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Abstract

During the last decade, research on income inequality has paid special attention to top income earners. At the same time, top marginal tax rates on upper income earners have declined sharply in many OECD countries. Discussions are still open on the relationship between the increase of the income share of the richest and to what extent the tax burden should be shifted towards top income earners. In this paper we analyse these questions by building and computing a theoretical framework which extends the Kakwani (1977) expression of the Reynolds-Smolensky (1977) redistribution index using the decomposition by income groups proposed by Lambert and Aronson (1993) and Alvaredo (2011). We show that for three types of revenue-neutral reforms based on Pfähler (1984) (that consist of allocating the tax shifts from "the poor" to "the rich" proportionally to tax liabilities, net income or gross income) the redistributive effect is always higher than before the reform. When the size of the rich group is sufficiently small we also find that the best option is allocating tax changes proportionally to net income, and the worst doing it proportionally to tax liabilities.

Keywords: inequality, redistribution, top incomes, decomposition, microsimulation

JEL Codes: D63, H23, H24

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Durante la última década, la investigación sobre la desigualdad de la renta ha prestado una atención especial a la concentración de rentas por parte de los individuos/hogares con más altos ingresos (generalmente identificados, en inglés, como "top incomes" o, comúnmente, como "el 1% -o menos- más rico"). La publicación del mediático y sugerente libro de Piketty (2014), El Capital en el Siglo XXI, ha estimulado el debate sobre cómo ha evolucionado la concentración de la renta y la riqueza entre esa pequeña proporción de los ciudadanos más ricos.

Una de las cuestiones abiertas y con una trascendencia a nuestro juicio importante en el ámbito de la hacienda pública tiene que ver con la relación existente entre ese incremento de la participación en la renta total de los individuos más ricos –p.e. el 1% de los individuos con mayores ingresos anuales– y el gravamen sobre la renta personal que debería establecerse sobre sus rentas, tanto en nivel como en grado de progresividad. Esta discusión no es ni nueva (aunque recientemente se ha puesto de nuevo en escena) ni simple (ya que combina los principios de equidad y eficiencia, siempre presentes en el diseño de los sistemas de impuestos). Sin embargo, desde hace algún tiempo y coincidiendo con las medidas de consolidación fiscal derivadas de la última crisis económica, se han realizado propuestas de reforma fiscal, algunas de ellas aplicadas, dirigidas a elevar los tipos impositivos de las rentas más elevadas.

Hasta el momento, la literatura dedicada a este tema es bastante escasa y centrada, esencialmente, en el análisis empírico (recientemente Gale et al., 2015). Una primera cuestión a abordar el análisis del potencial redistributivo que tienen este tipo de reformas. En concreto, en este trabajo, con el fin de acotar el estudio, nos centramos en las reformas neutrales desde el punto de vista recaudatorio consistentes en trasladar una parte de esa recaudación hacia los individuos con mayores rentas. Es decir, aumentar la recaudación proporcionada por 1% de los individuos más ricos y disminuir, en igual cuantía, la aportada por el 99% restante de los contribuyentes del impuesto. Conocer los efectos teóricos de este tipo de reformas sobre la progresividad global del impuesto y sobre sus efectos redistributivos ha de aportar luz sobre las potencialidades y límites de este tipo de medidas de reforma fiscal. Sin duda somos conscientes de la importancia de los efectos de comportamiento que este cambio en el gravamen de los individuos más ricos puede tener, de acuerdo con sus elasticidades de respuesta. Sin embargo, en un primer estudio parece razonable identificar las consecuencias distributivas dejando a un lado estos efectos sustitución. De esta forma obtenemos un marco de referencia, entroncado en la literatura teórica sobre redistribución impositiva, a partir del cual podremos evaluar posteriormente el impacto de esos cambios en el gravamen teniendo en cuenta las distorsiones sobre el comportamiento de los contribuyentes, tanto de los más ricos como del resto.

En concreto, en este trabajo estudiamos las reformas del impuesto sobre la renta personal denominadas en la literatura "reformas lineales à la Pfähler (1984)". Estas reformas combinan incrementos y rebajas en el gravamen previo aplicados de forma proporcional, alternativamente, a la cuota líquida pagada, a la renta gravada o a la renta neta del impuesto. Sus propiedades de neutralidad respecto de las medidas de progresividad local proporcionan atractivos resultados teóricos, que como se puede ver en este trabajo, permiten a su vez obtener otros específicos para estas medidas de gravamen del 1% más rico de los contribuyentes. El análisis realizado permite descomponer los cambios en la redistribución en sus componentes intra-grupo (para el 1% más rico y para el 99% restante) e inter-grupo, lo que permite identificar los trasvases de progresividad generados por la reforma.

Las proposiciones teóricas obtenidas permiten establecer ordenaciones de preferencia entre las distintas combinaciones de instrumentación de las reformas consideradas, así como su intensidad en función del volumen de recaudación transferido entre ambos grupos. Para ilustrar el análisis teórico, el trabajo incorpora diversos ejercicios de microsimulación a partir de la Muestra Anual de Declarantes del IRPF español proporcionada por el Instituto de Estudios Fiscales y la Agencia Estatal de Administración Tributaria, correspondiente al ejercicio 2011. Este análisis empírico permite verificar el cumplimiento de los resultados teóricos que se ofrecen.

1. INTRODUCTION

During the last decade, research on income inequality has paid special attention to top income earners. The publication of the thought-provoking book by Piketty (2014) has encouraged public debate by showing how in developed countries wealth has become concentrated in a very small proportion of citizens. As Atkinson *et al*. (2011) highlighted, the top percentile income share has more than doubled in the last decades (from less than 10 percent in the 1970s to over 20 percent in recent years). This trend is particularly noticeable in the United States, but it is also present in many other countries worldwide, including Southern European countries. The case of Spain is striking, since it shows one of the greatest rises in this concentration and also of income inequality in the European Union.

At the same time, top marginal tax rates on upper income earners have declined sharply in many OECD countries, particularly in Anglo-Saxon countries (Piketty *et al*., 2014). Discussions are still open on the relationship between the increase of the income share of the richest and the level at which they should be taxed. A traditional argument holds that tax increases for the richest taxpayers would mean high efficiency costs through several ways (reduction of labour supply, incentives to tax avoidance, etc.). But there are also analysis that stand up for a change in the shift of the tax burden towards the richest taxpayers. However we have to take into account that in a number of countries the measures of fiscal adjustment during the recent recession led to higher tax rates on personal income.

This discussion is neither new (although it has been recently put back in scene) nor simple (since it combines the principles of equity and efficiency, always present in the design of tax systems). We think that one of the first relevant questions to analyse is the redistributive potential that the mentioned tax shift may have: even in a revenue-neutral reform, and accepting no behavioural responses, shifting part of the tax burden of the PIT towards the top income earners (e.g. the top 1% usually mentioned in these discussions) has obviously effects of the global progressivity of the tax, and consequently on its redistributive effect. Nevertheless, even though this effect is pursued with these reforms, it is necessary to assess their limits and their impact on the reduction of income inequality. It is obvious that arguments of confiscatory taxation and efficiency will limit the concentration of a relevant part of the tax burden on the top income earners. As far as we know, there is only some empirical evidence based in simulation exercises (i.e. Gale *et al*., 2015), but without a theoretical framework that incorporates the main underlying relations between tax progressivity, tax burden, and income distribution.

In relation to the previous argument a second question to analyse is the most convenient way of allocating individually the tax increase to the "rich" and the corresponding tax decrease to the "poor". The way in which we implement the reform affects the structural progressivity of the tax and consequently its ability to reduce net income inequality, so it is necessary to analyse the implications of each possible alternative.

The aim of this paper is analysing the two aforementioned questions. We offer a theoretical framework which extends the decomposition of the Reynolds and Smolensky

(1977) redistribution index between progressivity and average tax rate proposed by Kakwani (1977), using the decomposition by income groups proposed by Lambert and Aronson (1993) and the relationship between top income shares and the Gini coefficient established by Alvaredo (2011). In order to analyse the alternatives for reforming the Personal Income Tax (PIT) with the aim of increasing its redistributive effect through a higher level of progressivity on the top income earners, we consider the linear tax reform alternatives studied in Pfähler (1984), both for implementing tax cuts and tax increases. The neutral local progressivity properties of this kind of PIT reforms *à la* Pfähler (1984) allow us to obtain several relevant results about top income taxation.

As an illustration of the theoretical results we evaluate the different reform types using microdata from the 2011 Spanish PIT Return Sample disseminated by the Spanish Institute for Fiscal Studies (Instituto de Estudios Fiscales, IEF) and the Spanish Tax Agency (Agencia Estatal de Administración Tributaria, AEAT). In order to carry out this exercise we use a stylized transformation of the Spanish PIT so that tax liabilities depend only on gross income, without taking into account any non-income attribute, but keeping the revenue and redistributive effect of the actual tax unchanged.

The paper is organised as follows. After this introduction, section 2 presents the theoretical framework and offers the theoretical results of the paper. Section 3 shows the empirical analysis, including a brief presentation of the data and the results of the microsimulation exercises carried out. The paper ends with a summary of the main conclusions.

2. THEORETICAL FRAMEWORK

2.1. Decomposing global PIT progressivity and redistributive effect by income level groups

The redistributive effect of the personal income tax (PIT) can be expressed as

$$\Pi^{RS} = G_Y - G_{Y-T} \qquad [1]$$

where Π^{RS} is the Reynolds-Smolensky (1977) redistribution index which is defined as the difference between gross (pre-tax) income inequality (G_Y) and net (after tax) income inequality (G_{Y-T}), both measured in terms of the Gini index.

In order to decompose the redistributive effect among different groups we can take as a starting point the expression proposed by Lambert and Aronson (1993) to split the Gini index for k groups:

$$G_Y = G_Y^B + \sum_k p_k s_{Y_k} G_{Y_k} + R \qquad [2]$$

where G_Y^B is a between-groups component, that expresses the inequality between the k groups assuming that all individuals within the group hold the same (average) income μ_{Y_k}, and $\sum_k p_k s_{Y_k} G_{Y_k}$ is the within-groups component that is calculated as the sum of the inequality indices within each group weighted by their share in the total population ($p_k = N_k/N$) and total income ($s_{Y_k} = Y_k/Y$). Finally R represents an extra term to make the decomposition work when the subgroup income ranges overlap. If the subgroups are defined as $k = 1,2, \dots, K$ non-overlappping subpopulations of N_K individuals then $R = 0$.

As a specific case of Lambert and Aronson (1993), Alvaredo (2011) proposes a computational expression for the Gini index that takes into account the existence of two groups only differentiated by their income level. Equation [3] shows that expression for a partition between the first 99 centiles of individuals according to their gross income (simply "99" for notation) and the remaining 1% ("100"):

$$G_Y = (s_Y - p) + (1-p)(1-s_Y)G_Y^{99} + p s_Y G_Y^{100} \qquad [3]$$

where p represents the population share of group 100 (i.e. 0.01) and s_Y is the share of gross income held by that group. This means that $1-p$ is the population share of group 99 (0,99) and $1-s_Y$ the gross income share of that group. G_Y^{99} and G_Y^{100} are the Gini indices of gross income within each group respectively. Comparing expressions [2] and [3] the correspondence between the components "between" ($G_Y^B = s_Y - p$) and "within" ($\sum_k p_k s_{Y_k} G_{Y_k} = p s_Y G_Y^{100} + (1-p)(1-s_Y)G_Y^{99}$) is straightforward, while R is not present in [3] because there is no overlapping effect.

Using the Gini decomposition presented in Equation [3] we can now write Equation [1] as follows

$$\begin{aligned} \Pi^{RS} = {} & [s_Y - p + (1-p)(1-s_Y)G_Y^{99} + p s_Y G_Y^{100}] - \\ & -[s_{Y-T} - p + G_{Y-T}^{99}(1-p)(1-s_{Y-T}) + p s_{Y-T} G_{Y-T}^{100}] \end{aligned} \qquad [4]$$

where s_{Y-T} is the proportion of the net (after-tax) income accumulated by group 100. Simplifying Equation [4] we obtain

$$\begin{aligned} \Pi^{RS} = {} & (s_Y - s_{Y-T}) + (1-p)(1-s_Y)\Pi_{RS}^{99} + p s_Y \Pi_{RS}^{100} \\ & -(s_Y - s_{Y-T})[(1-p)G_{Y-T}^{99} + p G_{Y-T}^{100}] \end{aligned} \qquad [5]$$

where Π_{RS}^{99} and Π_{RS}^{99} are the Reynold-Smolensky indices for groups 99 and 100 respectively, i.e. the difference between the Gini indices before and after taxes within each group. We assume that the applied tax has a structure $T = t(y)$ where tax liability T only depends (positively) on income. This ensures that the application of the tax does not produce re-ranking, therefore each group includes the same observations before and after tax.

Overall redistribution in [5] can be then understood as the sum of a between effect ($s_Y - s_{Y-T}$), two weighted within effects ($(1-p)(1-s_Y)\Pi_{RS}^{99}$ and $s_Y \Pi_{RS}^{100}$) and an interaction term ($-(s_Y - s_{Y-T})[(1-p)G_{Y-T}^{99} + p G_{Y-T}^{100}]$). Equation [5] is useful because the "between" and "within" effects can be easily interpreted since they are based in Gini indices, but the interaction effect has no direct interpretation.

As an alternative we can further develop [5] to embed the interaction term into the "within" terms, as follows:

$$\Pi^{RS} = (s_Y - s_{Y-T}) + (1-p)[(1-s_Y)G_Y^{99} - (1-s_{Y-T})G_{Y-T}^{99}] + p[s_Y G_Y^{100} - s_{Y-T}G_{Y-T}^{100}] \quad [6]$$

Now equation [6] expresses overall redistribution as the sum of the "between" effect ($s_Y - s_{Y-T}$, the same as in [5]) and two "within" effects ($(1-p)[(1-s_Y)G_Y^{99} - (1-s_{Y-T})G_{Y-T}^{99}]$ and $+p[s_Y G_Y^{100} - s_{Y-T}G_{Y-T}^{100}]$). These within effects are not expressed in terms of Reynold-Smolensky indices, but in terms of pseudo-Reynolds-Smolensky indices, since G_Y and G_{Y-T} are weighted respectively by the shares of gross income and net income of each group. Intuitively this weighting can be understood as a change of scale that reflects what happens when we merge the two subpopulations that have different sizes. Of course this is the same idea behind the interaction effect in Equation [5].

Our decomposition can be also expressed in terms of progressivity. Following Kakwani (1977), the redistributive effect in [1] can be decomposed as a product of a progressivity measure (the Kakwani index, Π^K) and a relative measure of revenue (the net effective average tax rate, $t/(1-t)$):

$$\Pi^{RS} = \frac{t}{1-t}\Pi^K = \frac{t}{1-t}(G_T - G_Y) \quad [7]$$

Now we can express overall redistribution Π^{RS} in Equation [5] in terms of the Kakwani decomposition for each group, just by expressing Π_{RS}^{99} and Π_{RS}^{100} in terms of Equation [7]:

$$\Pi^{RS} = (s_Y - s_{Y-T}) + (1-p)(1-s_Y)\frac{t^{99}}{1-t^{99}}\Pi_K^{99} + p s_Y \frac{t^{100}}{1-t^{100}}\Pi_K^{100} \\ -(s_Y - s_{Y-T})[(1-p)G_{Y-T}^{99} + pG_{Y-T}^{100}] \quad [8]$$

where Π_K^{99} and Π_K^{99} are the Kakwani indices of groups 99 and 100 respectively, and $t^{99}/(1-t^{99})$ and $t^{100}/(1-t^{100})$ are the net effective average tax rates of groups 99 and 100 respectively.

Again the advantage of this expression is that it offers a straightforward interpretation of the within effects, since they include Kakwani indices. But it also keeps the interaction term of [5], so we can alternatively use Equation [3] to decompose $(G_T - G_Y)$ in Equation [7]:

$$\Pi^{RS} = \frac{t}{1-t}\begin{Bmatrix}[s_T - p + (1-p)(1-s_T)G_T^{99} + p s_T G_T^{100}] - \\ [s_Y - p + (1-p)(1-s_Y)G_Y^{99} + p s_Y G_Y^{100}]\end{Bmatrix} \quad [9]$$

where s_T is the share of tax revenue borne by group 100. Simplifying we get:

$$\Pi^{RS} = \frac{t}{1-t}\{(s_T - s_Y) + (1-p)[G_T^{99}(1-s_T) - G_Y^{99}(1-s_Y)] + p(G_T^{100}s_T - G_Y^{100}s_Y)\} \quad [10]$$

In parallel with [6], there is no interaction effect in this expression, but it offers the results in terms of pseudo-Kakwani indices, i.e. the Gini indices are weighted by the shares of the corresponding variables, which can be again understood as a change of scale.

2.2. Shifting tax burden to top income earners through yield-equivalent linear PIT reforms

A reform of a progressive personal income tax $T^{(1)}(y)$ that implies an increase or a reduction for all taxpayers can be treated as a linear transformation of the original tax, where we can call λ to the relative reduction or increase, so the new tax $T^{(2)}(y)$ will obtain the following revenue:

$$T^{(2)} = (1 \pm \lambda)T^{(1)} \qquad [11]$$

Following Pfähler (1984), there is a set of linear reforms that are neutral in relation to different local progressivity measures[1]. There are three types of relevant reforms ($j = \{a, b, c\}$) that allow the comparison of their redistributive impact, their revenue elasticity and their ranking in terms of social preference:

a) The reduction (increase) of each taxpayer's tax liability is a constant fraction α of the original tax liability:

$$T_a^{(2)}(y) = T^{(1)}(y) \pm \alpha T^{(1)}(y) = (1 \pm \alpha)T^{(1)}(y) \qquad [12]$$

where $\alpha = \lambda$. For this reform the liability progression is kept constant, i.e. $LP^{(1)}(y) = LP^{(2)}(y), \forall y > 0$.

b) The reduction (increase) of each taxpayer's tax liability in a constant fraction β of the original net income:

$$T_b^{(2)}(y) = T^{(1)}(y) \pm \beta\left[y - T^{(1)}(y)\right] \qquad [13]$$

where $\beta = \lambda T^{(1)} / \left[Y - T^{(1)}\right]$. For this reform the residual progression is kept constant, i.e. $RP^{(1)}(y) = RP^{(2)}(y), \forall y > 0$.

c) The reduction (increase) of each taxpayer's tax liability in a constant fraction ς of gross income:

$$T_c^{(2)}(y) = T^{(1)}(y) \pm \varsigma y \qquad [14]$$

[1] Appendix I shows the definitions of the three progressivity measures that will be used: average rate progression (*ARP*), liability progression (*LP*) and residual progression (*RP*).

where $\varsigma = \lambda T^{(1)}/Y$. For this reform the average rate progression is kept constant, i.e. $ARP^{(1)}(y) = ARP^{(2)}(y), \forall y > 0$.

For the same revenue change it results that $\varsigma = \alpha\bar{t} = \beta(1-\bar{t})$, where $\bar{t}$ is the average effective rate of the original tax, $\bar{t} = T^{(1)}/Y$.

Keeping this class of reforms in mind we can define a generic revenue-neutral reform of the Personal Income Tax so that the revenue obtained from the 99% poorest taxpayers (group 99), is reduced in a fraction λ, so that this revenue reduction is now borne by the 1% richest taxpayers (group 100), so that:

$$T^{(2)} = \underbrace{(1-\lambda)T^{99(1)}}_{T^{99(2)}} + \underbrace{T^{100(1)} + \lambda T^{99(1)}}_{T^{100(2)}} \qquad [15]$$

where $T^{99(1)}$ and $T^{100(1)}$ express the total revenue paid by each group under the original tax.

If we express the revenue shifted from group 99 to group 100 ($\lambda T^{99(1)}$) as a fraction ℓ of total revenue ($\ell T^{(1)}$), we can express this reform as

$$T^{(2)} = \underbrace{(1-s_T-\ell)T^{(1)}}_{T^{99(2)}} + \underbrace{(s_T+\ell)T^{(1)}}_{T^{100(2)}} \qquad [16]$$

where s_T is the share of the original tax held by group 100, as in Equation [10].

The fundamental question here is: what changes in the total redistributive effect imposes this shift of a proportion ℓ of the tax burden from the "poor" to the "rich"? The result will depend on the way we apply the reduction to the poor and the increase to the rich. This can be done in an infinite number of ways, but if we limit the reform to the three types of linear changes explained before, we can implement it in nine different ways (the combination of a, b y c in the tax reduction of group "99" and in the increase of group "100").

2.2.1. Tax reduction for group 99

Following Pfähler (1984), options a, b and c can be ordered according with the different local progressivity measures, as shown in the following propositions[2]:

Proposition 1 (Pfähler, 1984)

The three alternatives a, b y c can be ordered according to their residual progression in terms of its inverse $RP^* = 1/RP$:

[2] Proofs of these propositions are presented in Appendixes II, III and IV.

$$RP^{*99(1)}(y) = RP^{*99(2)}_{b}(y) > RP^{*99(2)}_{c}(y) > RP^{*99(2)}_{a}(y), \forall y = (0, \min y^{100}) \quad [17]$$

As we already knew, reducing the tax for group 99 proportionally to net income (option b) keeps the residual progression of the original tax unchanged. But additionally we know that this option is better in terms of RP to, in this order, a reduction proportional to gross income (c) and a reduction proportional to the tax liability (a).

Proposition 2 (Pfähler, 1984)

The three alternatives a, b y c can be ordered according to their average rate progression in terms of its transformation $ARP^{*} = yARP$:

$$ARP^{*99(2)}_{a}(y) < ARP^{*99(2)}_{c}(y) = ARP^{*99(1)}(y) < ARP^{*99(2)}_{b}(y), \forall y = (0, \min y^{100}) \quad [18]$$

As we stated before, reducing the tax for group 99 proportionally to gross income (option c) keeps the ARP of the original tax unchanged. But additionally we know that a reduction proportional to tax liability (a) shows the lowest ARP, while a reduction proportional to net income (b) shows the highest ARP.

Proposition 3 (Pfähler, 1984)

The three alternatives a, b y c can be ordered according to their liability progression (revenue elasticity):

$$LP^{99(2)}_{a}(y) = LP^{99(1)}(y) < LP^{99(2)}_{c}(y) < LP^{99(2)}_{b}(y), \forall y = (0, \min y^{100}) \quad [19]$$

As we mentioned, reducing the tax for group 99 proportionally to tax liability (option a) keeps the LP of the original tax unchanged. And finally, we know that this option is better in lower in terms of LP to, in this order, a reduction proportional to gross income (c) and a reduction proportional to net income (b).

From the previous propositions 1 and 3, and applying the propositions 1 and 3 from Jakobsson (1976) and the well-known identity $L_Y \equiv (1-\bar{t})L_{Y-T(Y)} + \bar{t}L_T$ in terms of Lorenz curves, we obtain the following results that allow sorting the considered reforms according to their global redistribution and progressivity:

Proposition 4 (Pfähler, 1984)

Reducing the tax for group 99 proportionally to net income (option b) keeps the redistributive effect of the original tax unchanged, while reductions proportional to gross income (option c) or tax liability (option a) lead to a reduction in redistribution (larger in the latter case than in the former).

$$L^{99(1)}_{Y-T(Y)} = L^{99(2b)}_{Y-T(Y)} > L^{99(2c)}_{Y-T(Y)} > L^{99(2a)}_{Y-T(Y)} > L^{99}_{Y} \quad [20]$$

Proposition 5 (Pfähler, 1984)

Reducing the tax for group 99 proportionally to tax liability (option a) keeps the progressivity of the original tax unchanged, while reductions proportional to gross income (option c) or net income (option b) lead to an increase in progressivity (larger in the latter case than in the former).

$$L_Y^{99} > L_{T(Y)}^{99(1)} = L_{T(Y)}^{99(2a)} > L_{T(Y)}^{99(2c)} > L_{T(Y)}^{99(2b)} \qquad [21]$$

2.2.2. Tax increase for group 100

The previous results can be adapted immediately to linear tax increases proportional to tax liability (a'), net income (b') and gross income (c'). In the case of tax increases the relative positions of the local progressitity measures for each alternative would be in the reverse order (propositions 1 to 3), just as the ranking of the Lorenz curves (propositions 4 and 5). In our reform the application of these tax increases for group 100 will lead to the following results of redistribution and progressivity within this group:

Proposition 6 (Pfähler, 1984)

Increasing the tax for group 100 proportionally to net income (option b) keeps the redistributive effect of the original tax unchanged, while increases proportional to gross income (option c) or tax liability (option a) lead to an increase in redistribution (larger in the latter case than in the former).

$$L_{Y-T(Y)}^{100(2a\prime)} > L_{Y-T(Y)}^{100(2c\prime)} > L_{Y-T(Y)}^{100(2b\prime)} = L_{Y-T(Y)}^{100(1)} > L_Y^{100} \qquad [22]$$

Proposition 7 (Pfähler, 1984)

Increasing the tax for group 100 proportionally to tax liability (option a) keeps the progressivity of the original tax unchanged, while increases proportional to gross income (option c) or net income (option b) lead to an reduction in progressivity (larger in the latter case than in the former).

$$L_Y^{100} > L_{T(Y)}^{100(2b\prime)} > L_{T(Y)}^{100(2c\prime)} > L_{T(Y)}^{100(2a\prime)} = L_{T(Y)}^{100(1)} \qquad [23]$$

2.3. Assessing the redistributive effect of the 'linear' PIT reform alternatives

The combination of the three previous reductions and increases lead to nine alternative revenue-neutral PIT reforms. To evaluate these reforms in redistributive terms we apply the results of Propositions 4 and 6 to Equation [5], and we see that:

- The between effect $s_Y - s_{Y-T}$ is positive in the nine reforms, because the tax increase for group 100 makes its net income share (s_{Y-T}) smaller and therefore $s_Y - s_{Y-T}$ larger.
- Following proposition 4 the within effect of group 99 ($(1-p)(1-s_Y)\Pi_{RS}^{99}$) will be unchanged (reform *b*) or will decrease (reforms *a* and *c*).
- Following proposition 6 the within effect of group 100 ($ps_Y\Pi_{RS}^{100}$) will be unchanged (reform *b*) or will increase (reforms *a* and *c*).
- The change in the interaction term ($-(s_Y - s_{Y-T})[(1-p)G_{Y-T}^{99} + pG_{Y-T}^{100}]$) is always negative since all their terms ($s_Y - s_{Y-T}$, $(1-p)G_{Y-T}^{99}$ and pG_{Y-T}^{100}) are positive.

This implies an *a priori* ambiguous result, since positive and negative terms coexist in the nine possible combinations. However it is possible to obtain unambiguous conclusions for the three cases where we apply the same type of reforms to both groups (99 and 100).

Proposition 8

Any yield-equivalent reform of the progressive tax $T = t(y)$ that reduces all tax liabilities in group 99 proportionally to their original tax liabilities (a) and increases all tax liabilities in group 100 proportionally to their original tax liabilities (a') will increase the global redistributive effect.

Proof

Using Equation [10] we can write the global redistributive effect of the original tax as:

$$\Pi^{RS(1)} = \frac{t}{1-t}\left\{(s_T - s_Y) + (1-p)\left[G_T^{99(1)}(1-s_T) - G_Y^{99}(1-s_Y)\right] + p\left(G_T^{100(1)}s_T - G_Y^{100}s_Y\right)\right\} \quad [24]$$

According to propositions 5 and 7 we know that the Gini indices of tax liabilities for both groups ($G_T^{99(1)}$ and $G_T^{100(1)}$) are not affected by the reform, so using Equations [10] and [16] we can write the global redistributive effect of the new tax ($\Pi^{RS(2aa')}$) as:

$$\Pi^{RS(2aa')} = \frac{t}{1-t}\left\{(s_T + \ell - s_Y) + (1-p)\left[G_T^{99(1)}(1-s_T-\ell) - G_Y^{99(1)}(1-s_Y)\right] + p(G_T^{100}(s_T+\ell) - G_Y^{100}s_Y)\right\} \quad [25]$$

Isolating [24] in [25] we obtain:

$$\Pi^{RS(2aa')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\left\{1 - (1-p)G_T^{99(1)} + pG_T^{100(1)}\right\} \quad [26]$$

It is straightforward to show that $\Pi^{RS(2aa)} > \Pi^{RS(1)}$, since $\frac{t}{1-t}\ell > 0$, $1-(1-p)G_T^{99(1)} > 0$ and $+pG_T^{100(1)} > 0$.

Proposition 9

Any yield-equivalent reform of the progressive tax $T = t(y)$ that reduces all tax liabilities in group 99 proportionally to their original net income (b) and increases all tax liabilities in group 100 proportionally to their original net income (b') will increase the global redistributive effect.

Proof

Using Equation [6] we can write the global redistributive effect of the original tax as:

$$\Pi^{RS(1)} = (s_Y - s_{Y-T}) + (1-p)\left[(1-s_Y)G_Y^{99} - (1-s_{Y-T})G_{Y-T}^{99(1)}\right] + p\left[s_Y G_Y^{100} - s_{Y-T}G_{Y-T}^{100(1)}\right] \quad [27]$$

According to propositions 4 and 6 we know that the Gini indices of net income for both groups (G_{Y-T}^{99} and G_{Y-T}^{100}) are not affected by the reform, and neither the corresponding Reynolds-Smolensky indices. We also know that the new liability share of group 100 is $s_T + \ell$, so we can express their new net income share as $s_{Y-T} - \ell\frac{t}{1-t}$. Therefore using Equation [6] we can write the global redistributive effect of the new tax ($\Pi^{RS(2bb)}$) as:

$$\Pi^{RS(2bb')} = \left(s_Y - s_{Y-T} + \ell\frac{t}{1-t}\right) + (1-p)\left[(1-s_Y)G_Y^{99} - \left(1 - s_{Y-T} + \ell\frac{t}{1-t}\right)G_{Y-T}^{99(1)}\right] + p\left[s_Y G_Y^{100} - \left(s_{Y-T} - \ell\frac{t}{1-t}\right)G_{Y-T}^{100(1)}\right] \quad [28]$$

Isolating [27] in [28] we obtain:

$$\Pi^{RS(2bb')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\left\{1 - (1-p)G_{Y-T}^{99(1)} + pG_{Y-T}^{100(1)}\right\} \quad [29]$$

Since $\frac{t}{1-t}\ell > 0$, $1 - (1-p)G_{Y-T}^{99(1)} > 0$ and $+pG_{Y-T}^{100(1)} > 0$, the condition $\Pi^{RS(2bb')} > \Pi^{RS(1)}$ is fulfilled.

Proposition 10

Any yield-equivalent reform of the progressive tax $T = t(y)$ that reduces all tax liabilities in group 99 proportionally to their gross income (c) and increases all tax liabilities in group 100 proportionally to their gross income (c') will increase the global redistributive effect.

Proof

By Equation [14] we know that the new tax liability for each taxpayer in group 100 is the original tax liability plus a fixed proportion of her gross income. Following Rietveld (1990) we can express the Gini index of the new tax liability as the weighted sum of the Gini index of the original tax liability ($G_T^{100(1)}$) plus the Gini index of the tax increase, which equals the Gini index of gross income (G_Y):

$$G_T^{100(2c\prime)} = \frac{s_T}{s_T+\ell} G_T^{100(1)} + \frac{\ell}{s_T+\ell} G_Y \qquad [30]$$

Applying the same rule to group 99 we have:

$$G_T^{99(2c\prime)} = \frac{(1-s_T)}{(1-s_T-\ell)} G_T^{99(1)} - \frac{\ell}{(1-s_T-\ell)} G_Y \qquad [31]$$

Replacing Equations [30] and [31] in Equation [10] we get:

$$\Pi^{RS} = \frac{t}{1-t}\Big\{(s_T + \ell - s_Y) + (1-p)\Big[\Big((1-s_T)G_T^{99(1)} - \ell G_Y\Big) - G_Y^{99}(1-s_Y)\Big] + p\Big(\Big(s_T G_T^{100(1)} + \ell G_Y\Big) - G_Y^{100} s_Y\Big)\Big\} \qquad [32]$$

Isolating [24] in [32] we obtain:

$$\Pi^{RS(2cc')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\{1 - (1-p)G_Y^{99} + pG_Y^{100}\} \qquad [33]$$

Once more, it is straightforward to show that $\Pi^{RS(2cc')} > \Pi^{RS(1)}$, since $\frac{t}{1-t}\ell > 0$,
$1 - (1-p)G_Y > 0$ and $+pG_Y > 0$.

Proposition 11

Let aa', bb' and cc' be three yield-equivalent reforms that reduce all tax liabilities in group 99 and increases all tax liabilities in group 100 at the same rate ℓ, and share this rate proportionally to, respectively, their original tax liability, original net income and gross income, their ranking in terms of redistribution is ambiguous.

Proof

Consider that gross income in group 99 is distributed equally among all individuals. In this case $G_Y^{99} = G_T^{99} = G_{Y-T}^{99} = 0$, so Equations [26], [29] and [33] will be:

$$\Pi^{RS(2aa')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\{1 + pG_T^{100}\} \qquad [34]$$

$$\Pi^{RS(2bb')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\{1 + pG_{Y-T}^{100}\} \qquad [35]$$

$$\Pi^{RS(2cc')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\{1 + pG_Y^{100}\} \qquad [36]$$

Since $G_T^{100} > G_Y^{100} > G_{Y-T}^{100}$, then $\Pi^{RS(2aa')} > \Pi^{RS(2cc')} > \Pi^{RS(2bb')}$.

Consider now that gross income in group 100 is distributed equally among all individuals. In this case $G_Y^{100} = G_T^{100} = G_{Y-T}^{100} = 0$, so Equations [26], [29] and [33] will be:

$$\Pi^{RS(2aa')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\{1-(1-p)G_T^{99}\} \quad [37]$$

$$\Pi^{RS(2bb')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell(1-(1-p)G_{Y-T}^{99}) \quad [38]$$

$$\Pi^{RS(2cc')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\{1-(1-p)G_Y^{99}\} \quad [39]$$

Since $G_T^{99} > G_Y^{99} > G_{Y-T}^{99}$, then $\Pi^{RS(2aa')} < \Pi^{RS(2cc')} < \Pi^{RS(2bb')}$.

Given that these two cases give opposite results, the ranking of the three reforms in redistribution terms is generally ambiguous. Therefore, the relative order among the three tax reform alternatives remain an empirical issue.

Proposition 12

For asymmetric partitions of the population where $p \to 0$ the following order is fulfilled: $\Pi^{RS(2bb')} > \Pi^{RS(2cc')} > \Pi^{RS(2aa')}$.

Proof

Applying $p \to 0$ to Equations [26], [29] and [332] we obtain:

$$\lim_{p\to 0} \Pi^{RS(2aa')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\left(1 - G_T^{99(1)}\right) \quad [40]$$

$$\lim_{p\to 0} \Pi^{RS(2bb')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\left(1 - G_{Y-T}^{99(1)}\right) \quad [41]$$

$$\lim_{p\to 0} \Pi^{RS(2cc')} = \Pi^{RS(1)} + \frac{t}{1-t}\ell\left(1 - G_Y^{99(1)}\right) \quad [42]$$

Since $G_T^{99} > G_Y^{99} > G_{Y-T}^{99}$, then $\Pi^{RS(2bb')} > \Pi^{RS(2cc')} > \Pi^{RS(2aa')}$.

3. AN ILLUSTRATION FOR SPAIN

To illustrate the results of the previous sections we use Spanish PIT microdata from 2011 to simulate the three tax reforms of Proposition 11: aa' (changes proportional to tax liability), bb' (changes proportional to net income) and cc' (changes proportional to gross income).

In particular we use the 2011 Spanish PIT Return Sample disseminated by the Spanish Institute for Fiscal Studies (Instituto de Estudios Fiscales, IEF) and the Spanish Tax Agency (Agencia Estatal de Administración Tributaria, AEAT) which contains more than 2 million observations representative of more than 19 million tax returns[3]. To ensure microdata consistent with the methodology, we have made zero all negative incomes, and then have removed the observations that showed inconsistent results in the original tax (gross income < tax base, tax base < gross tax liability, gross tax liability < net tax liability, average legal rate > maximum marginal legal rate). Some of those cases where errors, while others are a consequence of the dual specifications of the Spanish PIT. As a result we use 2,031,577 observations (99.79% of the original observations) that represent 19,430,040 tax returns (99.82%).

Table 1 shows the descriptive statistics of the microdata regarding gross income.

Table 1. Descriptive statistics of the Spanish PIT 2011 (microdata sample)

Concept		Group 99	Group 100	Total
Number of observations		1,885,082	146,495	2,031,577
Population represented		19,235,740	194,300	19,430,040
Gross income	Total (EUR)	409,096,697,360	39,226,087,286	448,322,784,646
	Average (EUR)	21,268	201,885	23,074
	Standard dv. (€)	15,461	557,243	60,537
	Minimum (€)	0	102,063	0
	Maximum (€)	102,0630	96,182,743	96,182,743
	Gini index	0.37659546	0.35587053	0.41801533

Source: Own elaboration.

In order to assess the reforms we cannot use the 2011 Spanish PIT as a reference, since like any other real income tax it does not fit the $T = t(y)$ model, because tax liabilities depend not only on income but also on other variables (income type, age, personal and family characteristics, region, tax incentives, etc.). In order to stay as close as possible to the real tax we simulate a stylized tax $T = t(y)$ with the same revenue and redistribution effect as the real tax applied in 2011. To ensure that average rates are also distributed in a similar way we keep the basic structure of the real tax, i.e.

$$T = t(y) = f(y - d(y)) - c(y) \qquad [42]$$

where $f(\cdot)$ represents the tax schedule, $d(y)$ are tax deductions and $c(y)$ tax credits. All these parameters depend only on total income or are constant. In particular, in our microsimulation exercises $f(\cdot)$ is the real tax schedule applied in 2011 to "general income" (Spanish PIT also incorporates a different schedule for "savings income"), while $d(y)$ has a fixed part and a part that is proportional to income (but with a fixed limit), and $c(y)$ is constant (but limited to ensure that $T \geq 0$). All these values try to reproduce the real variability originated by tax treatments based on non-income attributes.

Table 2 shows the final parameters chosen.

[3] Detailed information on the database can be found in Pérez *et al.* (2014).

Table 2. Parameters of the stylized tax

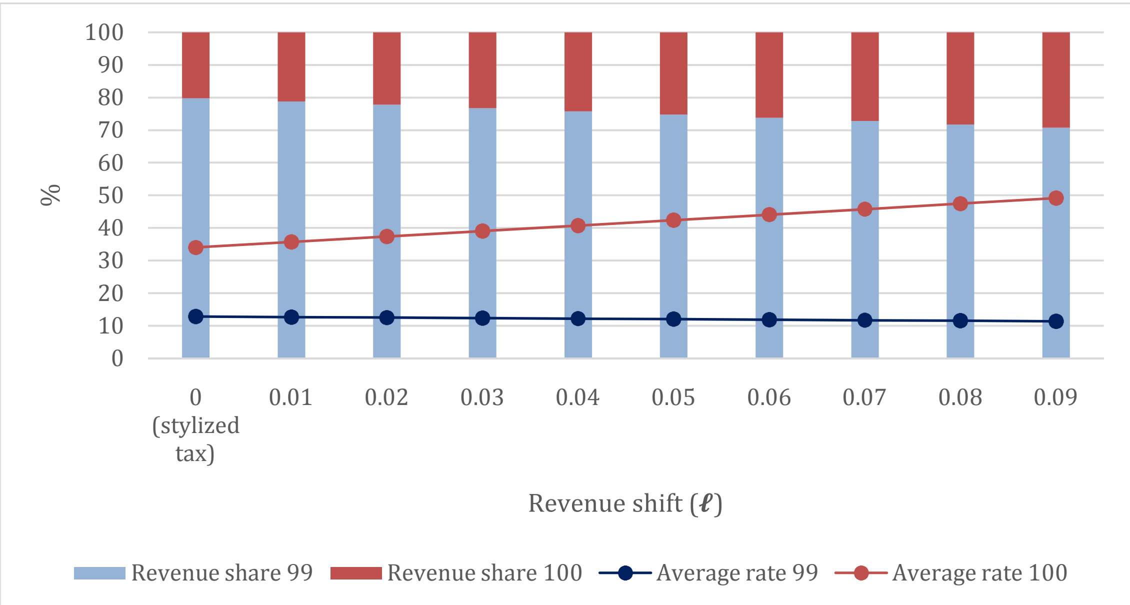
Parameter			Comments
	Taxable Income (EUR) – Marginal Rate		
$f(\cdot)$	0 - 17707.20	24%	Progressive tax schedule applied to "general income" in 2011
	17707.20 - 33007.20	28%	
	33007.20 - 53407.20	37%	
	53407.20 - 120000.20	43%	
	120000.20 - 175000.20	44%	
	> 175000.20	45%	
$d(\cdot)$	$\min(2500 + \min(.1372186y, 50000), y)$		EUR 2,500, plus 13.72186% of gross income with a limit of EUR 50,000. The deduction cannot be higher than gross income.
$c(\cdot)$	$\min(1591, f(y - d(y)))$		EUR 1,591 with the limit of the gross tax liability previously calculated

Source: Own elaboration.

Taking this stylized tax as a starting point we simulate the three types of reform (aa', bb' and cc') for several values of ℓ. We start by $\ell = 0.01$ (i.e. we shift 1% of the overall revenue from group 99 to group 100) and keep increasing this value (in steps of 0.01) while the effective average tax rate of group 100 is lower than 0.50. Although these simulations are only an illustration of the previous theoretical developments, we understand that this is a reasonable limit for the average tax rate of that group.

Table 3 shows all the shares, rates and Gini indices calculated in the simulations. Figure 1 shows graphically the revenue shares of group 99 ($1 - s_T$) and 100 (s_T) and their corresponding average tax rates (t^{99} and t^{100}) for all the simulated taxes.

Figure 1. Revenue shares and average effective tax rates



These results are the same for the three reforms, because the graph only shows the share of taxes between groups. The differences arise when we analyse the share of taxes within each group and the redistributive impact they have. To show these effects we use the decomposition in Equation [5]. Figures 2 to 4 show the results for each type of reform.

Figure 2. Redistributive effect of reforms type aa′

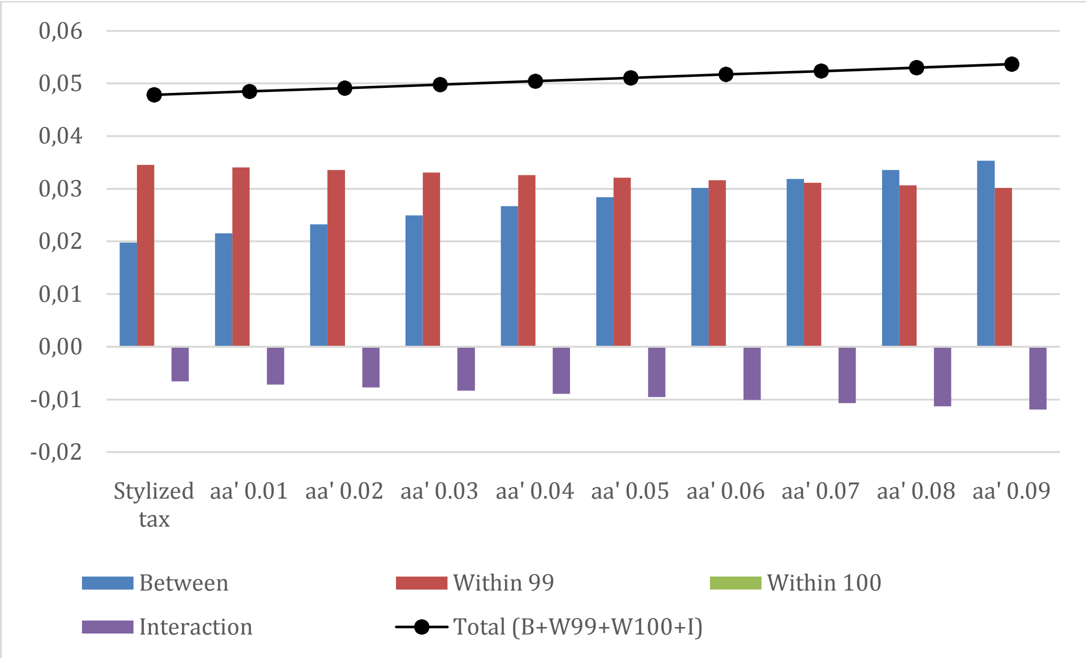


Figure 3. Redistributive effect of reforms type *bb′*

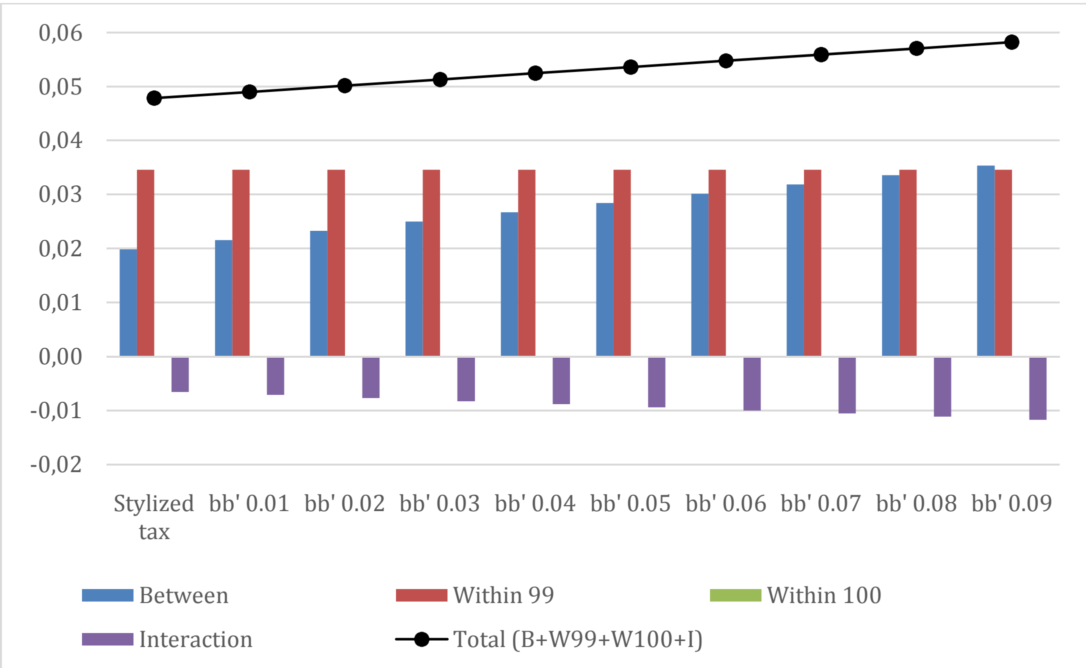
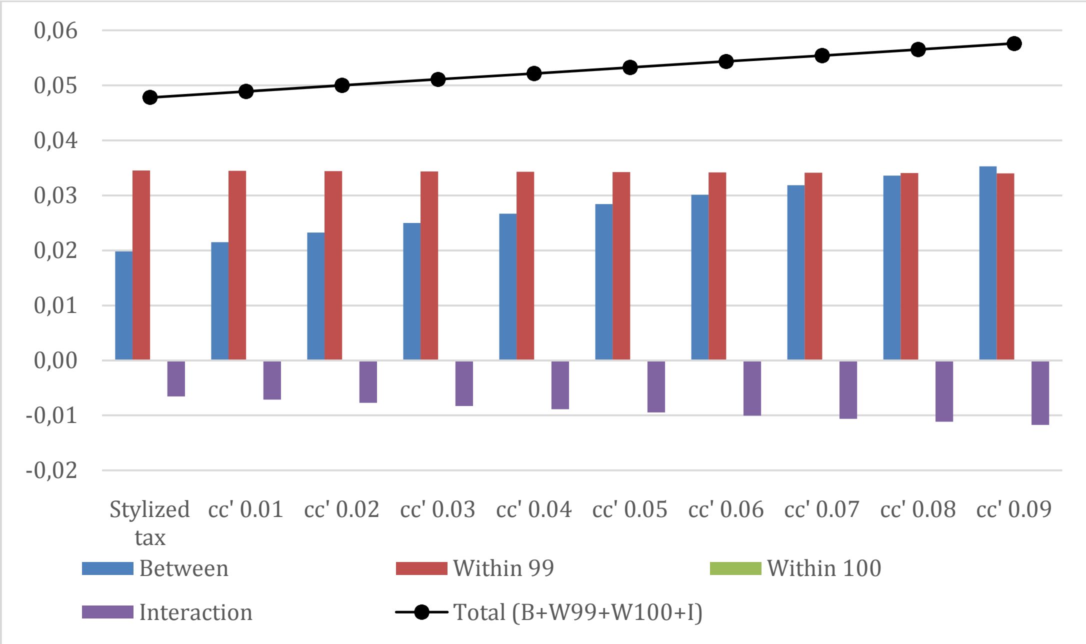


Table 3. Results of the simulations

				Stylized tax	Shift (ℓ)								
					0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Shares			s_Y	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495
			s_T	0.202389	0.212389	0.222389	0.232389	0.242389	0.252389	0.262389	0.272389	0.282389	0.292389
			s_{Y-T}	0.067691	0.065968	0.064244	0.062520	0.060797	0.059073	0.057349	0.055626	0.053902	0.052178
Tax rates			t^{99}	0.128512	0.126901	0.12529	0.123679	0.122068	0.120456	0.118845	0.117234	0.115623	0.114011
			t^{100}	0.340089	0.356893	0.373697	0.390501	0.407304	0.424108	0.440912	0.457715	0.474519	0.491323
			t	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024
Gross income			G_Y^{99}	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595
			G_Y^{100}	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871
			G_Y	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015
Reforms	aa'	Tax liabilities	G_T^{99}	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066
			G_T^{100}	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294
			G_T	0.695528	0.699275	0.703021	0.706767	0.710514	0.714260	0.718007	0.721753	0.725499	0.729246
		Net income	G_{Y-T}^{99}	0.338333	0.338882	0.339430	0.339975	0.340519	0.341060	0.341600	0.342137	0.342672	0.343206
			G_{Y-T}^{100}	0.315454	0.312349	0.309077	0.305625	0.301977	0.298116	0.294024	0.289677	0.285052	0.280122
			G_{Y-T}	0.370181	0.369536	0.368890	0.368245	0.367599	0.366955	0.366310	0.365667	0.365024	0.364383
	bb'	Tax liabilities	G_T^{99}	0.636066	0.638426	0.640810	0.643220	0.645654	0.648113	0.650596	0.653103	0.655636	0.658196
			G_T^{100}	0.434294	0.428699	0.423607	0.418953	0.414683	0.410751	0.407119	0.403754	0.400627	0.397714
			G_T	0.695528	0.701036	0.706508	0.711946	0.717347	0.722711	0.728037	0.733323	0.738571	0.743780
		Net income	G_{Y-T}^{99}	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333
			G_{Y-T}^{100}	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454
			G_{Y-T}	0.370181	0.369030	0.367878	0.366727	0.365576	0.364426	0.363276	0.362128	0.360981	0.359836
	cc'	Tax liabilities	G_T^{99}	0.636066	0.638125	0.640208	0.642319	0.644455	0.646617	0.648805	0.651020	0.653262	0.655531
			G_T^{100}	0.434294	0.430602	0.427241	0.424170	0.421352	0.418758	0.416361	0.414140	0.412077	0.410155
			G_T	0.695528	0.700814	0.706071	0.711302	0.716504	0.721677	0.726820	0.731933	0.737014	0.742063
		Net income	G_{Y-T}^{99}	0.338333	0.338404	0.338474	0.338544	0.338614	0.338683	0.338753	0.338822	0.338891	0.338959
			G_{Y-T}^{100}	0.315454	0.314398	0.313286	0.312112	0.310871	0.309558	0.308166	0.306688	0.305115	0.303438
			G_{Y-T}	0.370181	0.369094	0.368007	0.366920	0.365834	0.364748	0.363663	0.362579	0.361495	0.360414

Source: Own elaboration.

Figure 4. Redistributive effect of reforms type cc′



The three figures confirm empirically the results of Propositions 8 to 10 –all the reforms are more redistributive than the original tax– and 12 –the ranking from more to less redistributive is bb', cc', aa', what is consistent with the assumption $p \to 0$–. However, as can be seen in Table 3 the distance between aa' and cc' is much higher than between cc' and bb', which is related to the higher distance between G_T and G_Y than between G_Y and G_{Y-T} (see Proposition 12). We also see that within each reform type the redistributive effect rises as ℓ increases, which is a direct consequence of Propositions 8, 9 and 10.

Regarding the changes in the different partial effects considered we also see that in all the reforms the Between effect increases while the Interaction effect decreases. Both the Within 99 and Within 100 effects are constant for bb', which can be derived directly from Equation [5]; for aa' and cc' Within 99 decreases and Within 100 increases, but the latter effect is almost negligible due to the small population share of the last centile. In general the total redistributive effect is driven mostly by the Within 99 effect, although the Between effect exceeds it for high values of ℓ. Finally the Interaction effect grows in the opposite direction as ℓ increases, so it smooths the increase of total redistribution.

4. CONCLUDING REMARKS

Throughout this paper we have developed a methodology to assess PIT reforms that shift part of the tax burden towards the top 1% income earners, keeping overall revenue constant. Based on the Kakwani decomposition of the Reynolds-Smolensky index, and using the decompositions by income groups by Lambert and Aronson (1993) and Alvaredo (2011), we have developed a theoretical framework that allow us to obtain conclusions in terms of redistribution on a set of reforms based on Pfähler (1984). We also illustrate the results with an empirical exercise for Spain using a PIT microdata sample.

The main conclusions of this paper are the following:

- The overall redistribution of this type of reforms can be decomposed in a Between effect (that measures the pure effect of the shift) and a Within effect for each group (that measures how the distribution of tax changes within the group affect total redistribution). Depending on the way we make the decomposition there may be an additional interaction term.
- In principle, the redistributive result of this type of reforms is ambiguous in redistributive terms, since there are positive effects (Between and Within for the "rich") and negative effects (Within for the "poor" and Interaction).
- For three types of reforms based on Pfähler (1984) (that consist of allocating the tax changes proportionally to tax liabilities, net income or gross income) we show that the redistributive effect is always higher than before the reform.
- The ranking among those three types of reform is ambiguous except when the population size of the rich group is sufficiently small (empirically verified for Spain when p = 1%). In this case the best option is allocating tax changes proportionally to net income, and the worst doing it proportionally to tax liabilities.

As we exposed in the introduction, the motivation of this paper was to shed light on the potential capacity to reduce income inequality through a tax increase on the highest income individuals. The main objective of our study has been to develop a theoretical framework for accurately analysing the underlying drivers of the redistributive effects of this kind of reforms. Nevertheless, we think that there is still space for further research. In particular, we consider it crucial to extend our theoretical framework to incorporate behavioural responses to the tax changes proposed.

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APPENDIX I

Musgrave and Thin (1948) formally establish measures of progression along the income distribution that are widely used in the analysis of tax structures. Following Lambert (2001), these measures of local progression for a simplified tax $T(y)$ that only depends on income, whose marginal and average tax rates are $m(y)$ and $t(y)$ respectively, are defined as:

Average Rate Progression (ARP)

This measure shows the rate of increase of the average rate when gross income increases:

$$ARP(y) = \frac{d\,\frac{T(y)}{y}}{d\,y} = \frac{yT'(y)-T(y)}{y^2} = \frac{m(y)-t(y)}{y} > 0 \qquad \text{[A.I.1]}$$

For a strict progression it is necessary and sufficient that $m(y) > t(y)$ for all y. Lambert (1993) proposes the following useful transformation:

$$ARP^*(y) = yARP(y) = m(y) - t(y) > 0 \qquad \text{[A.I.2]}$$

Liability Progression (LP)

It is defined at any income level y for which $T(y) > 0$ as the elasticity of tax liability to pre-tax income:

$$LP(y) = \eta_{T(y),y} = \frac{d\,T(y)}{d\,y}\frac{y}{T(y)} = \frac{yT'(y)}{T(y)} = \frac{m(y)}{t(y)} > 1 \qquad \text{[A.I.3]}$$

Residual Progression (RP)

It is defined at all income levels y as the elaticity of post-tax income to pre-tax income:

$$RP(y) = \eta_{y-T(y),y} = \frac{d\,(y-T(y))}{d\,y}\frac{y}{y-T(y)} = \frac{y\left(1-T'(y)\right)}{y-T(y)} = \frac{1-m(y)}{1-t(y)} < 1 \qquad \text{[A.I.4]}$$

For interpretation consistency Lambert (2001) proposes expressing $RP(y)$ through its inverse $RP^*(y) = 1/RP(y)$, so that:

$$RP^*(y) = \frac{1-t(y)}{1-m(y)} > 1 \qquad \text{[A.I.5]}$$

Then, $RP^*(y)$ can be interpreted as the elasticity of pre-tax income to post-tax income, so that an increase in $RP^*(y)$ implies an increase in progression or that the tax is more residually progressive at y. In this paper, we use $RP^*(y)$ and $ARP^*(y)$ for comparing the tax reform alternatives considered.

APPENDIX II

For the proof of Proposition 1 we use Ruiz-Huerta *et al*. (1995).

Consider $RP^*(y) = 1/RP(y)$ so that the residual progression can be expressed as:

$$RP^*(y) = \frac{1-t(y)}{1-m(y)} > 1 \quad \text{[A.II.1]}$$

In the case of a reduction proportional to net income (b), the average and marginal rates of taxpayers included in group 99 after the tax cut will be:

$$t_b^{(2)} = \frac{T^{(1)}(y)}{y} - \beta\frac{y-T^{(1)}(y)}{y} = t^{99(1)} - \beta\left(1-t^{99(1)}\right) \quad \text{[A.II.2]}$$

$$m_b^{(2)} = T^{(1)\prime}(y) - \beta\left(1-T^{(1)\prime}(y)\right) = m^{99(1)} - \beta\left(1-m^{99(1)}\right) \quad \text{[A.II.3]}$$

From these definitions it is easy to prove that $RP^{*99(1)}(y) = RP^{*}{}_b^{99(2)}(y)$:

$$RP^{*}{}_b^{99(2)}(y) = \frac{1-t^{99(1)}+\beta\left(1-t^{99(1)}\right)}{1-m^{99(1)}+\beta\left(1-m^{99(1)}\right)} = \frac{(1+\beta)\left(1-t^{99(1)}\right)}{(1+\beta)\left(1-m^{99(1)}\right)} = RP^{*99(1)}(y) \quad \text{[A.II.4]}$$

If the tax cut is applied proportionally to gross income (c), the two tax rates will be:

$$t_c^{(2)} = \frac{T^{(1)}(y)-\varsigma y}{y} = t^{99(1)} - \varsigma \quad \text{[A.II.5]}$$

$$m_c^{99(2)} = \frac{d\left(T^{(1)}(y)-\varsigma y\right)}{dy} = T^{(1)\prime}(y) - \varsigma = m^{99(1)} - \varsigma \quad \text{[A.II.6]}$$

Now we prove that $RP^{*}{}_b^{99(2)}(y) > RP^{*}{}_c^{99(2)}(y)$:

$$RP^{*}{}_c^{99(2)}(y) = \frac{1-t^{99(1)}+\varsigma}{1-m^{99(1)}+\varsigma} < \frac{\left(1-t^{99(1)}\right)}{\left(1-m^{99(1)}\right)} = RP^{*99(1)}(y) = RP^{*}{}_b^{99(2)}(y) \quad \text{[A.II.7]}$$

Finally, if the tax cut is applied proportionally to tax liability (a), the two tax rates will be:

$$t_a^{(2)} = \frac{T^{(1)}(y)(1-\alpha)}{y} = t^{99(1)}(1-\alpha) \quad \text{[A.II.8]}$$

$$m_a^{99(2)} = \frac{d\left(T^{(1)}(y)(1-\alpha)\right)}{dy} = T^{(1)\prime}(y)(1-\alpha) = m^{99(1)}(1-\alpha) \quad \text{[A.II.9]}$$

Since $\varsigma = \alpha\bar{t}$, and for $m^{99(1)} > t^{99(1)}$, $\varsigma < \alpha\bar{t}$, then $RP^{*}{}_a^{99(2)}(y) < RP^{*}{}_c^{99(2)}(y)$:

$$RP^{*}{}_a^{99(2)} = \frac{1-t^{99(1)}+\alpha t^{99(1)}}{1-m^{99(1)}+\alpha m^{99(1)}} < \frac{1-t^{99(1)}+\varsigma}{1-m^{99(1)}+\varsigma} = RP^{*}{}_c^{99(2)}(y) \quad \text{[A.II.10]}$$

Consequently we prove that the following ranking applies to the three options considered:

$$RP^{*99(1)}(y) = RP^{*}{}_{b}^{99(2)}(y) > RP^{*}{}_{c}^{99(2)} > RP^{*}{}_{a}^{99(2)} \quad \text{[A.II.11]}$$

APPENDIX III

From the definitions of average and marginal tax rates in Appendix II for taxpayers in the group 99 we have:

$$ARP^{*}{}_{a}^{99(2)}(y) = m^{99(1)}(1-\alpha) - t^{99(1)}(1-\alpha) = (1-\alpha)\left(m^{99(1)} - t^{99(1)}\right) \quad \text{[A.III.1]}$$

$$ARP^{*}{}_{b}^{99(2)}(y) = \left[m^{99(1)} - \beta\left(1 - m^{99(1)}\right)\right] - \left[t^{99(1)} - \beta\left(1 - t^{99(1)}\right)\right] = (1+\beta)\left(m^{99(1)} - t^{99(1)}\right) \quad \text{[A.III.2]}$$

$$ARP^{*}{}_{c}^{99(2)}(y) = \left(m^{99(1)} - \varsigma\right) - \left(t^{99(1)} - \varsigma\right) = m^{99(1)} - t^{99(1)} = ARP^{*99(1)}(y) \quad \text{[A.III.3]}$$

It is trivial that for $\alpha, \beta > 0$ and for $m > t$ the following ranking applies:

$$ARP^{*}{}_{a}^{99(2)}(y) < ARP^{*}{}_{c}^{99(2)}(y) = ARP^{*99(1)}(y) < ARP^{*}{}_{b}^{99(2)}(y) \quad \text{[A.III.4]}$$

APPENDIX IV

Liability progression is defined at any income level y as the elasticity of tax liability to pre-tax income, so that:

$$LP(y) = \eta_{T(y),y} = \frac{d\,T(y)}{d\,y}\frac{y}{T(y)} = \frac{yT'(y)}{T(y)} = \frac{m(y)}{t(y)} > 1 \quad \text{[A.IV.1]}$$

From the definitions of average and marginal tax rates in Appendix II for taxpayers in the group 99 we get the values for liability progression:

$$LP^{*}{}_{a}^{99(2)}(y) = \frac{m^{99(1)}(1-\alpha)}{t^{99(1)}(1-\alpha)} = \frac{m^{99(1)}}{t^{99(1)}} = LP^{*99(1)}(y) \quad \text{[A.IV.2]}$$

$$LP^{*}{}_{b}^{99(2)}(y) = \frac{m^{99(1)} - \beta\left(1 - m^{99(1)}\right)}{t^{99(1)} - \beta\left(1 - t^{99(1)}\right)} \quad \text{[A.IV.3]}$$

$$LP^{*}{}_{c}^{99(2)}(y) = \frac{m^{99(1)} - \varsigma}{t^{99(1)} - \varsigma} \quad \text{[A.IV.4]}$$

Since in a progressive tax $m^{99(1)} > t^{99(1)}$, for $\varsigma > 0$ we verify that for any positive value of gross income $LP^{*}{}_{a}^{99(2)}(y) < LP^{*}{}_{c}^{99(2)}(y)$. Additionally, since $\beta > \varsigma$, the progressivity condition $m^{99(1)} > t^{99(1)}$ ensures that $LP^{*}{}_{c}^{99(2)}(y) < LP^{*}{}_{b}^{99(2)}$, therefore:

$$LP^{*}{}_{a}^{99(2)}(y) = LP^{*99(1)}(y) < ARP^{*}{}_{c}^{99(2)}(y) < ARP^{*}{}_{b}^{99(2)}(y) \quad \text{[A.IV.5]}$$