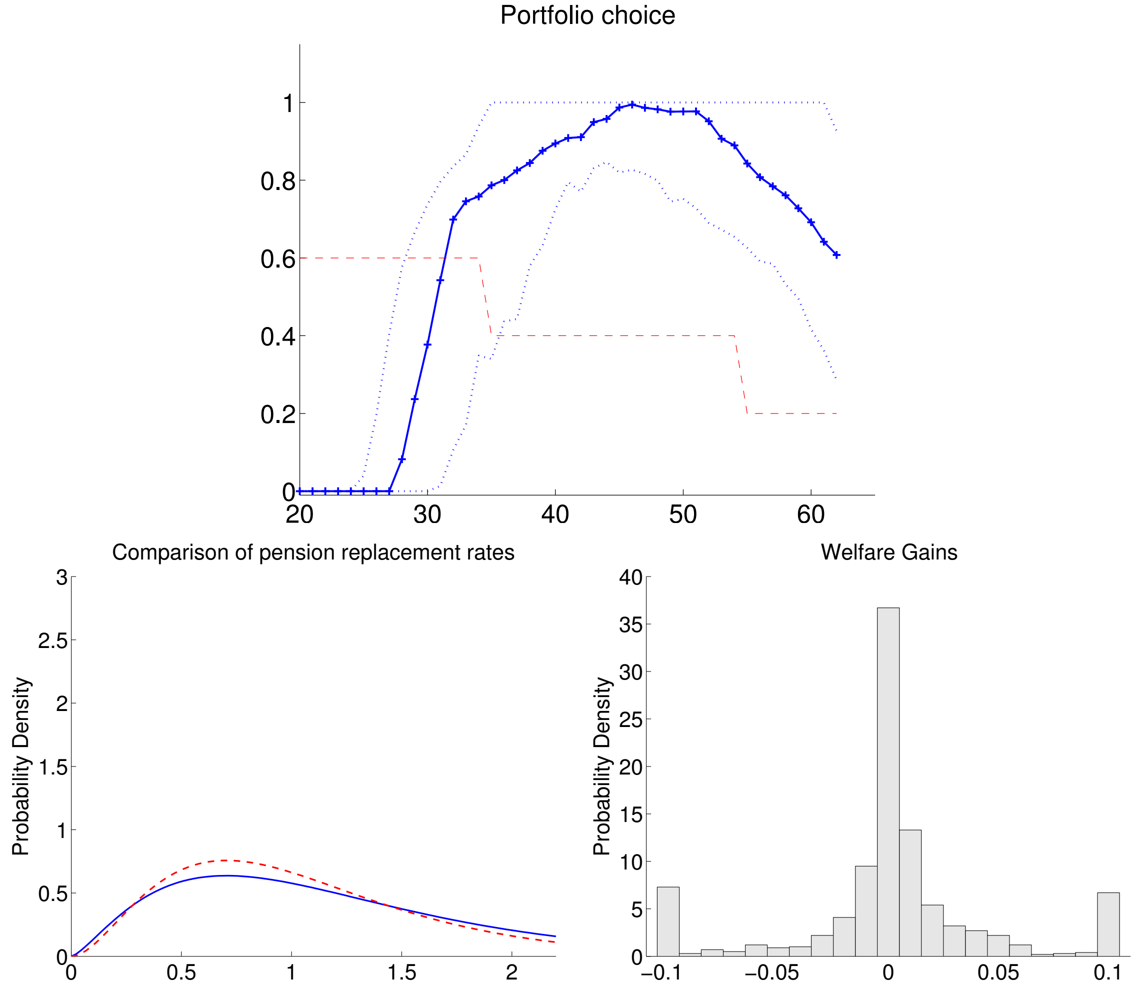

Figure 7. Simulation results for a male wage earner of high education. *Top panel*: median optimal portfolio choice (blue, continuous line) vs. current institutional default (red, dashed line); *Left-bottom panel*: distribution of the pension replacement rates obtained under the current portfolio default (red, dashed line) versus those with the optimal portfolio rule (blue, continuous line); *Right-bottom panel*: Distribution of the welfare changes induced by the change from the current portfolio default to the optimal.

High Education

Our simulations show large differences between the optimal and the default portfolio for male wage earners of high education (see fourth row of Table 3 and in Figure 7). This is not surprising, given how different these workers are with respect to our benchmark individual. The volatility of the permanent shocks is almost four times bigger and is correlated with shocks on aggregate returns. These two features clearly demand a more "conservative" portfolio allocation -- although the allocation also depends on the relative proportion of human and financial wealth, and on the average age-profile of labor income (Figure 1). Overall, our simulations suggest much less exposure to the risky asset. The average proportion of stocks is only 64% and the optimal life-cycle profile is hump-shaped, indicating that individuals should only start investing in stocks after almost one decade of professional activity. Then ω goes progressively up reaching nearly full exposure in the mid-40s (the average weight between ages 35 and 55 is 92%). Afterwards, the share of risky assets is similar to that in the base case: a marked reduction that leaves the average between ages 56 and 64 at 75%. The visual comparison of the distributions of replacement rates obtained under the current default vs. the optimal portfolio helps to explain the essence of the results. In sharp contrast to the benchmark case, our "safety-first" optimal rule can deliver smaller final pensions in a significant number of cases (the probability mass is redistributed both to the right and to the left of the mode under the current default). The welfare gains of following our optimal portfolio are also smaller, with an average gain of 0.91% of annual consumption and a median gain only slightly positive (0.06%). Some people gain a great deal -- the 95% percentile of the welfare distribution displays a staggering 37.7% increase in life-time consumptions. But the losses in the opposite case (i.e., the potential gains that are missed due to the lower exposure to risk) are even higher at 57.6%. The proportion of individuals that do not benefit from the optimal portfolio (47.2%) is also much higher than in the base case. We can only conclude that the application of the optimal rule may generate a lot of *ex post* acrimony.

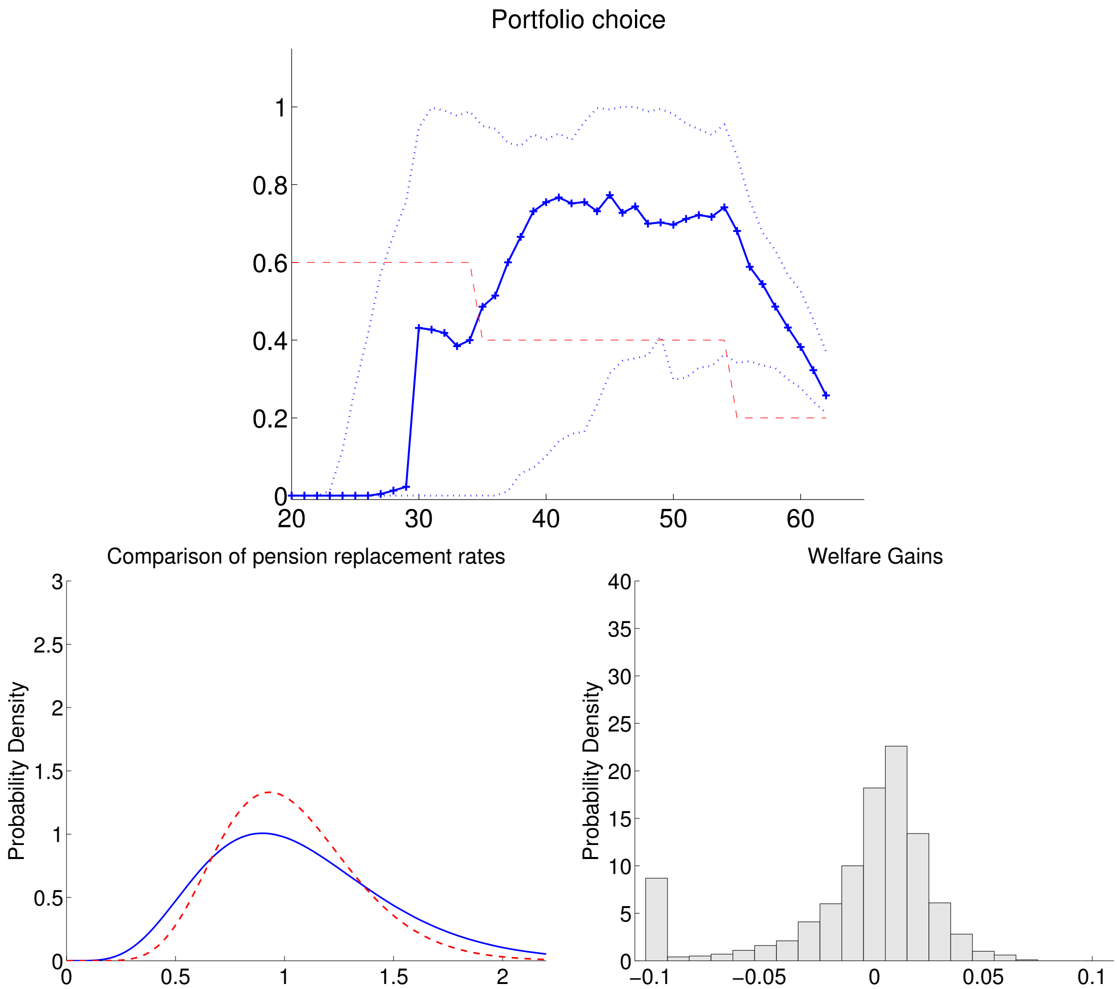

Figure 8. Simulation results for an employer of average education. *Top panel*: median optimal portfolio choice (blue, continuous line) vs. current institutional default (red, dashed line); *Left-bottom panel*: distribution of the pension replacement rates obtained under the current portfolio default (red, dashed line) versus those with the optimal portfolio rule (blue, continuous line); *Right-bottom panel*: Distribution of the welfare changes induced by the change from the current portfolio default to the optimal.

Employers

We conclude our analysis with the exploration of the optimal default for Employers. As with highly educated workers, our *a priori* would have favored a very conservative portfolio, given the extremely high correlation between return shocks and their idiosyncratic income shocks. Only the relatively small volatility of their permanent income shocks would temper this conjecture. The simulated results (displayed in Figure 8 and in the bottom row of Table 3) clearly confirm our intuition. The optimal average exposure to risky assets is only 46% and, as with the highly educated workers, it is hump-shaped and concentrated in the age interval 35 to 50. Quantitatively, our optimal portfolio allocation is safer in this case, although there is a huge difference between the 5% and 95% band (see Figure 8). Overall, all the same issues discussed for highly educated workers reappear in this case. The shift towards more extreme replacement rates is more acute in this case (left-down panel of Figure 8). The median welfare gain is somewhat bigger, but still very small (0.34% increase in life-time consumption). The proportion of losers is high (42.4%) and the welfare losses are also very substantial (the 5% percentile of the distribution of welfare changes by -42.5%). The maximum welfare gains derived from the extra protection against excessive risk taking are smaller in this case (slightly above 4%). The application of this rule would also result in *ex post* regrets.

5. Conclusions

Our research is a contribution to the analysis of optimal portfolios in mandatory pension systems. We show how the current default rules for portfolio allocation in Chile can be improved in an *ex ante* way by taking into account observable individual characteristics that affect labor income. Based on these observable characteristics, we develop a set of new rules for the life-cycle allocation of pension wealth that, on average, results in a higher *expected* welfare than that obtained by following the current defaults. Depending on the characteristics of the individual, our conditional rules can prescribe both higher exposure to risky assets (as in the case of male wage earners of average education) and lower exposure (as in the case of employers).

The policy implications are important. The results suggests that, beyond default portfolios for financial assets, governments should consider observable workers characteristics (age, gender, education, occupation) when defining the level of the contribution rate to the pension system, and the share allocated to pay-as-you-go vs. financial assets. We argue that doing so could not only increase welfare, but also improve incentives to enroll and contribute among individuals who today are outside the system by choice, either working as informal wage earners of self-employed.

At this stage, however, two important caveats should be remembered for the real-world applicability of our methods.. First, it must be clearly understood that the optimal rules are a guarantee of improvement in only an *ex ante* way. It is always the case that, ex post, some individuals would have found themselves in a better situation under the current portfolio default. For instance, our optimal rule stipulates a less risky default portfolio for employers than the actual “one-size-fits-all” default allocation. This lead to welfare improvements for this group *on average*, but there are always combinations of idiosyncratic and aggregate shocks under which a more “aggressive” portfolio would have performed better. Our simulations indicate that the proportion of individuals that fall in this situation can be large for some type of workers. This, of course, does not invalidate our conclusions, but emphasizes the idea that further research on how to compensate ex-post losers is needed.

A second, more technical, caveat results from how we construct the optimal portfolio rule. Note that we do not propose an automatic modification of the proportion invested in risky stocks with the age of the individual (as with, for example, the *Target-date-Funds* offered by default by US employers). Our optimal portfolio is a function, at each age, of the current realization of the stochastic labor income and of the current values of the stocks of accumulated private and pension wealth. With our rules, similar workers (in terms of age, education, gender and type of contract) get different advice for their portfolio choice depending on their present and past record of labor income shocks. In the real world, private wealth and the permanent component of income shocks can only be observed by the policy maker approximately (e.g. within brackets and with measurement error). This would reduce the gains from the application of the optimal rule. Our results should, therefore, be taken as upper bounds on the gains that can be feasibly expected in a real world reform.

We conclude the analysis with some suggestions for improvements and future research. An important step forward to complete the current analysis (applied to some selected groups of workers) would be to develop a nationwide welfare evaluation of the outcome of our experiment. This can be done by taking a partition of our representative sample, evaluating the distribution of welfare gains for each group in the partition and aggregating the results (weighted by the population share of each group). A second promising line of research would focus on exploring feasible *ex post* compensation mechanisms for the workers that experience welfare losses from the change in the default rules. Improving the quality of the key empirical inputs (probably by incorporating more information from other databases) is another avenue for improving the robustness and relevance of the results of our experiments in the future.

References

AFP (2012, feb). Aporte previsional solidario de vejez, APS-vejez (online resource). http://www.safp.cl/573/article-5786.html.

Benartzi, S and R H. Thaler (2007). Heuristics and biases in retirement savings behavior. *Journal of Economic Perspectives*, **21**, 81–104.

Berstein, Solange, Olga Fuertes, and Nicolas Torrealba (2011a). Esquema de multifondos en chile. Documento de Trabajo 43, Superintendencia de Pensiones.

Berstein, Solange, Olga Fuertes, and Nicolas Torrealba (2011b). La importancia de la opción por omisión en los sistemas de pensiones de cuentas individuales: Estrategias de inversión de ciclo de vida. Documento de Trabajo 44, Superintendencia de Pensiones.

Bodie, Zvi and J Treussard (2007). Making investment choices as simple as possible but not simpler. *Financial Analysts Journal*, **63**, 42–47.

Bovenberg, L, R Koijen, T Nijman, and C Teulings (2007). Saving and investing over the life cycle and the role of collective pension funds. *De Economist*, **155**.

Campbell, John, Joao Cocco, Francisco Gomes, and Pascal Mahenhout (2001). Investing retirement wealth: a life-cycle model. In John Campbell and M Feldstein (Eds.), *Risk aspects of Investment-based Social Security Reform*. University of Chicago Press.

Campbell, J and L Viceira (2002). *Strategic Asset Allocation*. Oxford University Press.

Carroll, C (2006). The method of endogneous gridpoints for solving dynamic stochastic optimization problems. *Economics letters*, 312–320.

Carroll, C (2009, March). Lecture notes on solution methods for microeconomic dynamic stochastic optimization problems.

Carroll, C and A Samwick (1997). The nature of precautionary wealth. *Journal of Monetary Economics* (40), 41–71.

Cocco, Joao, Francisco Gomes, and Pascal Mahenhout (2005). Consumption and portfolio choice along the life cycle. *The review of financial studies*, **18**, 491–533.

Davis, SJ and P Willen (2002). Income shocks, asset returns and portfolio choice. In O S Mitchell, Z Bodie an d PB Hammond, and S Zeldes (Eds.), *Innovations in Retirement Financing*, pp. 20–49. Pension Reseach Council. The Wharton School of the U. of Pennsylvania University of Pennsylvania Press.

Estelle, J, G Martínez, and A Iglesias (2005, diciembre). The payout stage in chile: who annuitizes and why? Documento de trabajo 14, Superintendencia de AFP.

Gomez, F, L Kotlikoff, and L Viceira (2008). Optimal life-cycle inventing with flexible labor supply: a welfare analysis of life-cycle funds. *American Economic review: papers and proceedings* (98:2), 297–303.

Human Mortality Database (Data downloaded on May 2011). University of california, Berkeley (usa) and max planck institute for demographic research (germany). Technical report, available at www.mortality.org or www.humanmortality.de.

Merton, R C (1971). Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economics Theory*, **9**, 373–413.

Miles, David and Alex Cerny (2006). Risk, return and portfolio allocation under alternative pension systems with incomplete and imperfect financial markets. *The Economic Journal*, **116**, 529–557.

Repetto, A and C Huneeus (2005). The dynamics of earnings in chile. In J Restrepo, A Tokman, N Loayza, and K Schmidt-Hebbel (Eds.), *Labor markets and Institutions*, Volume 8 of *Central Banking, analysis and Economic policies book series*, pp. 383–410. Central Bank of Chile.

Storesletten, K., C. Telmer, and A. Yaron (2004). Consumption and risk sharing over the life cycle. *Journal of Monetary Economics* (51), 609–633.

Appendix 1. The details of the model

A. Labor income, asset allocation and portfolio defaults

This section provides a rather informal review of the key principle underlying our calculations in this paper: that differences in the properties of labor income would lead to different portfolio choices over the life-cycle. We resort to well-known expressions of optimal portfolio choices to highlight the links between asset allocation and human capital.

It is very well established that the age of a person is an irrelevant factor for the portfolio choice of an investor who only possess financial wealth[25]. His/her asset allocation (summarized in the proportion of risky assets ω) will be identical to that of a short-term investor, known from the classical Markovitz mean/variance analysis:

$$\omega = \frac{E_t r_{t+1} - r_{t+1}^f + \frac{1}{2}\sigma^2}{\eta\, \sigma^2} \tag{1}$$

where we assume log-normal returns to the risky asset $r_{t+1} = Ln\,(1 + R_{t+1})$ and $r_{t+1}^f = Ln\,(1 + R_{t+1}^f)$, constant variances and a constant degree of relative risk aversion, η, on the part of the investor. In this case, all investors should hold the same combination of risky assets, independently of age. This finding is, however, largely irrelevant for individual investors because human wealth is the most important form of wealth available during large parts of the individual life.[26] Actually, for a large majority of the population, it is *the only* form of wealth available early in life (the exception being people with large inheritances). If the stream of future labor income where riskless, the resulting portfolio choice is still quite straightforward. The optimal portfolio share of risky stocks on the individual's financial wealth should be:

$$\omega = \frac{E_t r_{t+1} - r_{t+1}^f + \sigma^2/2}{\eta\, \sigma^2}\left(1 + \frac{H_t}{A_t}\right) \tag{2}$$

with H_t standing for the value of the stock of Human Wealth and A_t representing the stock of financial wealth. The fundamental conclusion from a life-cycle perspective is that ω *should decline along* the working career of a typical individual (as H_t is progressively exhausted (in absolute terms) and also because of the drop in the relative weight of human vs. financial wealth derived from savings and the accumulation of retirement wealth. It is also evident from (2) that workers with different endowments of human capital may follow different portfolio rules along their life.

To complete the analysis we must acknowledge the uncertain nature of labor income.[27]. We cannot get a closed-form expression in the most general case, but the basic intuitions can be derived in a two-period model with log-normal shocks to income and asset returns.[28]. In this context the optimal risky assets weight is:

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[25]This is the classical result of Merton (1971), which assumes constant relative risk aversion and an environment characterized by constant investment opportunities.

[26]The formula is still valid in a context of complete markets where investors can capitalize human wealth and hedge its risk (as shown in, again, Merton (1971)). This is obviously not the case in real world markets.

[27]Due to both idiosyncratic and aggregate shocks, that can be permanent or transitory and that may be correlated with other sources of uncertainty.

[28]See chapter 6 in Campbell and Viceira (2002) for a summary of the model and detail references.

$$\omega = \frac{1}{\phi}\left(\frac{E_t r_{t+1} - r^f_{t+1} + \sigma^2/2}{\gamma\,\sigma^2}\right) + \left(1 - \frac{1}{\phi}\right)\frac{\sigma_y\,\rho}{\sigma} \tag{3}$$

where σ_y is the variance of the labor income shocks, ρ is the correlation with return shocks and ϕ is the elasticity of consumption with respect to financial wealth.[29] The first term is the usual solution for a short-run investor, now inversely weighted by the wealth-elasticity of consumption. The second component is an income hedging component. If $\rho < 0$ the risky asset offers a good hedge against negative income shocks, leading to a larger optimal allocation (and the opposite if the correlation is positive).
The properties of labor income are, then, of paramount important for asset allocation. As before, ω should systematically change with age according to how quickly is human capital exhausted. Furthermore, workers with very volatile labor incomes (large σ_y) and workers whose income is highly synchronized with the returns of the risky assets (large ρ) should have far more "conservative" portfolio allocation than otherwise.

B. Life cycle problem

This section provides a more detailed description of the formulation, theoretical solutions and numerical simulation of our model of rational life-cycle behavior.

B1. Bellman equation (active worker)

The recursive version of the individual problem presented in section 2.1 has the key property that its associated value function is homogeneous with respect to the permanent component of labor income, P. As a result, a new equivalent problem with one less state variable can be formulated by expressing all variables as ratios to P. The recovery of the original variables from the transformed ones (denoted by lower case letters) is straightforward (eg. $M = m\,P$, $\Pi = \pi\,P$, and so on). Similarly, the original value function is obtained from the value function of the transformed problem, $v_i(m,\pi)$, after a direct transformation: $V(i,M,\Pi,P) = \Gamma^{1-\eta}\,v_i(m,\pi)$. The problem that is effectively solved with our numerical procedure is as follows:

$$v_i(m,\pi) = \max_{\{c,\omega\}}\{\,u(c) + \beta_{i+1}E_i[(\Gamma_{i+1}\,\Psi')^{1-\eta}\,v_{i+1}(m',\pi')]\,\}$$
$$a = m - c$$
$$m' = a\,\mathcal{R}_f{}' + \varsigma\,\Theta'$$
$$\pi' = \pi\,\mathcal{R}' + (1-t)(1-\varsigma)\,\Theta'$$
$$\mathcal{R}' = \frac{R^f + e'\,\omega}{\Gamma_{i+1}\,\Psi'}$$
$$\mathcal{R}_f{}' = \frac{R^f}{\Gamma_{i+1}\,\Psi'}$$

where the discount factor $\beta_i = \beta\,S_i$ and we denote the excess return $e = R - R^f$. The expectation operator E reflects labor and returns uncertainty.

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[29] The optimal consumption function is this context is $c_{t+1} = k + \phi\,(a_t + r_{t+1}) + (1-\phi)\,y_{t+1}$ $\phi = e^{\overline{a}_{t+1}-\overline{l}}/(1 + e^{\overline{a}_{t+1}-\overline{l}}) < 1$where a stands for private wealth, $\overline{y}$ is permanent income and $\overline{a}$ represent long term assets. To link (2) and (3) note that form the definition of ϕ it follows that $1/\phi = 1/(e^{\overline{a}_{t+1}-\overline{y}}) + 1 = e^{\overline{y}}/e^{\overline{a}_{t+1}} + 1 = E_t[H_{t+1}]/E_t[A_{t+1}] + 1$ (which in continuous time converges (as $\sigma_l \to 0$) to $(1 + H_t/A_t)$.

B2. Theoretical solutions

Rational behavior is summarized in the first order conditions necessary conditions emerging from the problem above:

Optimal consumption at age i:

$$u'(c_i) = \beta_i\, E_i\left[\frac{R^f}{(\Gamma_{i+1}\,\Psi_{i+1})^\eta}\,\frac{d\,v_{i+1}}{d\,m}(m_{i+1},\pi_{i+1})\right]$$

which, applying a standard envelop theorem, leads to:

$$u'(c_i) = \beta_i\, R^f\, E_i[u'(\Gamma_{i+1}\,\Psi_{i+1}\,c^*_{i+1}(m_{i+1},\pi_{i+1})\,)]$$

where c^*_{i+1} is the optimal consumption function available from the immediately preceding iteration.

Optimal portfolio choice

$$0 = \beta_i\, E_i\left[\left(\frac{e_{i+1}}{(\Gamma_{i+1}\,\Psi_{i+1})^\eta}\right)\frac{d\,v_{i+1}}{d\,\pi}(m_{i+1},\pi_{i+1})\right]$$

where the derivative of the value function with respect to the IRA balance is also obtained via an envelop theorem:

$$\frac{d\,v_i}{d\,\pi}(m_i,\pi_i) = \beta_i\, E_i\left[\left(\frac{R^f + e_{i+1}\,\omega^*_{i+1}}{(\Gamma_{i+1}\,\Psi_{i+1})^\eta}\right)\frac{d\,v_{i+1}}{d\,\pi}(m_{i+1},\pi_{i+1})\right]$$

Note that, in order to solve this problem, the next-period functions ω^*_{i+1} and $\frac{d\,v_{i+1}}{d\,\pi}(m,\pi)$ should be available from a preceding iteration.

B3. Numerical solution method

The model is solved using the method of “endogenous grid point” developed by C. Carroll (see, for instance Carrol (2006)). The method is a set of procedures for solving dynamic stochastic optimization problems intended to provide highly accurate solutions with large savings in the computational burden implied by more traditional techniques. Although the methods involves several choices (like the particular discrete approximation techniques used for numerical integration or the use of linear interpolation for the approximation of functions outside the solution grid), its key ingredient is the avoidance of root-finding algorithms in the solution of the first order conditions of the problem. This is achieved by solving the problem on an exogenous set of values for the end-of-period savings, rather than for the cash-on-hand (m) state variable. The values of m are recovered endogenously as a product of the solution method.

B.4. Properties of the solutions

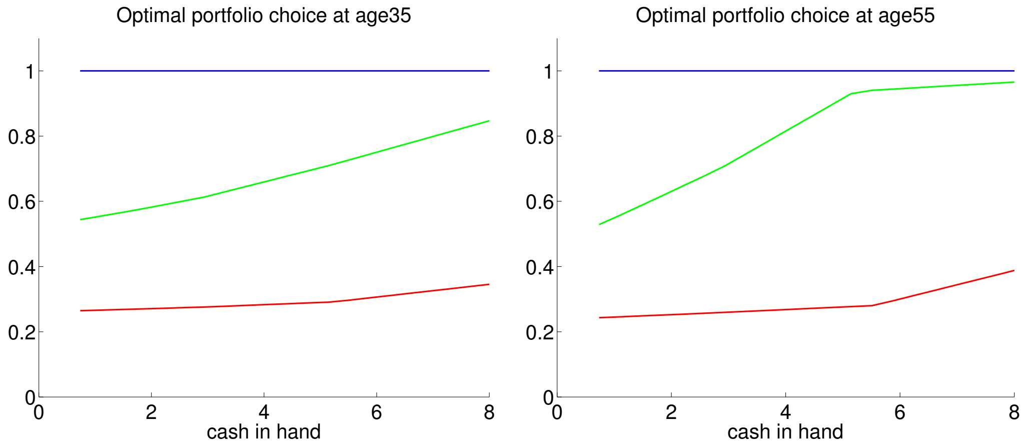

Figure 9. Optimal portfolio choice as a function of cash in hand for three level of accumulated pension rights: low (blue line); average (green line) and high (red line).

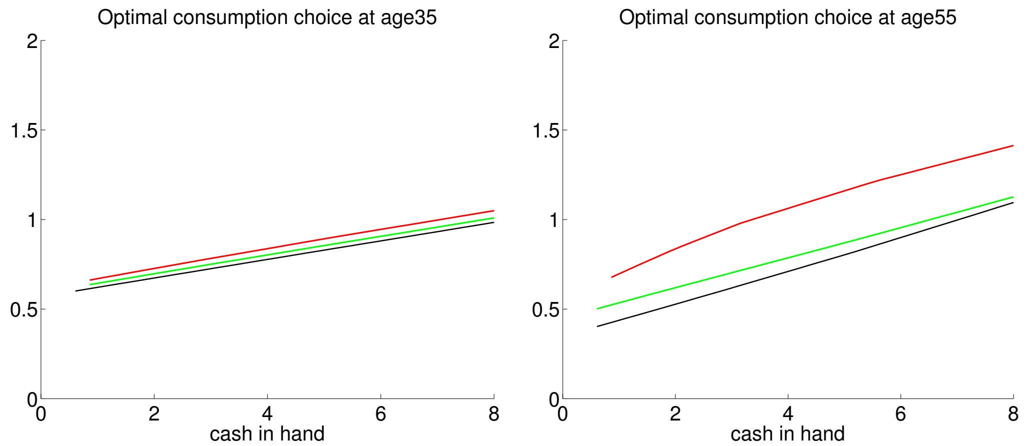

Figure 10. Optimal consumption choice as a function of cash in hand for three level of accumulated pension rights: low (blue line); average (green line) and high (red line)

Figures 9 and 10 illustrate some of the basic properties of the optimal behavioral rules of our problem (for our benchmark calibration). The optimal portfolio choice as a function of the amount of liquid resources available (cash in hand) is illustrated in Figure 9 at two different ages and for three levels of accumulated pension wealth. The proportion of risky assets in the optimal portfolio is increasing in the size of liquid resources available (or constant if the individual is constraint at the corner with a 100% allocation to stocks). Ceteris-paribus, the exposure to risky assets tends to decrease with the size of accumulated pension assets and to be smaller later in life. Extremely risky portfolio choices are, therefore, optimal early in life or for workers with small pension assets (which means that only a small proportion of total wealth is at risk with those bets).
Figure 10 displays the optimal consumption decisions of our benchmark agent, arranged in a similar way as with portfolio choices. In accordance with the standard theory, our simulations uncover consumption functions that are, ceteris paribus, increasing in cash in hand and the level of pension wealth. Note how liquidity considerations are more important early in life (optimal consumption varies with cash in hand but is roughly constant with pension assets), while pension wealth takes a more prominent role as retirement approaches.

C. Sensitivity analysis

Our simulations assume specific values for some of the unobservable preference parameters of the model. To test the robustness of our findings, we have repeated the experiments under an alternative set of preference parameters. The results in the three most critical dimensions (degree of risk aversion, degree of time discounting of future events and intensity of the preference for leisure --revealed by an early retirement decision) are displayed in Table 4. To simplify the comparison, the table also reproduces the results for our benchmark simulation. It emerges that, although there are some appreciable numerical differences, the main findings of the paper seem robust to the particular value assumed for these unobservable parameters.

Name	**Gender**	**Educ**	**type**	**Rep Rate**	**Avr**	**Q5%**	**Welfare Median**	**Q95%**	**% losers**
				Base					
Base	Male	Avrg	Wage E	1.01/0.87	1.82	-0.63	1.73	4.45	7.17
HighE	Male	High	Wage E	0.93/0.87	0.91	-57.6	0.06	37.3	47.19
				Extreme risk aversion					
Base	Male	Avrg	Wage E	0.95/0.87	2.50	-0.58	1.17	14.4	12.1
HighE	Male	High	Wage E	0.79/0.87	1.20	-73.1	-0.11	7.0	72.5
				Impatient individuals					
Base	Male	Avrg	Wage E	0.99/0.87	0.11	-0.13	0.1	0.37	10.1
HighE	Male	High	Wage E	0.74/0.87	0.63	-1.9	-0.57	0.43	73.6
				Early Retirement					
Base	Male	Avrg	Wage E	0.91/0.79	2.52	-2.64	2.56	7.25	12.9
HighE	Male	High	Wage E	0.84/0.79	5.32	-59.1	0.14	151	45.8

Table 4. Sensitivity analysis. Simulation results for the benchmark individual with high risk aversion (η=8), high discount factor (β=0.9) (welfare figures multiplied by 100) and early retirement at 60. As with Table 3, we reproduce the pension replacement rates (under the optimal vs. current portfolio default) and selected statistics from the distribution of welfare changes induced by the substitution of the optimal for the current portfolio default

Appendix 2. The Chilean Pension System

The public sector exerts a very large influence on individual savings decisions in Chile through the regulation of its pension system. Consequently, we pay special attention to reproduce the most fundamental features of this important public/private set of institutions. The core of the system is a set of privately managed individual accounts that operates under the *defined-contribution* principle (ie. participants bear the risk of poor performance of their retirement investments). The system is, however, strongly regulated by the public sector, who also provides a guaranteed minimum pension to all participants[30]. It follows a review of the main characteristics of the system (as they have

[30]The Chilean PAYG pension system was converted in 1981 into a "Fully Funded" (FF) system. It subsequently became a worldwide reference for funded (and private) pension systems. However, it can hardly be characterized as a pure FF pension system nowadays. After the 2008 reform the system is, in effect, a multi-pillar one, with a significant degree of protection to the eventuality of very low returns in

been reproduced in our model of section 2). A summary of the values selected for the most important parameters of the model is provided in Table 1.

Retirement accounts, portfolio choice and portfolio defaults

The system is based on mandatory individual retirement accounts (IRAs) managed by private providers (Administradoras de Fondos de Pensiones, AFP). Registered individuals contribute a 10% of their gross labor income to their retirement accounts and pay a 1.5% commission to the AFP for the management of the account.[31] They also pay a 1.5% contribution charge to finance the invalidity and survival's pensions provided by the system. Finally, individuals pay another 7% of their income to finance the health system, but we model that contribution as part of the general income tax.

Individuals can choose the proportion of risky assets in their retirement accounts. Since 2002 all AFP offer their affiliates a choice among five funds ("Multifondos") featuring clearly different asset compositions (ie. risk/return profiles). The funds are identified by a capital letter and arranged according to their riskiness in a decreasing order: A's funds are mostly invested in variable-income while funds in E are invested almost entirely in fixed income products; Statistical information on the historical composition of each "Multifondo" can be found in page 15 of Berstein et al. (2011a).[32]

By default, workers are assigned a "Multifondo" (referred simply as a "Fund" in the text) conditional on age and gender. Fund B is selected for both men and women up to 35 years of age; Fund C is selected for men between 36 and 55 and for women between 36 and 50. Finally, Fund D is the default for men aged 56 and older and for women of 51 years of age or older. Effectively, these restrictions implement a life-cycle portfolio choice characterized by a decreasing proportion of risky assets (starting at 60% and ending at 20%, given the composition of Funds B/D). This pattern is similar to that provided by the "Target-date funds" or "life-cycle" funds normally offered by the private asset-management industry (see, for example, Bodie and Treussard (2007)). In section 4.3 we discuss the convenience of adjusting these defaults according to other observable characteristics of the enrolled workers.

Pension benefits and retirement age

The legal retirement age is 65 for men (60 for women), although earlier retirement is possible for workers with large enough accrued pension assets. Actually, early retirement was the most popular option before the 2004 reform (see, for example, Estelle et al (2005)). The conditions for early retirement, however, became considerably more restrictive after 2004 and we focus on the case of workers who retire at the age of 65 in our simulations (checking the robustness of our findings to this assumption in appendix 1

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private retirement accounts. This protection represents a contingent liability backed by the general tax revenue of the State (i.e., it implies the existence of a PAYG component on the system).

[31]Commissions vary with the AFP chosen, but 1.5% is a representative figure.

[32]"Multifondo" A can have up to 80% (and no less than 40%) of its assets invested in risky assets. These figures go down progressively along the sequence of "Multifondos", with fund E featuring a maximum 5% allocation to variable-income. Note also that affiliates can contribute to a maximum of 2 "Multifondos". This means that they can (approximately) implement any possible split between risky/riskless assets in their retirement accounts (eg. by weighting "Multifondos" A and E in the correct proportions). This justify our modeling of the portfolio choice, ω, as a continuous variable.

section C, devoted to sensitivity analysis). Retirement from the labor force and the claim of the pension benefit are assumed to take place simultaneously at the legal retirement age.

The initial value of the individual pension benefit depends on the amount of funds contributed to the IRA during the life-cycle and the history of returns achieved (by the combination of Funds selected during the working ages). In reality, workers can withdraw their funds in three alternative ways: as programmed withdrawals, by conversion to a constant annuity or through a mixture of both mechanisms. The annuitization option is the most popular and we take it as our base case. In our model, we implement a simplified version of the pension formula used by insurance companies to determine the value of the annuity payout. Given a IRA balance at retirement of Π, the (age-dependent) old-age and survival pension $\overline{B}_i$ is set such that:

$$\Pi = E_R\left[\sum_{i=R}^{T} \left(\frac{1}{R^f}\right)^{i-R} \overline{B}_i\right]$$

where $E_R[.]$ stands for the expected value, at the age of retirement, of the sum in brackets, given a set of survival probabilities for husband and wife. For married men (or spouses with dependent children) $\overline{B}$ represents a joint annuity that secures a 60% of the husband annuity, B, to the surviving widow (ie, $\overline{B}_i = S_i\, B + (1 - S_i)\, S_i^f\ 0.6\, B$).[33]

Minimum and maximum pensions

We reproduce the system of minimum pensions resulting after the 2008 overhaul of the preexisting safety nets. The most outstanding impact of the reform was a large increase in non-contributive pensions with the introduction of the Basic Solidarity Pension. In accordance with our focus on the behavior of affiliates to the AFP system, we model the details of the APS (*Aporte Previsional Solidario*) benefit, in particular the top up of the contributive pension of retirees age 65 or older to an annually legislated maximum (called PMAS).[34] The value of the PMAS benefit has seen successive increases as the new program was progressively phased in since 2008 (see AFP (2012)). In our simulations we calibrate to the value projected for 2011 (255.000 pesos or 8% of the average wage in 2008 according to the ESI data described below). We also model a maximum pension benefit, resulting from the existence of a legal ceiling on the amounts that can be paid into the retirement account. This upper constraint in the size of benefits is rarely binding in our simulations with representative cases.

Pension assets risk and returns

[33]Insurance companies apply their own assumptions when setting the survival probabilities and discount factors in the annuity formula. Estelle et al. (2005) estimate the MWR (*money's worth ratio* or the ratio of the present expected value of the annuity streams to the initial payment) to vary (for men) between 91.6 and 98.1% in 1999 and between 95.8 and 99.3% in 2003. We model a prudent behavior by setting the MWR in our model to the smallest of those figures.

[34]To qualify for the top-up, the claimant must be a resident in the country for 20 years and his/her family income must be below some specific threshold. We assume that these conditions are met by our simulated households.

We simplify the real institutional structure of the system of Multifondos by assimilating the properties of the risky asset in our model to the historical performance of Fund A. Consequently, we calibrate $E[R]$ and σ_R^2 to the mean and variance of the annual real returns estimated from the monthly data available from *Superintendencia de Pensiones* (covering the period from the end of 2002 to the beginning of 2011).[35] In a similar fashion, the riskless rate of interest is assimilated to the return of Fund E. The resulting parameter values are displayed in Table 1.

The second leg of our representation of public institutions is the fiscal system. Here we really cut to the very basics of the system. We consider two constant tax rates: an 18% rate levied on capital income (inspired in the rate imposed on business profits) and a 25% rate applied to labor income (including the 7% contribution rate to the health system and the 1.5% collected for invalidity pensions). Social contributions are exempted on the tax base of the income tax.

[35]The estimated values differ slightly according to the particular AFP considered, but the differences are very small and do not change the conclusions of our analysis in any relevant way.

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