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Abstract

This paper analyzes the driving factors behind the business cycle dynamics of carbon emissions in the U.S. economy from 1975Q1 to 2023Q3. We first identify some key stylized facts regarding the correlation between carbon emissions and the different components of the Kaya decomposition, some of which exhibit a sharp change in sign around the trend reversal of the environmental Kuznets curve in the late 20th century. From the estimated distribution of shocks in a dynamic stochastic general equilibrium environmental model, we find that: (a) innovations in green energy production play a marginal role in U.S. emissions cycles; (b) barely 17 percent of total emissions cycles are explained by aggregate shocks like those to total factor productivity or household consumption, while the rest stem from innovations in the efficiency in production of brown energy (brown energy productivity shock) and emissions per unit of brown energy; (c) since 2000, brown energy shocks have positively affected (increased) emissions growth, while emissions technology shocks have negatively impacted emissions, particularly following a structural break around 2007; and (d) without these shocks, the U.S. would have experienced a negative emissions gap for over 40 years. Since 2007, emissions reduction has accelerated, leading to convergence of observed and counterfactual Kuznets curves at around $16,000 per capita GDP. Our findings explain the intriguing negative correlation between emissions and the share of dirty energy observed over the past twenty years. They suggest a connection to innovations in shale oil and gas production, highlighting both the limited potential for emission reduction through advances in producing a "cleaner" brown energy mix, and the urgent need for a decisive shift to renewable energy to achieve long-term climate goals.

Keywords: emissions, cycles, energy, Kuznets Curve, Kaya Identity, E-DSGE model, counterfactuals.
JEL Classification: E32, E37, Q43.

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1. Introduction

The global shift toward a low-carbon economy is a top priority for policy makers worldwide, highlighting the urgency of addressing climate change and reducing emissions. According to the Intergovernmental Panel on Climate Change, "Climate change is a threat to human well-being and the health of the planet" (IPCC, 2023). This urgency has motivated the signing of numerous environmental treaties, most notably the 2015 Paris Agreement that aims at keeping "the increase in the global average temperature well below 2 ° C above preindustrial levels," to mitigate many climate change risks (UNFCCC, 2015). In parallel, the International Energy Agency (IEA) has set the target of Net Zero Emissions by 2050, consistent with the goals of the Paris Agreement (IEA, 2020).

As most countries have intensified their commitment to environmental protection, a deeper understanding of the determinants of climate change has become essential for designing effective environmental policies. Climate change is a pressing issue that, if not effectively addressed, threatens living standards and social welfare. According to the World Bank, sustainable development encompasses three fundamental pillars: economic growth, environmental protection, and social inclusion (World Bank, 2015). Therefore, environmental policies should not impede economic development or exacerbate income inequality.

The interaction between the environment and the economy has mainly been examined from a long-term perspective, either to study the environmental damage that accompanies economic activity or to understand how different aspects of environmental policy impact economic growth. Fewer studies have adopted a shorter-term perspective, analyzing the relationship between economic activity and environmental degradation at the frequency of the business cycle. From this perspective, understanding the relationship between factors underlying the production cycle and the carbon emissions cycle can be relevant for guiding economic policies that seek to maximize the decoupling of GDP and emissions during expansions and minimize it during recessions. In summary, a more detailed understanding of the relationship between economic fluctuations and emissions is crucial to developing effective environmental policies that align with international goals without compromising economic growth.

In this paper, we analyze the behavior of the carbon emissions cycle and the various factors into which it can be decomposed. The study focuses on the U.S. economy over the period from the first quarter of 1975 to the third quarter of 2023. We first describe the relationship between emissions and their determinants and describe the evolution of emissions and the production of various energy sources in the U.S. economy through the lenses of alternative versions of the environmental Kuznets curve (EKC), which relates carbon emissions to economic activity. Next, we turn our attention to the different elements of the Kaya Identity that decompose carbon emissions into emissions intensity per unit of brown (dirty) energy used, the demand for brown energy as a share of total energy consumption, the energy intensity or total energy consumption per unit of GDP, and GDP itself. This decomposition allows us to obtain a series of stylized facts about the emissions cycle, based on second moments such as the correlation of each factor with emissions and the volatility of these factors.

To explain the observed cyclical pattern of emissions and their determinants, in the second part of the paper,, we conduct a structural analysis estimating a dynamic stochastic general equilibrium (E-DSGE) model. The cyclical analysis of emissions finds a natural tool in E-DSGE models (see Nordhaus, 1991, or Annicchiarico, *et al.*, 2021). These models extend DSGE models to capture the interrelations of environment and economic activity. With the estimated model, we obtain the response of emissions and GDP to different structural shocks, calculate the variance decomposition of the forecast error for the emissions cycle and the GDP cycle, perform a historical decomposition analysis for emissions, and conduct a series of counterfactual exercises to assess the relevance of the estimated shocks to explain the changing pattern of emissions since 1976.

The reference model is based on the work of Andrés, Boscá, Doménech, and Ferri (2024a, 2024b) — hereafter *ABDF* — which considers energy as a specific production factor, alongside labor and capital, and distinguishes between environmentally responsible energy generation or "green" energy and energy generated from fossil fuels, or "brown" energy, whose use generates carbon emissions. We make two key modifications to this model. First, while the structural parameters of the original model are calibrated for the Spanish economy, in this paper the calibration is based on the main ratios and estimated elasticities of the U.S. economy. Second, we introduce various shocks whose posterior distributions are are estimated following a Bayesian method: technological shocks to total factor productivity (TFP), consumption preference shocks, efficiency shocks in green and brown energy production, and shocks to emissions per unit of brown energy.

Unlike some previous empirical studies that consider only standard productivity shocks to TFP or investment (see Khan *et al.*, 2019), or where the exact nature of the shocks remains indeterminate (see Jo and Karnizova, 2021), the use of a rich E-DSGE model makes it possible to account for other identified shocks that admit a precise interpretation. This approach allows us to provide a more structural explanation of a larger portion of the emissions cycle, which remains unexplained for two-thirds in Khan *et al.* (2019).

It is widely recognized that carbon emissions and GDP are closely linked. However, this relationship has gradually weakened over time, with a progressive — albeit still insufficient — decoupling of economic growth from emissions. In this paper, we show that this long-term decoupling coincides with other technological changes of various kinds, which affect the emissions and GDP cycles differently and, therefore, alter the dynamic relationship between the two variables. Considering these effects is crucial for the evaluation and design of environmental policy and its interaction with macroeconomic policies at the frequency of the business cycle.

The main result from the descriptive analysis is that the correlation at the cyclical frequency between emissions and the intensity of dirty energy in the energy mix has become negative and significant since the early 21st century, coinciding with the period during which the decoupling between economic growth and per capita emissions in the US has materialized in the Kuznets curve. It is important to note that this correlation was positive or zero before around 2000, and it has become negative thereafter, suggesting that increases (decreases) in the share of dirty energy over the total amount of energy used have coincided with reductions (increases) in emissions. At first glance, this finding may seem counterintuitive, as emissions might increase as the share of polluting energies in total energy production increases. However, this observation may provide relevant insights into the dynamics of technological innovation in the polluting energy production sector, due to the changing composition of the brown energy mix.

This result is further supported by the structural analysis derived from the estimated model, where we find that about four-fifths of the variance in the emissions cycle is explained by specific technological shocks affecting dirty energy and emissions per unit of brown energy use, complementing the partial result obtained by Khan, *et al.* (2019). It is not the macroeconomic factors driving the production cycle (TFP and consumption shocks) that mainly account for emissions variance, but rather those related to technology in brown energy production and its emissions. Notably, for the U.S. economy, shocks affecting green energy production efficiency play a very marginal role in the emissions cycle.

A closer examination of the structural shocks reveals two significantly different periods within the downward-sloping trend of emissions that started around 2000, with a clear breaking point around 2007. Between 2000 and 2007, the change in the slope of the Kuznets curve with respect to its pre-2000 profile is associated with negative shocks affecting both total factor productivity (TFP) and brown energy production, the two shocks contributing to moderate carbon emissions. From 2008 onward, the increasing use of new extraction technologies in shale production, such as hydraulic fracturing (fracking) and horizontal drilling, reduced the price of oil and, particularly, gas, thereby limiting the substitution of brown energies with green energies. Our model identifies this process through a sequence of shocks that contribute to a relative increase in emissions compared to a scenario without such shocks. Alongside the reduction in gas and oil prices, the share of these in brown energy production increased while the share of coal decreased. Our model interprets this shift as a sequence of shocks that led to a reduction in emissions per unit of dirty energy used.

According to our counterfactual exercises, the sequence of shocks affecting both brown technology production and emissions generated per unit of dirty energy kept emissions higher than expected without these shocks for 40 years. However, they also contributed to the pronounced acceleration in emissions reduction since 2007, leading to current emissions per unit of GDP converging with those that would be observed in the absence of such shocks. Nevertheless, this time evolution has resulted in a higher stock of greenhouse gasses than the one we would have observed in the absence of these energy shocks.

The remainder of the paper is organized as follows: Section 2 reviews the literature on the study of the emissions cycle. Section 3 presents our descriptive analysis. Section 4 briefly describes the model by Andrés, *et al.* (2024a, 2024b) and details the main modifications made. Section 5 discusses the calibration and estimation of the model. Section 6 presents a set of results derived from the estimated model. Section 7 concludes.

2. Related Literature

Several studies have addressed the long-term evolution of emissions and the factors that determine it. According to Känzig and Williamson (2024), and Teives-Henriques and Borowiecki (2017), the main factor contributing to mitigating the growth of emissions is energy intensity. Allcott and Greenstone (2012) find that the U.S. economy has improved its energy productivity 2.4 times since 1949, although they caution that the U.S. remains relatively more energy-intensive than other countries. This intensity is determined by economic growth. Zhou *et al.* (2021) estimate that a 1% increase in per capita GDP leads to a 0.62% to 0.78% reduction in energy intensity, though they warn that this effect weakens in advanced stages of economic growth. Technological progress also plays a crucial role in the continuous improvement of energy efficiency, along with changes in the energy mix, urbanization, and industrial structure, as shown by Voigt *et al.* (2014). Even the development of the financial system can be an influential factor in energy intensity, especially in developing or emerging countries, as highlighted by Chen *et al.* (2019).

The study of factors behind the emissions cycle is relatively recent. Among the pioneering works is Heutel (2012), which marks a significant precedent in the study of the relationship between economic cycles and CO2 emissions in the United States. The author concludes three clear facts: 1) Emissions are procyclical, meaning they increase during expansions and decrease during recessions; 2) Energy intensity decreased during the period 1981–2003; 3) Emissions are inelastic with respect to GDP, but the elasticity value can vary over time and with the level of production.

Khan *et al.* (2019) conduct an empirical and theoretical analysis of the emissions cycle in the United States. These authors identify six disturbances within a structural vector autoregression (SVAR): an anticipated and unanticipated neutral technology (TFP) shock, an anticipated and unanticipated investment-specific shock, a government spending shock, and a monetary policy shock. The authors find that the joint contribution of all these aggregate shocks to the forecast error variance (FEV) of emissions is approximately 33%, which implies that around 66% of the emissions cycle dynamics cannot be accounted for by the standard shocks that drive the business cycle dynamics of most macroeconomic aggregates. Therefore, they warn that a significant part of the variation in emissions is due to yet unidentified structural shocks.

These studies suggest the existence of a trade-off between decarbonization policies and economic activity, either because the empirical analysis based on correlations between emissions and GDP pointed in that direction, or because the nature of the more traditional shocks considered in the models generated a positive correlation between emissions and the economic cycle. However, a recent study by Jo and Karnizova (2021) points to other types of shocks that can generate a negative correlation between GDP and emissions that explain nearly half of the volatility in emissions on average. This implies that achieving a certain level of emissions reduction does not necessarily require that economic activity must contract. Thus, models that omit disturbances to energy production efficiency overestimate the costs of environmental policy. However, Jo and Karnizova (2021) do not identify the precise nature of these shocks nor embed them into a more general macroeconomic framework, which would enlighten the joint analysis of environmental and other macroeconomic policies.

The main contribution of this paper is to analyze the drivers of the emissions cycle, in a structural framework that combines aggregate and energy-specific shocks. In particular we identify the shocks that may result in positive or negative correlations between economic activity and total emissions and assess their relative importance in the emission cycle in the U.S. during the sample period.

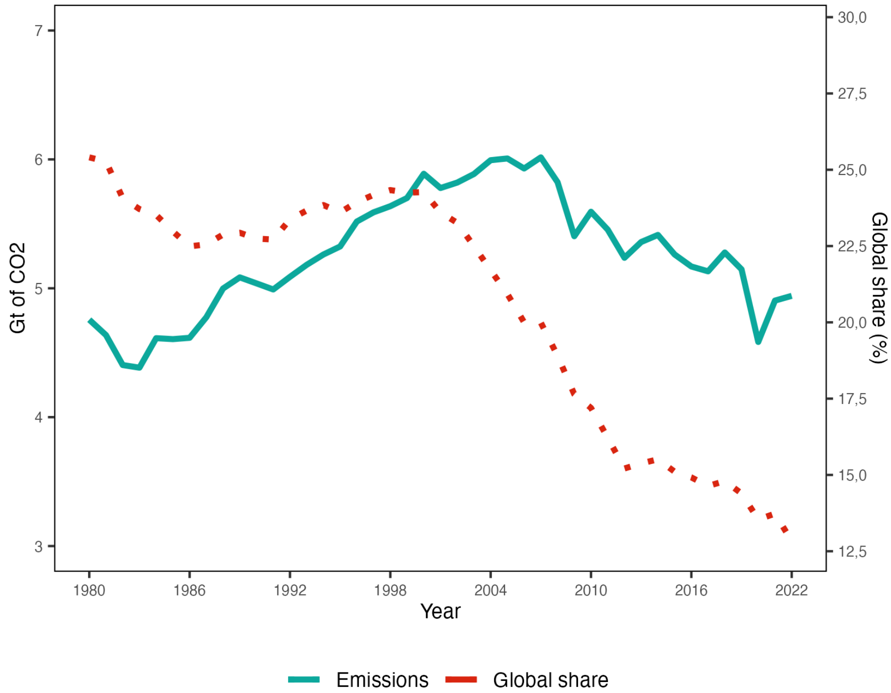
3. US Carbon Emissions Stylized Facts

In this section, we provide evidence of some empirical facts related to carbon emissions in the United States. First, to frame the analysis of the cycle, which is the main objective of this work, we will present long-term evidence on the evolution of emissions and energy consumption. In the second part of this section, we will conduct a second-moment analysis, which will allow us to extract some novel stylized facts about carbon emissions cycles in the United States.

3..1 Long-Term evolution of emissions and energy consumption

Before analyzing the cyclical frequency, we examine CO2 emissions and energy consumption trends over the past four decades, comparing the U.S. with other economies. Figure 1 shows that between 1980 and 2007, U.S. CO2 emissions increased by 27%, from 4.76 Gt to 6.02 Gt. Despite this rise, the U.S. share of global emissions decreased from 25% to 20% due to higher increases in other economies' emissions. The Great Recession

Figure 1: *Evolution of CO2 emissions*



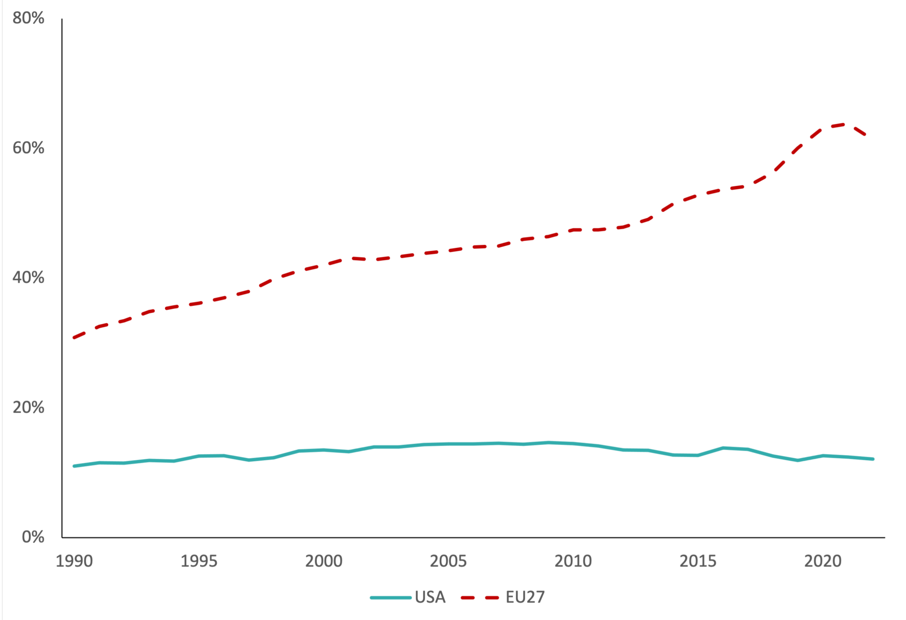
Source: Own elaboration based on Energy Information Administration (EIA).

caused a structural break, leading to a continuous decline in U.S. emissions. By 2022, emissions had dropped to 5 Gt, a 17% decrease from 2007, reducing the U.S. share of global emissions from 25.4% to 12.8%. This reduction in the U.S. share coincides with increases in emissions from other major economies. China's share surged from 8% in 1980 to 33% in 2022, and India's rose from 1.5% to 7%. In contrast, the EU-27 reduced its share by 13.6 percentage points, the most significant reduction among major emitters.

The structure of the energy sector is a critical determinant of emissions. Figure 2 represents the share of green energy on total energy produced. Green energy includes all sources that do not create carbon emissions when used: nuclear, hydroelectric, geothermal, solar, and wind[2]. Brown energy includes coal, gas, and oil. The difference in green energy production between the US and Europe is striking. Whereas the share of green energy production in the US remained almost constant and consistently below a meager 15%, in Europe it increased from 30% to more than 60%.

[2] We have removed biofuels from total energy produced

Figure 2: *Share of green energy production (excluding biofuels): USA vs EU-27 (from 2020)*



Source: Own elaboration based on data on primary energy production by source (EIA and European Commission) https://www.eia.gov/totalenergy/data/annual/ *and* https://energy.ec.europa.eu/data-and-analysis/eu-energy-statistical-pocketbook-and-country-datasheets_en

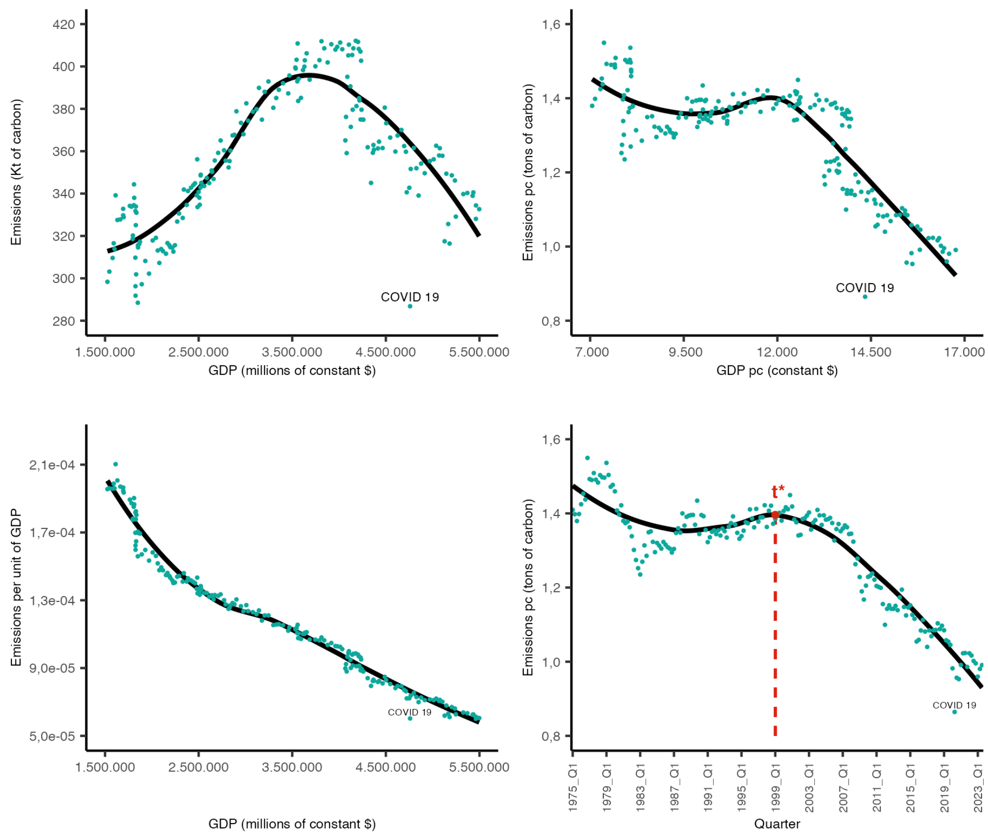
Environmental Kuznets curve

The EKC illustrates the relationship between GDP and emissions. Figure 3 shows different versions of the EKC: a) Emissions versus GDP, b) Emissions per capita versus GDP per capita, and c) Emission Intensity (Emissions/GDP) versus GDP. Additionally, the figure displays the evolution of per capita emissions over time, with non-linear fits to the scatter plots.

The representation of the EKC displays a clear hump-shaped relationship. In the US, per capita emissions decreased from approximately 1.4 tons per quarter to less than 1 ton, while real per capita GDP increased from $7,000 to $17,000 per quarter. This trend is consistent with a continuous reduction in emission intensity per unit of GDP, as shown in the lower left panel of Figure 3. Currently, emission intensity per unit of GDP is one-fifth of what it was in the first quarter of 1975, confirming that the U.S. economy has improved its environmental efficiency.

The non-linear fit of per capita emissions over time identifies the turning point of the EKC as the first quarter of 1999. From this point onward, the economy has been on the downward-sloping segment of the curve that captures the decoupling between emissions and GDP, as well as between per capita emissions and GDP per capita. This

Figure 3: *Environmental Kuznets curve (Quarterly flows)*



critical point t^* will play an important role in the analysis of the cycle frequency in the following subsection.

3..2 Cyclical frequency statistics

Next, we discuss the short-term dynamics of emissions along the business cycle. To extract the cyclical component of the different series, we use a filter consisting of taking 4-period log differences as an approximation to the year-over-year (YoY) growth rate and then subtracting the sample mean of these log differences to ensure that the filtered series ($\hat{x}_t$) have a zero mean over the whole period. Algebraically,

$$\hat{x}_t = (\ln x_t - \ln x_{t-4}) - \frac{\sum_{j=5}^{T} (\ln x_j - \ln x_{j-4})}{T-4} \tag{1}$$

The period considered spans from the first quarter of 1974 ($j = 1$) to the third quarter of 2023, providing a sample of $T = 199$ observations. This interval incorporates significant economic events, such as the energy crisis of the 1970s, the Great Recession, the COVID-19 pandemic, and the war between Russia and Ukraine.

We use data series on GDP, aggregate private consumption, carbon emissions, energy production that generates carbon emissions, and energy production that does not generate greenhouse gases. All series are divided by population.

The real, seasonally adjusted GDP and consumption series are obtained from the Federal Reserve Economic Database (FRED) of St. Louis, while the CO2 emissions and energy production series are sourced from the U.S. Energy Information Administration (EIA). The X-12-ARIMA method from the US Census Bureau (Findley *et al.*, 1998) was used to seasonally adjust the emissions and energy production series.

Kaya identity and emissions cycles

To explore the decoupling detected in the Kuznets curve between GDP and emissions since the end of the 20th century, we relate the evolution of emissions to GDP and other variables that influence their behavior. Thus, we decompose emissions as the product of four components: the ratio of emissions to brown energy (emission intensity, $\frac{emis}{en\_br}$); the ratio of brown energy to total energy (brown energy intensity, $\frac{en\_br}{en\_tot}$); the ratio of total energy to GDP (energy intensity, $\frac{en\_tot}{PIB}$); and GDP. The following identity represents this decomposition,

$$emis_t = \left(\frac{emis}{en\_br}\right)_t \times \left(\frac{en\_br}{en\_tot}\right)_t \times \left(\frac{en\_tot}{GDP}\right)_t \times GDP_t \tag{2}$$

This accounting decomposition method for emissions, commonly used in studies related to carbon emissions, is known as the *Kaya Identity* (Kaya, 1989). By taking logarithms of equation (2), lagging the remaining expression by 4 periods, and subtracting both logarithmic expressions, the Kaya Identity can be represented in terms of the cyclical component of the various variables:

$$\widehat{emis_t} = (\widehat{Iemis_t}) + (\widehat{Ienbr_t}) + (\widehat{Ientot_t}) + (\widehat{GDP_t}) \tag{3}$$

where the variables with hats represent the cyclical component according to equation (1). Specifically, $Iemis_t$, $Ienbr_t$, and $Ientot_t$ denote emission intensity, brown energy intensity, and total energy intensity, respectively. These intensities are defined as the ratios on the right-hand side of equation (2).[3]

[3] This approach is closely related to the Logarithmic Mean Divisia Index (LMDI) Decomposition Analysis[4] commonly used in studies related to the Kaya Identity and emission cycles (see Zhang, *et al.*, 2023, or Koilakou, *et al.*, 2023).

Non-stationary correlations

Using the cyclical version of the Kaya identity, we conduct a moving correlation analysis. Specifically, we calculate centered contemporaneous correlation coefficients over a 20-quarter window, denoted as $\rho(20)_t^{\widehat{emis},\widehat{x}_i}$, with t moving continuously from 1977:Q3 to 2021:Q1. Here, $\widehat{x}_i$ represents any of the right-hand side components in equation (3).[5]

Figure 4 illustrates the moving correlations, offering insight into the stability of the cyclical relationships between emissions and the components of the Kaya Identity, as well as identifying potential structural changes in these relationships. It reveals that correlations can vary significantly over time. A notable example of this volatility is observed in the relationship between the GDP cycle and the emissions cycle. While the correlation is predominantly positive and exhibits high values—resulting in an overall positive correlation when estimated jointly for the entire period—it fluctuates considerably, even becoming negative during certain periods, such as between 1991 and 1997. This evidence highlights the different nature of the shocks that affect carbon emissions over time, which we will study in Section 6.

The correlation between total emissions and emissions intensity cycles is more stable, although there are important variations at the beginning and the end of the sample period. Regarding the correlation between emissions and total energy intensity, the moving correlation shows a downward trend from 1975 to the end of the 20th century, followed by an increase, suggesting a higher synchronization between both cycles since the year 2000.

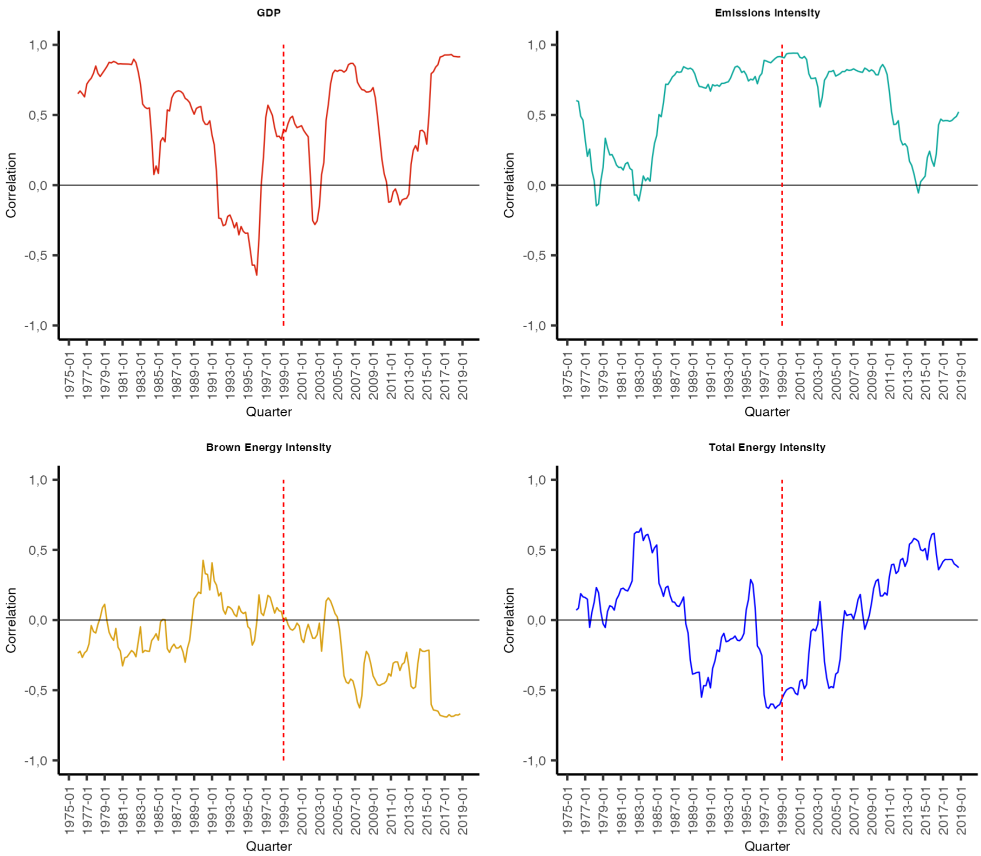
One of the most interesting results is that, since the beginning of the 21st century, coinciding with the turning point observed in the EKC Curve, there appears to be a structural change in the correlation between emissions and brown energy intensity per unit of energy produced. Indeed, until the end of the 20th century, the correlation between these two variables fluctuated around values close to zero. However, from the start of the 21st century, the correlation began a downward trajectory, reaching values close to -0.7 today. This result is not intuitive and rather surprising, as when the growth of polluting energy per unit of total energy rises above its average, indicating that polluting energies become more abundant in relative terms, emissions growth falls below its average. We will come back to this feature in Section 6. to better understand the underlying causes of this pattern.

Cross-correlations across subperiods

Table 1 presents cross-correlations for three periods: (1) the complete period (1975:Q1 - 2023:Q3); (2) the subperiod prior to the detected turning point in the Kuznets Curve

[5] Equivalently, we have estimated simple rolling regressions using each of the Kaya Identity components on the right-hand side, obtaining similar results.

Figure 4: *Moving linear correlation between the emissions cycle and the components of the Kaya Identity*



(1975:Q1 - 1999:Q1); and (3) the subperiod corresponding to the downward phase of the Kuznets Curve.

Over the entire period, the correlation between emissions and GDP cycles is the most significant positive one and has intensified over time, consistent with Doda's (2014) findings that emissions are procyclical. Furthermore, the intensity of this procyclicality increases as countries develop economically. We cannot rule out the hypothesis that the correlation between emissions and energy produced per unit of GDP (total energy intensity) differs from zero for both the entire period and the two subperiods considered around the peak of the EKC.

However, the result we wish to emphasize is the correlation between emissions and the brown energy share in total energy, particularly its shift to a significantly negative value since the end of the last century. This result, along with previous graphical analysis, suggests that a key event related to brown energy occurred in the last twenty years,

Table 1: *Cross-correlations by subperiods*

Correlations	1975:1-2023:3	1975:1-1999:1	1999:1-2023:3
$\widehat{emis}, \widehat{GDP}$	0.694***	0.627***	0.735***
$\widehat{emis}, \widehat{Iemis_t}$	0.542***	0.468***	0.556***
$\widehat{emis}, \widehat{Ienbr_t}$	-0.210***	-0.086	-0.321***
$\widehat{emis}, \widehat{Ientot_t}$	0.105	0.066	0.146

****: significant at 1%; **: significant at 5%; *: significant at 10%*

causing brown energy and carbon emissions to diverge. It appears that the United States has managed to decouple emissions from dirty energy production. A more structural analysis is required to identify the nature and size of the shocks that are ultimately driving the emissions series, which begins with the formulation of an E-DSGE model.

4. Economic Model

Our model is a version of the environmental dynamics general equilibrium (E-DGE) model developed by Andrés, *et al.* (2024a, 2024b) with four major modifications: first, while the original model was calibrated for the Spanish economy, we recalibrate it for the U.S. economy. Second, we adjust the original import equation for fossil fuels, taking into account that the U.S. economy, unlike the Spanish one, is not a net importer of fossil fuels. Third, we introduce a set of stochastic variables representing shocks of different natures. Fourth, we estimate the standard deviation and persistence of the shocks using a Bayesian methodology. The resulting estimated model will allow us to calculate the smoothed shocks, which will form the basis for a series of exercises aimed at enhancing our understanding of the underlying components of the carbon emissions cycle in the U.S.

In the following subsections, we will first provide a description of the basic functioning of the model. Subsequently, we will present the main equations, including those affected by the modifications we have introduced. A more detailed description of the model can be found in Andrés, *et al.* (2024a, 2024b).

4..1 General description

The model describes a closed economy that produces goods and services using labor, capital, and energy. It distinguishes four sectors or levels of production. The first sector consists of energy producers, which are divided into two types based on the technologies they use: environmentally responsible technologies and polluting technologies. Specifically, brown energy is produced from fossil fuels and capital, while green energy is produced using only capital without fossil fuels. Brown energy producers have the option to invest in technology to reduce their emissions, which incurs an economic cost.

The next level of production involves energy distributors. These companies purchase various quantities of green and brown energy from energy producers and package them into an "energy mix." They then sell the packaged energy in the market for use as a production factor. The third level consists of non-energy intermediate goods producers. This sector is composed of numerous firms operating under monopolistic competition that use labor, capital, and energy from distributors to produce a variety of differentiated intermediate goods. In the final production level, we find the final goods producers, who combine various inputs of intermediate goods to sell a homogeneous product intended for consumption, investment, and public spending under perfect competition. The price of this final product is sticky and responds to shocks with some delay. Households own the firms and can invest in three types of capital: capital for the production of green energy, capital for brown energy, and capital to produce the rest of goods and services.

Since the production of goods and services uses both green and brown energy in the energy mix, the model allows for the possibility of reducing emissions not only by decreasing final production but also by altering the combinations of energy inputs. Additionally, two further avenues for technological progress are considered for reducing emissions: an improvement in the productive efficiency of each type of energy, which would imply a reduction in production costs and change in the incentives for production and energy demand, and a reduction in emissions per unit of brown energy produced. These technological innovations are incorporated into the model through the introduction of specific disturbances or shocks.

Economic damages caused by environmental degradation due to the accumulation of carbon in the atmosphere are incorporated into the model through total factor productivity, which is negatively affected by the level of atmospheric carbon stock at any given moment. This carbon stock is the accumulation of the current and past flows of domestic emissions in the United States and the rest of the world, adjusted for the Earth's natural carbon capture capacity, primarily provided by forests and oceans.

4..2 Model shocks

We will now discuss the shocks that have been introduced, emphasizing their interpretation. It is important to note that these shocks affect a set of model equations and first-order conditions, which we will not detail here.

Shock on consumption preferences

The representative household maximizes its lifetime utility, which is determined by its consumption (c_t) and working hours (h_t). We assume that consumption preferences are affected by a stochastic shock ς_t^c, which generates exogenous changes in relative consumption preferences over time. This is interpreted as a pure aggregate demand shock. Thus, the representative household solves the following optimization problem:

$$\max_{\{c_t,h_t,i_t^y,i_t^g,i_t^b,k_t^y,k_t^g,k_t^b,b_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\varsigma_t^c \frac{c_t^{1-\sigma}}{1-\sigma} - \kappa_L \frac{h_t^{1+\varphi}}{1+\varphi} \right) \qquad \text{s.t.} \tag{4}$$

where σ and φ represent the intertemporal elasticities of substitution, and β is the intertemporal discount rate of preferences.

We assume that all shocks in the model follow an AR(1) process, and thus the stochastic shock affecting consumption preferences (ς_t^c) can be expressed as

$$\ln \varsigma_t^c = \rho_c \ln \varsigma_{t-1}^c + \epsilon_t^c \qquad \epsilon_t^c \sim N(0, \sigma_c) \tag{5}$$

where the parameter ρ_c measures the persistence or inertia of the shock, and ϵ_t^c is white noise following a normal distribution with mean 0 and constant variance σ_c.

In line with expressions (4) and (5), a positive value of ϵ_t^c would indicate an increase in consumption preferences (to the detriment of leisure) above its trend, leading to higher demand for goods and services.

Shocks on Technological Efficiency in Green and Brown Energy Production

The production of green energy (v_t^g) and brown energy (v_t^b) is carried out by firms using a specific type of capital in each case. The technology used to produce each type of energy is represented by a Cobb-Douglas function, as shown in Expressions (6) and (7).

$$v_t^g = \varsigma_t^g A^g \left(k_{t-1}^g\right)^{\alpha^g} \tag{6}$$

$$v_t^b = \varsigma_t^b A^b \left(k_{t-1}^b\right)^{\alpha^b} \left(m_t^b\right)^{(1-\alpha^b)} \tag{7}$$

For green energy production, only one specific type of capital (k^g) is used, where the parameter α^g represents the elasticity of green energy production with respect to capital. In brown energy production, specific capital (k^b) is combined with fossil fuels (m_t^b), such as oil or gas. The parameters α^b and $1-\alpha^b$ indicate the elasticity of brown energy production with respect to specific capital and fossil fuels, respectively. A^g and A^b represent the efficiency levels, or total factor productivity, in the production of green and brown energy, respectively. These parameters indicate how efficiently capital (along with fossil fuels in the case of v_t^b) is used to generate energy. An increase in any of these parameters would lead to a reduction in the production cost of the corresponding type of energy.

The specific technology for the production of green and brown energy can be affected by disturbances over time, ς_t^g and ς_t^b, which specifically impact efficiency in energy production. The dynamic behavior of these shocks is modeled in Expressions (8) and (9).

$$\ln \varsigma_t^g = \rho_g \ln \varsigma_{t-1}^g + \epsilon_t^g \qquad \epsilon_t^g \sim N(0, \sigma_g) \tag{8}$$

$$\ln \varsigma_t^b = \rho_b \ln \varsigma_{t-1}^b + \epsilon_t^b \qquad \epsilon_t^b \sim N(0, \sigma_b) \tag{9}$$

A positive value of ϵ_t^g or ϵ_t^b indicates an increase in the efficiency of green or brown energy production above its normal or trend level, leading to a reduction in production costs.

Shock on Emissions Efficiency

Carbon emissions are an increasing function of the amount of brown energy produced:

$$e_t^b = \left(1 - \mu_t^b\right) \varsigma_t^e \gamma_1^b \left(v_t^b\right)^{1-\gamma_2^b} \tag{10}$$

where $0 \leq \gamma_2^b < 1$ and γ_1^b are parameters controlling the curvature and the marginal effect on emissions from the production of brown energy, respectively. μ_t^b is a variable capturing the technological effort made by firms to reduce emissions. The relationship between emissions and energy production is mediated by a shock ς_t^e, which evolves according to an AR(1) stochastic process:

$$\ln \varsigma_t^e = \rho_e \ln \varsigma_{t-1}^e + \epsilon_t^e \quad \epsilon_t^e \sim N(0, \sigma_e) \tag{11}$$

A *negative* realization of ϵ_t^e indicates that carbon emissions per unit of brown energy production are reduced below their mean, reflecting some form of technological improvement in carbon emissions.

Since carbon emissions depend on a specific type of energy production, the model allows for reductions in emissions not only through decreased aggregate production but also by shifting production inputs to cleaner alternatives or implementing technological innovations in polluting energy production that reduce emissions.

Shock on Total Factor Productivity

Economic damage caused by environmental degradation is captured in the model through its effect on Total Factor Productivity (TFP), which refers to the joint efficiency with which labor, capital, and energy are combined to produce goods and services. According to Equation (12), TFP evolves negatively with the atmospheric carbon stock x_t and positively with $\varsigma_t^a \widetilde{A}^y$, which represents the shock-affected TFP if atmospheric carbon accumulation were not an issue (for example, if carbon were reduced to pre-industrial levels).

$$A_t^y = [1 - d_0 d_1^{x_t}] \varsigma_t^a \widetilde{A}^y \tag{12}$$

In the above expression, $d_0 d_1^{x_t}$ characterizes the damage function caused by accumulated atmospheric carbon. We assume that the constant $\widetilde{A}^{y}$ fluctuates around its mean due to the influence of a stochastic technological shock, ς_t^a, whose dynamics are described in Expression (13).

$$\ln \varsigma_t^a = \rho_a \ln \varsigma_{t-1}^a + \epsilon_t^a \quad \epsilon_t^a \sim N(0, \sigma_a) \tag{13}$$

A positive value of ϵ_t^a reflects a period during which TFP is above its average or trend value. In summary, the effective Total Factor Productivity is determined by the TFP that would be achievable in the absence of atmospheric carbon, adjusted for technological innovations, and reduced by the negative effects caused by environmental degradation.

5. Calibration and Estimation

5..1 Calibration

We calibrate the model for the United States at a quarterly frequency, using the average of a series of economic, environmental, and energy ratios for the period 1975:Q1 – 2023:Q3. Following the original model, a clear distinction is made between "green" and "brown" energies. Within the group of green energies, we consider hydropower, nuclear, and renewables (wind and solar), as they are clean sources of energy in terms of CO2 emissions. Energy generation from coal, oil, and gas is classified as brown energy.

In terms of units of measurement, emissions are expressed in kilotons of carbon, energy in kilotons of oil equivalent, and GDP in millions of constant dollars. All series are adjusted for population. Since per capita GDP is normalized to 1 million dollars, most variables are interpreted in terms of millions of dollars of production.

Below, we provide an overview of the model's calibration strategy. Some parameters are sourced directly from the literature, others have been calculated based on empirical evidence and equations from the static model, and the remaining generic parameters are drawn from the original *ABDF* model. A detailed view of the energy and macroeconomic ratios that align with the static solution of the model can be found in Appendix A.

Macroeconomic parameters

Regarding the parameters that affect the utility function of individuals, the intertemporal elasticity of consumption is set at $\sigma = 2.0833$, consistent with the estimates of Guvenen (2006). Similarly, the intertemporal elasticity of leisure, φ, is adjusted to a value of 1.1765, based on the work of Keane (2011). Finally, the discount factor β, which measures the impatience of individuals, is calibrated to replicate a real interest rate of 1% per quarter, equivalent to 4% per year. This value is chosen to reflect a realistic rate of return on capital, following the recommendation of Nordhaus (2007).

The parameters for the Taylor rule are derived from Carvalho, *et al.* (2021), who set a value of 0.7 for the parameter controlling inertia in monetary policy decisions, as well as values of 1.25 for the coefficient weighting the output gap and 0.95 for the sensitivity of the interest rate to deviations in the inflation rate. In terms of fiscal policy, a baseline scenario is considered where both taxes and subsidies are set to zero, indicating no fiscal intervention.

Price rigidity is captured in the model by establishing an equivalence between the Calvo price rigidity parameter in the literature and the Rotemberg quadratic adjustment costs used in this model. Thus, the parameter κ_p is set at 2.4074. To calculate the elasticity of substitution for intermediate goods producers, the average profit margin in the U.S. between 1960 and 2014 was utilized, which was reported to be 1.31 according to Demirer (2020). Based on this estimate, a value of $\sigma_r = 4.2258$ was calibrated. The remaining macroeconomic parameters are calibrated using ratios derived from the FRED database to ensure consistency with the model's static solution and units.

Energy and Environmental Parameters

The quarterly depreciation rates for capital used in the production of goods (δ_y), green energy (δ_g), and brown energy (δ_b) were calculated by taking the annual depreciation rates from the *ABDF* model and dividing them by 4 to obtain their quarterly equivalents. Thus, a quarterly depreciation rate of 1.1% is derived for capital used in the production of goods, 0.52% for green energy, and 0.09% for brown energy. Additionally, the adjustment costs for capital employed in the production of goods are set at $\kappa_I^y = 15$, while for green energy (κ_I^g) and brown energy (κ_I^b), they are set at 20, reflecting a higher adjustment cost for energy production installations, following the calibration from the *ABDF* model.

The contribution of capital income in brown energy production can be calculated using the equation shown below, derived from the brown energy production function (see Equation 7).

$$1 - \alpha^b = \frac{m_t^b}{v_t^b} = \frac{\left(\frac{m_t^b}{v_t^b + v_t^g}\right)}{\left(\frac{v_t^b}{v_t^b + v_t^g}\right)} \tag{14}$$

The elasticity of brown energy production to fossil fuels is equal to the ratio of the energy used to produce energy to the proportion of installed brown energy. Using data from tables 2.6 and 7.1 of the Annual Energy Review and table 1.2 of the Monthly Energy Review from the EIA (2023), a value of $\alpha^b = 0.68$ is set.

For the elasticity of substitution between green and brown energy within the energy mix, a value of $\sigma^x = 1.8$ was chosen based on the estimates of Papageorgiou, *et al.* (2017). Additionally, we use the equation of the model to obtain the value for the CES distribution parameter between green energy production and brown energy production,

θ^g, which we calculate to be 0.35. To calibrate the proportion of energy expenditure relative to GDP, we relied on data from Table 1.7 of the Monthly Energy Review from the EIA (2023). Accordingly, we set $1 - \alpha^y - \beta^y = 0.0810$.

The atmospheric carbon stock depends on domestic emissions e_t and emissions from the rest of the world e^{row}, as shown below:

$$x_t = \eta x_{t-1} + e_t + e^{row} \tag{15}$$

where $1 - \eta$ represents the carbon decay rate. To calculate the parameter η, considering that natural carbon sinks absorb around 30% of global emissions in the long run,[6] a value of $\eta = 0.9983$ is adjusted, consistent with a carbon half-life of 400 quarters or 100 years $(-\log(2)/\log(\eta))$, slightly higher than the 83 years assumed in Heutel (2012).

To calibrate the parameters that control the elasticities in the emissions function (see Equation 10), the parameter γ_2^b is set to 0, following the work of Annicchiarico and Di Dio (2015), while the value of γ_1^b is calculated from Equation (16), derived from the model equations:

$$\gamma_1^b = \frac{e^b_{1975:1-2023:3}}{\left(v^b_{1975:1-2023:3}\right)^{1-\gamma_2^b}} \tag{16}$$

where $e^b_{1975:1-2023:3}$ and $v^b_{1975:1-2023:3}$ represent the average value of emissions and brown energy production during the period 1975:Q1-2023:Q3 in the United States. The average carbon emissions for the United States during this period were 0.1054 kilotons (kt) per million dollars of production, while brown energy production reached 0.2113 kt of oil equivalents. Consequently, a value of $\gamma_1^b = 0.4991$ is obtained.

Following the *ABDF* model, an exponential environmental damage function of the form $d(x_t) = d_0 d_1^{x_t}$ is adjusted, consistent with Golosov, *et al.* (2019). The calibrated parameters result in $d_0 = 0.0010$ and $d_1 = 1.0126$.

5..2 Estimation

Using real data from the U.S. economy, we estimate the persistence parameters and the variance of the white noise involved in the stochastic processes of the five shocks included in the E-DSGE model. We will begin by describing the source of the data used in the estimation, the methodology employed for extracting the cycle, and the estimation strategy followed to obtain the shock parameters. We solve and estimate the model using Bayesian techniques with Dynare 5.4[7].

[6] Brienen et al. (2020).
[7] See Adjemian, *et al.*, 2024

Data

Some of the data used for estimation has been presented in Section 3., although a brief review is provided here for clarity. For the model estimation, we utilized time series data from 1975:Q1 to 2023:Q3 for real GDP, aggregate private consumption (also in real terms), CO2 emissions, green energy production, and brown energy production. These five series were selected to correspond to the five disturbances considered in the model, aiming for the closest possible affinity between the observables and shocks to facilitate parameter identification.

The emissions data for the period come from the Energy Information Administration (EIA) via the Monthly Energy Review, section 11.1, *"Total Energy CO2 Emissions."* This data corresponds to emissions generated by all energy sources, representing the vast majority of emissions in the economy. Similarly, the energy production data is sourced from section 1.2, *"Primary Energy Production by Source,"* of the same publication. The GDP, consumption, and population series are obtained from the Economic Database of the Federal Reserve Bank of St. Louis (FRED).

The methodology employed to remove the trend component from the time series and extract the cyclical component begins by dividing the variables by the population. Next, the natural logarithms are taken, and fourth-order differences are calculated as an approximation of the year-over-year (YoY) growth rate of per capita variables. Finally, the mean growth rate is subtracted to center the new series around zero. Figure 5 represents the cyclical component of all series using the described methodology.

Estimation results

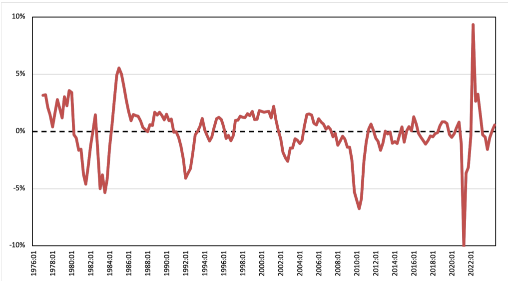
We estimate the model using Bayesian methods. To calculate the posterior distributions, we employ the Metropolis-Hastings (MH) algorithm within the Monte Carlo Markov Chain (MCMC) framework. This algorithm generates samples from the posterior distribution by exploring the parameter space with a Markov chain, accepting or rejecting new values based on a probability criterion. We use 500,000 replications and two parallel Metropolis chains. The acceptance ratio of the algorithm is 30% for both chains, which is well within the standard interval in the literature.

Convergence is assessed using the Geweke (1992) convergence test, which examines the differences between the means of the first and last parts of the chain. Evidence of different means indicates potential convergence issues. We do not find evidence against the equality of the means except, perhaps, for σ_c, for which the p-value for the corresponding statistics in the χ^2 distribution is 7.1% in the first chain.[8] However, the Brooks and Gelman (1998) test, shown in Figure 6, provides additional insight. This graphical test

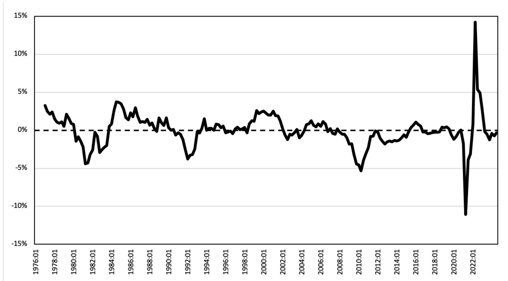
[8] Standard errors are corrected for serial correlation using a tapered window of 15% (see Geweke, 1998).

Figure 5: *Cyclical Component of the Observables*

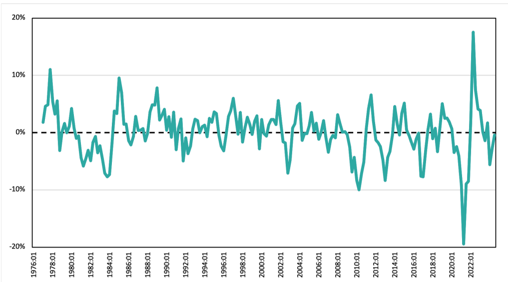
(a) *GDP*



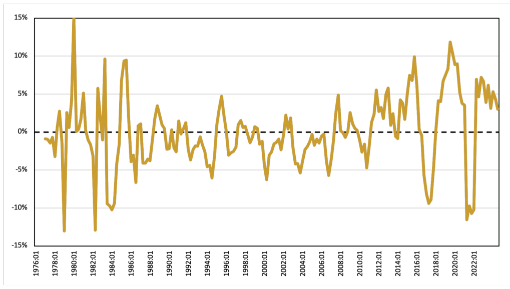
(b) *Consumption*



(c) *CO2 Emissions*



(d) *Brown Energy Production*



(e) *Green Energy Production*

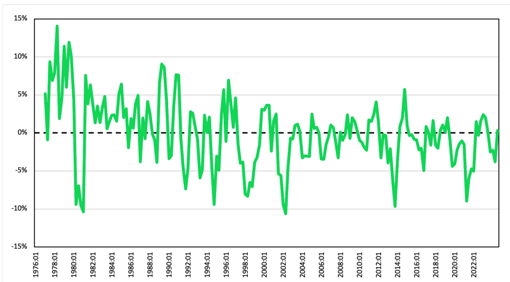
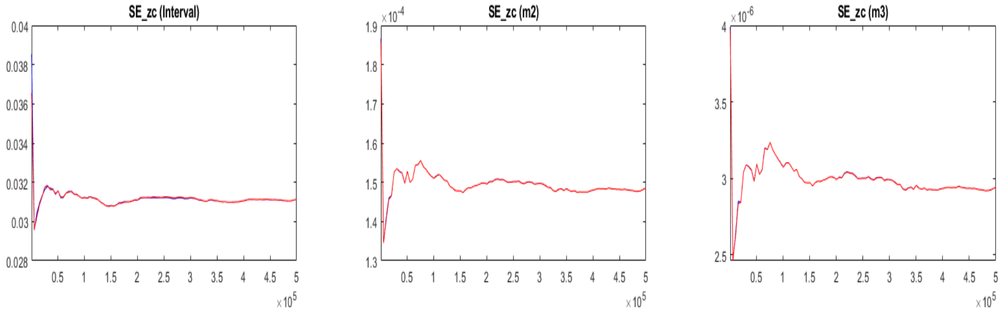


Figure 6: *Brooks and Gelman (1998) test for σ_c*



compares the between- and within-chain variances and indicates convergence when the two series in each subplot converge to the same value, which occurs approximately after 100,000 iterations. Hence, we conclude convergence also for σ_c.

Table 2: *Priors and posteriors*

Parameter	Prior			Posterior	
	Distribution	Mean	Std	Mean	90% HPDI
ρ_c	*Beta*	0.60	0.20	0.9368	[0.9207; 0.9531]
ρ_g	*Beta*	0.50	0.20	0.8812	[0.7400; 0.9954]
ρ_b	*Beta*	0.50	0.20	0.9378	[0.8956; 0.9777]
ρ_e	*Beta*	0.50	0.25	0.9795	[0.9627; 0.9982]
ρ_a	*Beta*	0.80	0.08	0.9498	[0.9397; 0.9600]
σ_c	*Inv-Gamma*	0.05	0.15	0.2166	[0.1969; 0.2365]
σ_g	*Inv-Gamma*	0.05	0.15	0.0249	[0.0225; 0.0273]
σ_b	*Inv-Gamma*	0.05	0.15	0.0414	[0.0378; 0.0450]
σ_e	*Inv-Gamma*	0.05	0.15	0.0351	[0.0321; 0.0380]
σ_a	*Inv-Gamma*	0.05	0.15	0.0896	[0.0813; 0.0976]

Table 2 shows the decisions made regarding the prior distributions and the results of the estimation in terms of the posterior distributions. Columns labeled "Prior" describe the prior distribution along with its mean and standard deviation (Std). Columns labeled "Posterior" provide the mean of the posterior and the 90 percent highest posterior density interval (HPDI), which contains 90% of the mass of the posterior distribution around the mean.

In general, we use a very diffuse set of priors, with an identical distribution for the standard deviations of the shocks. This approach allows differences in our posterior distribution of the shocks to be strongly influenced by the various observables. For the autocorrelation coefficients, we acknowledge our limited knowledge about most of them, except for ρ_a, for which we anticipate a relatively high value. Consequently, we assign it a higher mean and a lower standard deviation in the prior distribution.

As indicated by the table, the data proves to be very informative about most of the estimated parameters. Specifically, the estimated autocorrelation coefficients of shocks are higher than initially guessed, and there is a wide range of values for the standard deviations of the shocks.

6. Results

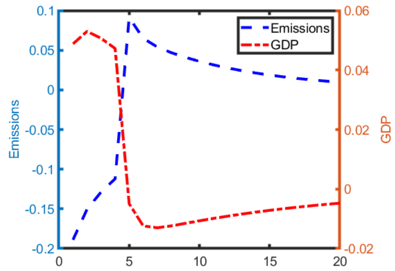
In this section, we analyze the dynamics of emission and GDP cycles using the estimated E-DSGE model, focusing on the impact of various shocks included in the model.

6..1 Impulse-response functions

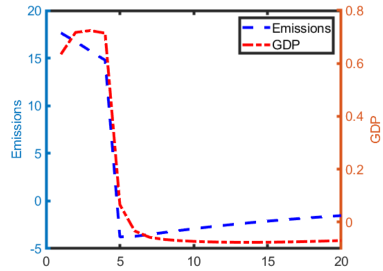
Figure 7 illustrates emissions' and GDP impulse-response (IR) functions in response to the five considered shocks. These IRs show the percentage YoY growth resulting from a 100% change in the shock size. Green technology shocks (ϵ_t^g), which affect green energy productivity, have a negative but modest impact on the carbon emissions growth rate, reducing it by slightly less than 0.2% in response to a 100% shock. Since green energy does not produce emissions, this indicates that a technology shock originating in the U.S. green production sector has minimal general equilibrium effects on emissions. In contrast, the shock increasing brown energy production efficiency (ϵ_t^b) and the shock affecting emissions per unit of brown energy used (ϵ_t^e) both have significant and positive effects on carbon emissions growth following a 100% shock. Notice that ϵ_t^e appears positively in equation (10), so a positive shock must be interpreted as deteriorating the emissions technology or the increase in emissions for a given use of brown energy. Consequently, as shown in Subplot 7c, a 100% positive realization of this shock would lead to an increase of almost 13 percentage points in the emissions growth rate. Interestingly, for all energy shocks (ϵ_t^g, ϵ_t^b, and ϵ_t^e), the effect on GDP growth rate is relatively small compared to their impact on emissions. Thus, the IR results highlight the potential of both the green technology shock and, particularly, the emissions technology shock to decouple GDP growth from carbon emissions.

The image displayed in Figure 7 reveals that the responses to a positive total factor productivity (TFP) shock and a shock that boosts aggregate consumption differ significantly from those induced by energy shocks. Both the TFP and consumption shocks drive emissions and GDP in the same direction on impact, with substantial increases in both growth rates during the initial quarters. However, while the GDP rate responds immediately to a TFP shock, the increase in emissions growth rates occurs with a delay, showing a moderate effect in the first period. Beyond the initial impact, the positive shock to consumption causes a non-negligible protracted decline in GDP and emissions. The strong

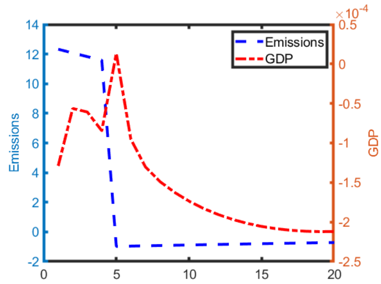
Figure 7: *Impulse-Response of GDP and Emissions (in percentage points relative to the steady state) to a 100% Variation in Shock Size*



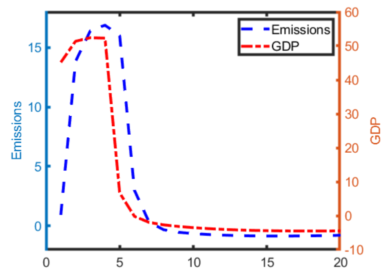
(a) *Green technology shock*



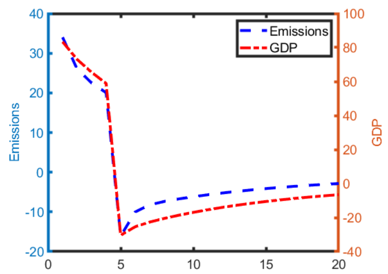
(b) *Brown technology shock*



(c) *Emissions shock*



(d) *TFP shock*



(e) *Consumption shock*

shift in preferences toward current consumption leads to a sharp decline in investment, which subsequently reduces the demand for capital and other factors of production, including energy.

6..2 Forecast Error Variance decomposition

The forecast error variance decomposition (FEVD) shown in Table 3 helps to identify which specific shocks are most significant in explaining both the emission and GDP cycles. Differently from the IR functions of the previous section, here the estimated sizes of the shocks are taking into account.

We study both the contribution of each shock for a given period and the evolution of this contribution over time. The time horizon in forecast error variance decomposition refers to the number of quarters over which the forecast variance is evaluated. The unconditional variance represents the long-term variance assuming an infinite prediction horizon. For a 1-quarter forecast horizon, the variance of forecast errors reflects the impact of shocks occurring in the next period (the next quarter). In contrast, for a 40-quarter forecast horizon, the variance of forecast errors accounts for the cumulative impact of shocks occurring over the next 40 quarters.

More than 55% of the variation in emissions growth rates on impact (p=1), is explained by changes in technological efficiency in brown energy production. The shock that directly affects carbon emissions per unit of brown energy used accounts for another 37% of the total variance in emissions growth rates, while the aggregate demand shock (consumption shock) contributes 7.5%. Both the green energy shock and the TFP shock have no impact at this horizon. This finding aligns with the minimal impact of the TFP shock on emissions observed in Subplot 7d. However, the influence of the TFP shock grows significantly over time, reaching 9.4% of the emissions variability in growth rates after 20 quarters. Conversely, the contributions from the brown energy and consumption shocks decrease over time, stabilizing around 9.5% and 5.5%, respectively, after five years. The green energy shock remains negligible throughout.

Using the forecast error variance (FEV) decomposition for GDP reveals that TFP and consumption preference shocks account for the entirety of GDP business cycle, with energy technological shocks playing a negligible role. In the short term, the influence of TFP shocks increases at the expense of the consumption preference shock, but over the long run, these contributions stabilize. TFP shocks account for approximately 72% of the cyclical variation in GDP, while consumption shocks explain the remaining 28%.

In contrast, the emissions cycle in the United States is primarily driven by energy-related shocks (affecting energy and emissions efficiency). Only 15% of the emissions cycle is attributed to macroeconomic shocks (TFP and consumption), which are the main determinants of the GDP cycle. Green energy-related shocks have an insignificant impact on both GDP and emissions cycles.

Table 3: *Forecast Error Variance Decomposition*

Emissions					
Time Horizon	ς_t^g	ς_t^b	ς_t^e	ς_t^a	ς_t^c
Unconditional	0.01	50.12	34.86	9.50	5.50
Conditional p=1	0.02	55.10	37.37	0.03	7.48
Conditional p=4	0.01	50.08	37.62	7.55	4.74
Conditional p=20	0.01	50.36	34.82	9.35	5.46
Conditional p=40	0.01	50.43	34.80	9.28	5.49

GDP					
Time Horizon	ς_t^g	ς_t^b	ς_t^e	ς_t^a	ς_t^c
Unconditional	0.00	0.07	0.00	71.97	27.96
Conditional p=1	0.00	0.06	0.00	63.14	36.80
Conditional p=4	0.00	0.07	0.00	74.75	25.18
Conditional p=20	0.00	0.07	0.00	71.44	28.49
Conditional p=40	0.00	0.07	0.00	71.79	28.14

6..3 Historical shock decomposition

The forecast error variance decomposition might not be very informative if the relationship between cycles and forecast errors is weak.[9] In this subsection, we present a historical decomposition of the YoY carbon emissions growth rate.[10] For clarity and ease of reading, Figure 8 presents the decomposition exercise in four subperiods. The solid black line represents the emissions growth rate, while the different colored bars illustrate the quarterly estimated contributions of the five shocks in the model.[11] For an interpretation of these bars, consider the first quarter of 1988 in Figure 8b. The YoY emissions growth rate was 7.8% above the mean growth. This figure resulted from positive contributions to emissions from the shocks related to the technology affecting emissions per unit of brown energy produced (ςe), which alone accounted for 5% of emissions growth. Additionally, the consumption shock (ςc) contributed 3.1%, and the TFP shock (ςa) increased emissions by 2.3%. However, shocks affecting brown energy production (ςb) reduced emissions by 3.3%. Among all the shocks, only those affecting green energy production (ςg) had virtually zero effect on emissions growth during that quarter. Finally, there is a component associated with initial conditions (*i.c*) or deterministic trends that which accounts for a small portion of the variable's behavior not directly attributed to the iden-

[9] See Seymen (2008).

[10] As explained in subsection 5..2, the sample mean of the growth rate over the complete sample has been removed from the YoY growth rate of all observables, so that the average of the values shown in the figure over the entire sample period is zero.

[11] The notation ς_t^j has been replaced with ςj, where j refers to green energy ($j = g$), brown energy ($j = b$), emissions technology ($j = e$), TFP ($j = a$), and consumption ($j = c$), respectively.

tified shocks. This component plays a minor role, shows some positive values after a few quarters since the start of the period in 1976:1, and diminishes over time as the influence of initial conditions fades.

The exercise clearly shows that, in the vast majority of quarters, shocks affecting brown energy production and brown emissions technology are dominant in explaining the rate of growth of emissions.[12] In subsection 6..1, we analyzed the responses of emissions to normalized energy shocks and compared these with traditional consumption and TFP shocks, which are crucial for economic growth over the business cycle. The historical decomposition results reflect not only the influence of IR functions depicted there but also the specific series of smoothed shock sizes estimated from the model.

A concise summary of the main results from Figure 8 is provided in Table 4, which shows both the mean of the smoothed shocks (or the innovations obtained from the data using the Kalman filter) and the mean of the contributions of the shocks to the YoY growth rate of emissions over different periods. Innovations and contributions are displayed at the top and bottom parts of the table, respectively. The first row covers the entire period from 1976 to 2023, where the mean of the smoothed shocks has been normalized to zero[13]. However, the mean of the observed variable over the full period is zero by construction. Hence, the subperiods' observed emissions and estimated innovations means should be interpreted as the differential mean with respect to that of the whole period. The next two rows break down the full period into two subperiods: up to 2000 and after 2000.

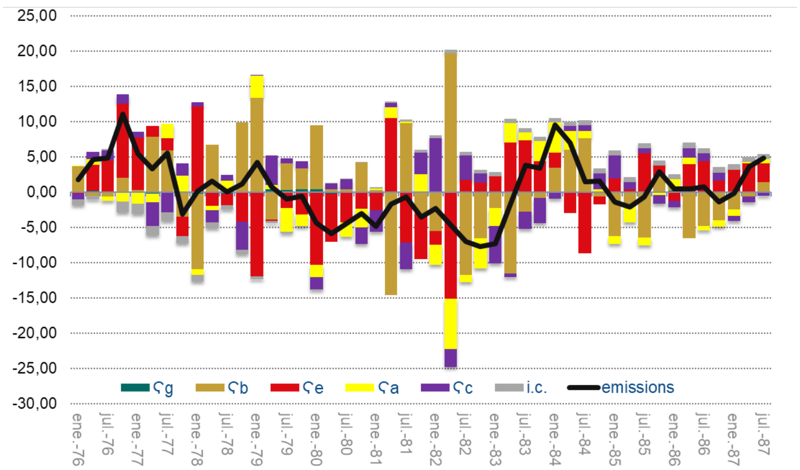
Compared to the pre-2000 period, the post-2000 period is characterized by higher consumption shocks, lower TFP shocks, lower shocks to the volume of emissions from brown energy use, lower shocks to brown energy productivity, and lower shocks to green energy productivity. However, we find notable differences in the size and sign of the shocks when we further divide the post-2000 period into 2000-2007 and 2008-2023. Negative shocks to green energy production are predominant and strong from 2000 up to 2007, but they are much smaller thereafter. Similarly, while negative shocks to brown energy production productivity predominated before 2007, they significantly reversed signs from 2008 to 2023. This pattern is the opposite of the one displayed by the shock to the emissions function, indicating once again a significant reversal in emissions efficiency, which substantially improves (negative values of ϵ_t^e) after 2007.

The contributions at the bottom part of the table show what the average rate of growth of the filtered emissions would have been if the economy had been influenced

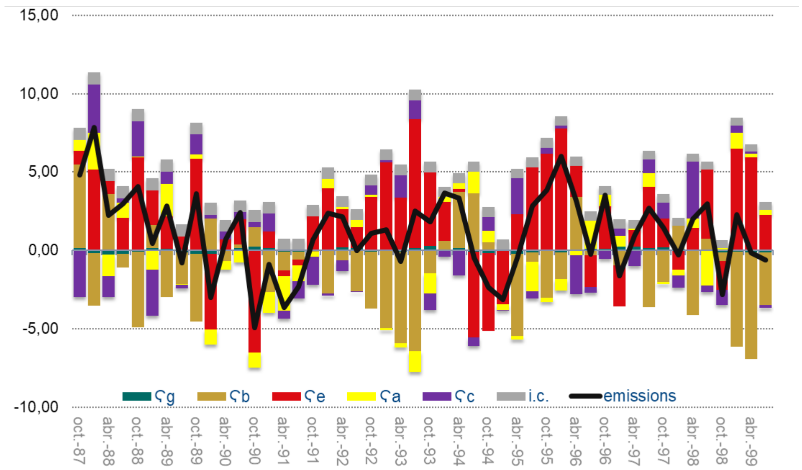
[12] The main exception is the third quarter of the pandemic crisis, when emissions fell by 9%, mainly explained by consumption (-3.6%) and TFP (-5.6%) shocks.

[13] Although these estimated shocks fluctuate up and down around zero with an asymptotic zero mean, no constraints were introduced in the estimation to guarantee that the sample estimated mean is zero.

Figure 8: *Historical Decomposition of YoY Emissions Growth (pp)*



(a) *Period 1976:1 - 1987:3*

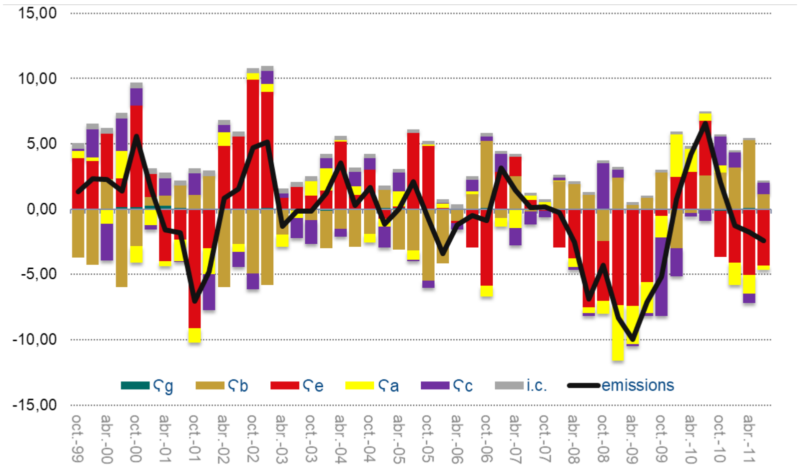


(b) *Period 1987:4 - 1999:3*

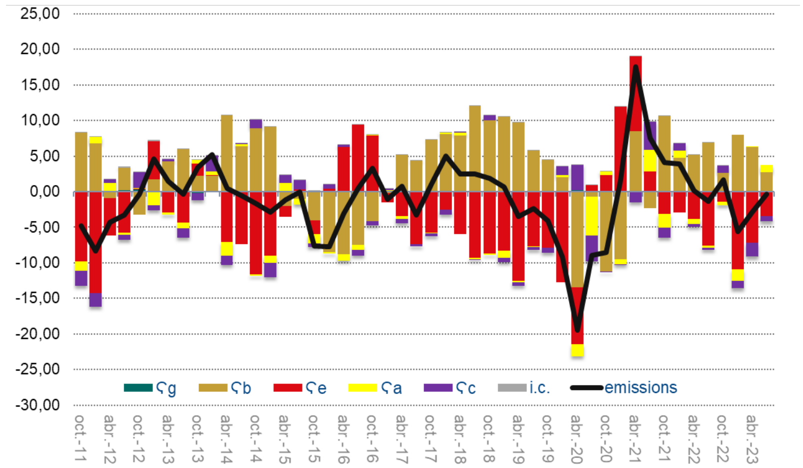
by each of the shocks in isolation. Over the complete period, brown energy shocks have contributed to an increase in the average rate of emissions growth by 0.30%, while innovations in emissions technology of brown energy have reduced it by 0.32%, and the TFP shock has contributed a reduction of 0.23%. Thus, during this complete period, the contributions of individual shocks to the average emissions growth rate have been relatively moderate, with TFP shocks contributing similarly to each of the energy shocks.[14] The

[14] Moreover, the contributions of the consumption and green energy shocks have been neutral. We do not

Figure 8: *Historical Decomposition of YoY Emissions Growth (pp) – Continued*



(c) *Period 1999:4 - 2011:3*



(d) *Period 2011:4 - 2023:3*

emissions growth rate in the 1976-2000 period was 0.72% higher than the average for the entire period, whereas it was 0.73% lower after 2000. Before the year 2000, the contribution of the brown productivity shock was significantly negative, but since the beginning of the 21th century, its contribution changed to positive. Together with the contribution from emissions technology shocks, these results suggest a shift in the sources of carbon emissions. The U.S. economy has transitioned from using a lot of pollutant fossil fuels for production up to the year 2000 to using more fossil fuels but with significantly lower

present the contribution of the component that accounts for initial conditions and that makes the difference between the horizontal sum of contributing factors and the total in the last column.

Table 4: *Mean of the shocks and their contributions to emissions (%)*

Smoothed shocks						
Period	ϵ_t^g	ϵ_t^b	ϵ_t^e	ϵ_t^a	ϵ_t^c	Emissions
1976-2023	0.00	0.00	0.00	0.00	0.00	0.00
1976-1999	0.34	0.15	0.22	0.17	-0.63	0.72
2000-2023	-0.34	-0.15	-0.22	-0.17	0.64	-0.73
2000-2007	-0.44	-1.30	0.86	0.10	1.37	0.45
2008-2023	-0.29	0.43	-0.77	-0.31	0.27	-1.33
Contributions						
Period	ς_t^g	ς_t^b	ς_t^e	ς_t^a	ς_t^c	Emissions
1976-2023	0.00	0.30	-0.32	-0.23	-0.03	0.00
1976-1999	0.00	-0.59	1.03	-0.13	0.03	0.72
2000-2023	0.01	1.19	-1.68	-0.33	-0.10	-0.73
2000-2007	0.02	-1.33	1.56	-0.16	0.00	0.45
2008-2023	0.00	2.46	-3.32	-0.42	-0.15	-1.33

pollution incidence thereafter.

The last two rows further divide the 2000–2023 period into two subperiods: from 2000 to 2007, coinciding with the onset of the financial crisis and the Great Recession, and from 2008 to 2023. These results make it clear that between 2000 and 2007, the slowdown in the growth rate of emissions was driven by negative shocks affecting productivity in brown energy production (and, hence, increasing brown energy production prices) and, to a lesser extent, total TFP. However, the most pronounced change since 2000 occurred after 2007. In this subperiod, emissions reached the lowest rate of growth (-1.33% below the average of the entire period), and although the US experienced a significant shock that resulted in intensive use of fossil fuels, this was more than compensated for by a reduction in emissions produced per unit of fossil fuel use.

6..4 Counterfactual evolution of emissions

In this section, we conduct a comprehensive analysis of the long-term implications of our study by examining the determinants of emissions in the U.S. economy since 1976. Figure 9 presents a counterfactual analysis in which we remove each of the three environmental shocks (ϵ_t^g, ϵ_t^b, and ϵ_t^e) individually and collectively to illustrate the emission trajectories in their absence. For clarity, we reverse the filtering transformation applied to the series by adding the average over the entire period and integrating the simulated and observed data.

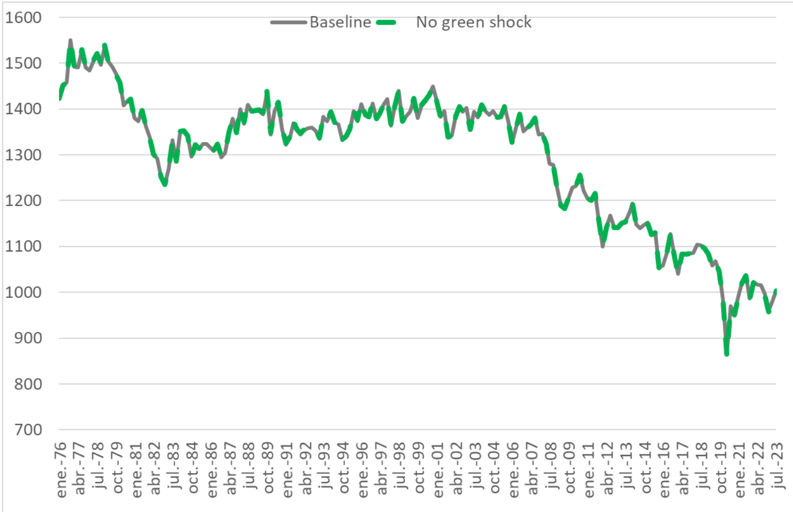
Consistent with the results found in the previous subsections, removing the green shocks from the first quarter of 1976 onward does not have a significant effect on the dynamics of emissions (see Figure 9a).

More importantly, had the brown technology shock not occurred over the entire period, emissions would have increased at a significantly higher rate until 2006 (Figure 9b). Without the shocks affecting brown energy production, emissions per person would have been about 33% higher at that point. Since then, counterfactual emissions would have decreased more markedly than the observed series. In other words, efficiency shocks in brown energy production, being predominantly negative seem to have moderated emissions growth until approximately the financial crisis but later slowed the reduction in emissions. By the end of 2023, without the brown technology shock, emissions would have been 149 kg per person lower than observed. This pattern can be traced back to the numbers in Table 4. From 2008 onwards, the realization of positive shocks to the production of brown energy contrasted with the previous years in which these shocks were either small and positive (1979-2000) or large and negative (2000-2007). This can be attributed to inefficiency in the extraction of fossil fuels that prevented substantial reductions in their price in those years. Over the recent fifteen years, key technological advances in drilling techniques in the oil and gas industry have pushed down the relative price of oil and gas thus causing an increase in their use and consequently in emissions.

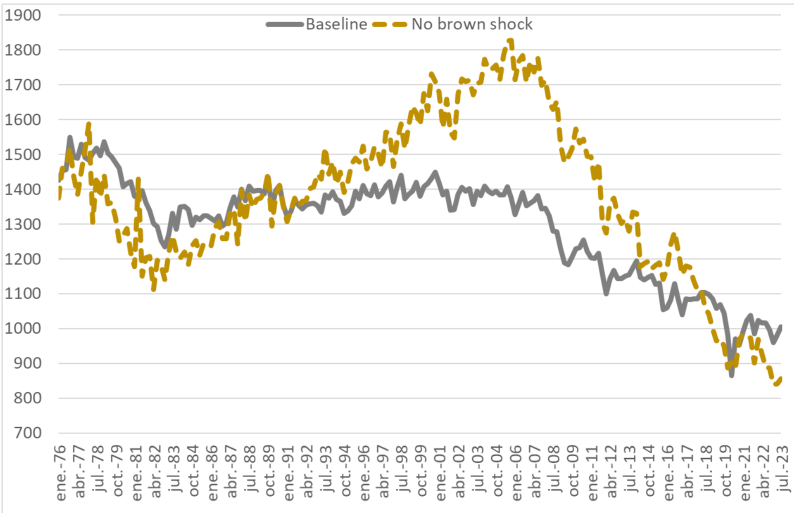
The shock that controls for carbon emissions due to brown energy use plays the opposite role compared to the previous one, as already indicated by Table 4. Without this shock, emissions would have decreased steadily until around 2007, which contrasts with the observed constant dynamics of the series during the same period. This implies that the shock responsible for carbon emissions from brown energy use was associated with approximately 50% more emissions per person by 2006 (Figure 9c). However, since then, emissions would have experienced a pronounced increase without this shock. Overall, by the end of 2023, emissions were 14% lower due to the recent trajectory of the emissions technology shock. The interpretation of this shock is somewhat more tentative since it is not easy to identify significant changes in the cleanliness of oil, gas, and coal in terms of emissions. As we discuss later, we attribute this to a change in the mix of brown technologies that resulted from the previously mentioned reduction in the relative price of oil and gas that rendered many coal mines unprofitable.

In the final subplot (Figure 9d), we remove all three environmental and energy shocks simultaneously, although the counterfactual results are primarily influenced by ϵ_t^b and ϵ_t^e. Essentially, without these specific shocks, the US economy would have exhibited a negative emissions gap compared to the observed levels for over 30 years. However, since around 2007, the combined effect of these two shocks has accelerated emissions reductions, suggesting that the emission reductions associated with the energy mix of oil, gas, and carbon have more than offset the increased use of these energy sources. After nearly 50 years, the US economy would have reached the current emission levels even without the energy/environmental shocks. Nonetheless, along the way, these shocks

Figure 9: *Counterfactuals of per capita emissions (kg per person)*



(a) *No green technology shock*



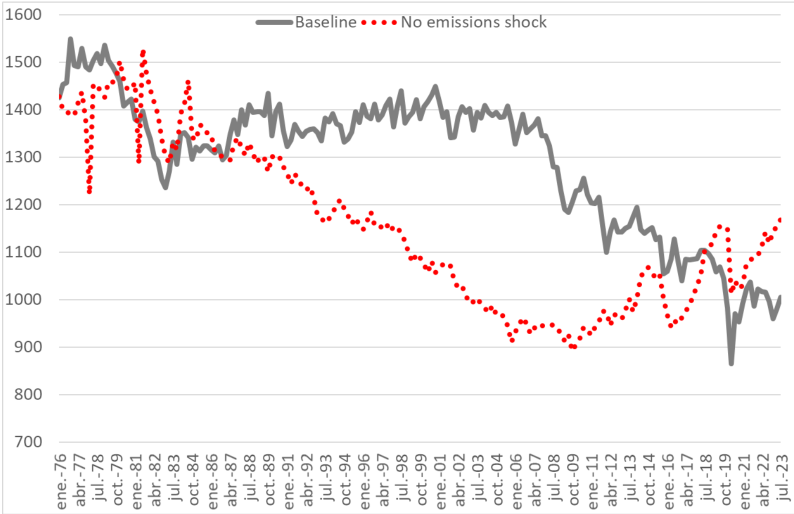
(b) *No brown technology shock*

have been responsible for a substantial accumulation of carbon in the atmosphere.[15]

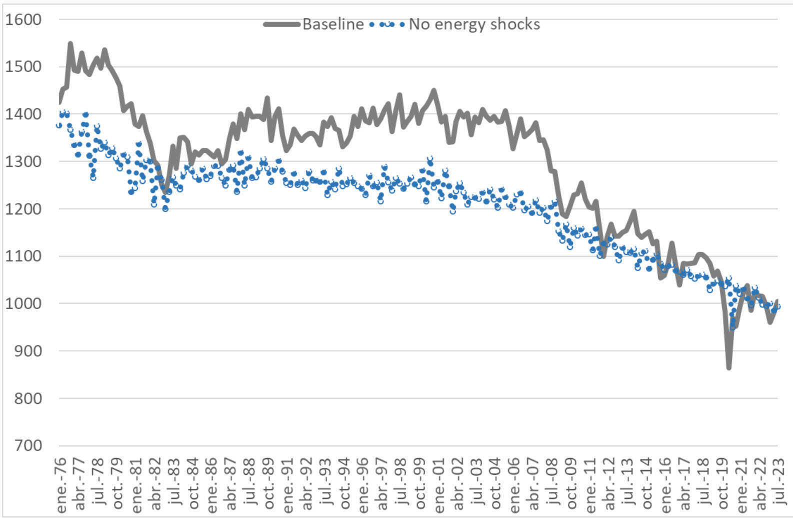
Figure 10 shows the observed and counterfactual Kuznets curves, offering an al-

[15] Our counterfactuals suggest that since 1975, energy shocks have been responsible for 7% of all carbon accumulated by the U.S. in the atmosphere.

Figure 9: *Counterfactuals of per capita emissions (kg per person)-Continued*



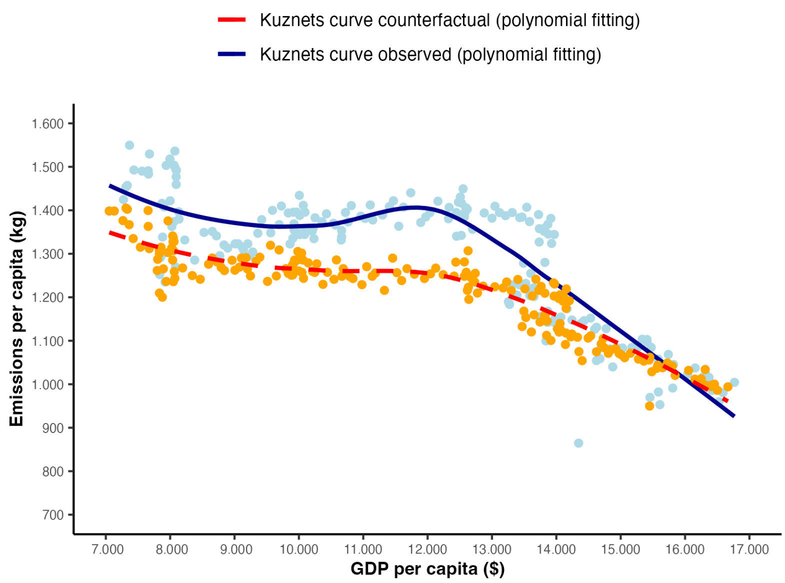
(c) *No emissions technology shock*



(d) *No energy shocks*

ternative perspective that supports the conclusion drawn from Figure 9d. This figure depicts the relationship between per capita GDP and per capita emissions. The counterfactual scenario suggests that, without the energy/environmental shocks, emissions would have remained lower up to a per capita GDP level of $15,000. However, since the U.S. economy reached approximately $12,000 per capita GDP, the accelerated rate of emissions reduction per unit of GDP, compared to the counterfactual scenario, has led to

Figure 10: *Kuznets curve: observed and counterfactual*



the convergence of the observed and counterfactual Kuznets curves.

The question then arises: what could explain the shocks related to energy and environmental factors occurring in the U.S. after approximately 2007? We dedicate the next section to providing a tentative answer to this question.

6..5 Cheaper and Less Pollutant Fossil Fuels

Our results indicate a structural change around 2007, consistent with: (a) a technological improvement in the production of dirty energy that slowed the reduction in emissions; and (b) a reduction in emissions per unit of brown energy used. We argue that both shocks are related to shale oil and gas production in the US.

According to the Council of Economic Advisers of the US (2019), from 2007 to 2019, innovations in shale production led to an eight-fold increase in extraction productivity for natural gas and a nineteen-fold increase for oil. Beyond its impact on fossil fuel production, shale production innovations reduced the price of oil and, particularly, gas, thereby restraining the substitution of brown energies for green energies. Our model identifies this process through a sequence of ϵ_t^b shocks that contribute to a relative increase in emissions compared to the scenario without such shocks.

As the share of gas in brown energy production increased, and the price of gas and oil decreased, coal extraction became noncompetitive and many coal mines closed.[16] Thus, the mix of brown energy changed to one with a higher share of gas, and a lower share of coal, which our model identifies as a sequence of shocks ϵ_t^e that reduced the emission per unit of dirty energy produced. In a similar vein, Lindequist and Selent (2025) apply the synthetic control method to the U.S. economy and find that the substitution of coal with shale gas in the brown energy mix played a significant role in reducing carbon emissions from 2007 onward.

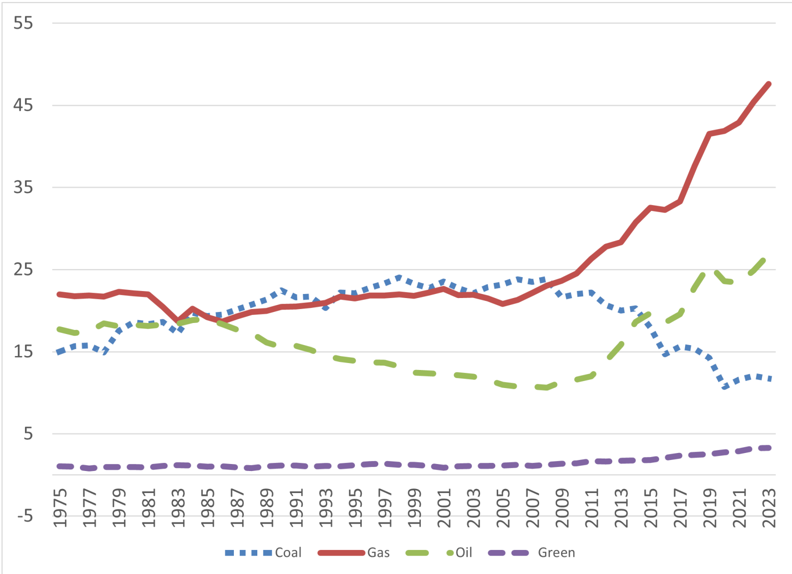
Figure 11 confirms the substantial impact that shale innovations had on gas and oil production after 2007, which coincided with a decline in coal production. Additionally, green energy production—encompassing nuclear, hydroelectric, geothermal, solar, and wind energy—remained at very low levels and did not keep pace with the growth of gas and oil production.

The general picture that emerges from our results is one of stark contrasts. The optimistic interpretation is that the US has significantly reduced carbon emissions in favor of cheaper and less polluting, but still brown, energies investing in technologies that make the mix of brown energy used cleaner. This has made economic activity in the U.S. not to increase emissions per capita despite the easier access to gas and oil. Should this trend continue in the future it may help to close the gap with its 2050 targets.

Unfortunately, relying on a cleaner brown energy mix ("greening through browning") will make it difficult for the U.S. to follow an emissions path consistent with the 2050 objective. No matter how effective improvements in the brown energy mix may be in reducing emissions, their potential is inherently limited. Therefore, despite the progress this strategy has made in lowering carbon emissions, it will not achieve net-zero emissions by 2050—or in the foreseeable future—unless the country makes a decisive commitment to renewable energy and succeeds in implementing it.

[16] According to Feaster (2023), by the end of 2026, the US economy will have closed half of its coal generation capacity, which peaked in 2011

Figure 11: *Primary Energy Production by Source (Quadrillion Btu). Source: EIA and own elaboration:* `https://www.eia.gov/totalenergy/data/annual/`



7. Conclusions

While the long-term relationship between economic growth and emissions has been extensively studied, detailed analyses of the cyclical fluctuations in emissions and their driving factors are less common. This paper addresses that gap through a combination of empirical facts and structural analysis, focusing on the United States. The results obtained not only expand upon previous findings in this area of literature but are also relevant for designing optimal policies that consider the environmental component.

The first part of the paper presents some stylized facts. In the second part, we use an environmental stochastic dynamic general equilibrium model (E-DSGE) to estimate structural shocks and analyze their influence on emissions. We can explain and interpret some of the stylized facts detected, such as the negative correlation, appearing in the last 20 years, between the cycles of emissions and the cycles of the brown energy share over total energy.

The model incorporates specific technological shocks related to the production of clean and dirty energy, as well as the efficiency in emissions generation, in addition to traditional aggregate demand and supply shocks. With the model calibrated to the U.S. and estimated using Bayesian methods, we obtain a series of results that point in the same direction: a structural change around 2007, consistent with (a) a technological improvement in the production of dirty energy that slowed the reduction in emissions, and (b) a reduction in emissions per unit of brown energy used. We estimate that these two types of shocks account for about 80% of the variance of emissions, significantly more than the contribution of standard macroeconomic shocks, which does not reach 20%.

We argue, and offer evidence, that these two energy/environmental shocks detected as crucial are related, since 2007, to important innovations emerging in shale oil and gas production in the U.S. Shale production innovations reduced the price of oil and gas, thereby restraining the substitution of brown energies for green energies. Our model identifies this process through a sequence of shocks that positively affected the productivity of brown energy production and augmented carbon emissions.

However, as the share of gas in brown energy production increased, and the prices of gas and oil decreased, coal extraction became non-competitive, leading to the closure of many coal mines. Thus, the mix of brown energy changed to one with a higher share of gas and a lower share of coal, which our model identifies as a sequence of shocks that reduced emissions per unit of dirty energy produced.

The overall findings present a dichotomy of interpretations. On the one hand, a hopeful perspective suggests that the U.S. has made significant strides in reducing carbon emissions by investing in technologies that enhance the environmental performance of brown energy production. This progress enables the country to maintain current carbon emission levels comparable to what they would have been without the extensive use of brown energy. If this trend persists, it could facilitate the U.S. in bridging the gap toward its 2050 emissions reduction targets.

Conversely, a more critical viewpoint underscores that the lack of adequate development of green energy sources in the past has led to the U.S. contributing to a considerable accumulation of carbon in the atmosphere over the last four decades. Furthermore, there are inherent limitations in the strategy of greening brown energy. For instance, generating 100% of energy from natural gas still results in emissions. This constraint could impede the objectives outlined in the Paris Agreement in the near future unless the U.S. commits decisively to transitioning toward renewable energy sources.

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Appendix A Calibration Parameters

This appendix presents the selected energy and macroeconomic ratios (Table A1), as well as the details of the parameters and exogenous variables used in the model (Table A2).

Table A1: *Energy and macroeconomic ratios (average 1975:Q1-2023:Q3)*

Energy Ratios	**Value**
Energy Intensity (kt of oil equivalents per million $ of GDP)	0.1323
Emissions (kt of carbon per million $ of GDP)	0.1054
Carbon Stock (kt of carbon per million $ of GDP)	127.5854
Carbon Intensity (kt of carbon per kt of oil equivalent)	0.7971
Green Energy Production as a Share of Brown Energy Production	0.1820
Percentage of Energy Used to Produce Energy	0.2700
Share of Green Energy in the Energy Mix	0.1540
Share of Brown Energy in the Energy Mix	0.8460
Macroeconomic Ratios	**Value**
Consumption as a Percentage of GDP	0.6530
Investment as a Percentage of GDP	0.1270
Public Spending as a Percentage of GDP	0.222
Ratio Hours Worked to GDP	0.0170
Investment in Green Energy as a Percentage of Total Investment	0.0113
Investment in Brown Energy as a Percentage of Total Investment	0.0105
Return on Capital for Final Goods	0.0211
Return on Capital for Green Energy	0.0152
Return on Capital for Brown Energy	0.0109

Source: Author's own elaboration

Table A2: *Model Calibration Parameters*

Parameter	Value	Description
β	0.9901	Discount Rate
σ	2.0833	Intertemporal Elasticity of Consumption
φ	1.1765	Intertemporal Elasticity of Leisure
δ_y	0.0111	Depreciation of Capital in Goods Production
δ_g	0.0052	Depreciation of Capital in Green Energy Production
δ_b	0.0009	Depreciation of Capital in Brown Energy Production
κ_I^y	15.0000	Adjustment Costs of Capital in Goods Production
κ_I^g	20.0000	Adjustment Costs of Capital in Green Energy Production
κ_I^b	20.0000	Adjustment Costs of Capital in Brown Energy Production
α^g	0.6800	Capital Elasticity in Green Energy Production
α^b	0.6800	Capital Elasticity in Brown Energy Production
γ_1^b	0.4824	Emissions Function Adjustment Parameter
γ_2^b	0.0000	Emissions Function Elasticity Parameter
θ_1^b	0.5679	Emissions Cost Reduction Function Adjustment Parameter
θ_2^b	2.8000	Emissions Cost Reduction Function Elasticity Parameter
σ^x	1.8000	Substitution Elasticity in the Energy Mix
θ^g	0.3537	Distribution Parameter in the Energy Mix
κ_L	7824.1	Disutility of Labor
$\bar{\pi}$	1.0000	Steady-State Inflation Rate
σ^r	4.2258	Substitution Elasticity of Intermediate Goods
κ_p	2.4074	Price Rigidity Parameter
α^y	0.3098	Capital Elasticity in Final Goods Production
β^y	0.6092	Labor Elasticity in Final Goods Production
η	0.9983	Natural Absorption of Atmospheric Carbon
d_0	0.0010	Damage Function Parameter
d_1	1.0126	Damage Function Parameter
τ	0.0000	Emissions Tax per Unit
t^{ig}	0.0000	Subsidy for Investment in Green Energy
t^m	0.0000	Subsidy for Demand for Green Energy
A^x	1.0000	Total Factor Productivity in Energy Mix Production
$\tilde{A}^y$	6.6350	Total Factor Productivity in Final Goods Production
ν^g	0.0500	Total Factor Productivity in Green Energy Production
ν^b	0.2567	Total Factor Productivity in Brown Energy Production