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JAVIER FERRI

FRANCISCA HERRANZ-BÁEZ

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Carbon Pricing in Residential and Non-Residential Sectors: Household Inequality and Compensation Strategies*

Javier Ferri[a,b] Francisca Herranz-Báez[a]

[a] *University of Valencia, Spain*
[b] *Fedea, Spain*

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Abstract

This paper analyzes the macroeconomic and distributional impacts of carbon pricing policies targeting both residential and non-residential sectors. Using a model that incorporates nominal price rigidities, sectoral labor adjustments, and financial frictions tied to housing collateral, we uncover critical transmission mechanisms affecting household welfare. Our analysis highlights the distinct effects on borrowers and lenders: carbon pricing in the non-residential sector reduces labor demand and wages, disproportionately impacting borrowers, while residential carbon pricing lowers housing prices, tightening credit constraints for borrowers but imposing higher welfare costs on lenders who own more housing assets. We explore compensation strategies, particularly the allocation of carbon dividends. An equal distribution initially stabilizes consumption for borrowers but becomes less effective as emissions decline and carbon revenues shrink. Robustness checks show that factors like labor mobility, firm markups, and debt distribution significantly influence inequality. Flexible prices can mitigate early consumption inequality, while a higher share of debt among borrowers affects policy efficiency and the required carbon price levels. Our findings underscore the need for sector-specific carbon pricing and adaptive compensation mechanisms to balance efficiency and equity.

Keywords: carbon emissions, TANK, inequality, compensation policies.

JEL Classification: E27, H23, Q58, D63

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1 Introduction

The transition to climate neutrality stands as one of the central macroeconomic challenges for advanced economies. The European Union and other developed regions have committed to ambitious emission reduction targets that are essential for mitigating the effects of climate change and for reaching net zero emissions by 2050. Carbon pricing has emerged as a key instrument for advancing these objectives. However, the design and quantitative evaluation of climate policies continue to focus predominantly on emissions from the production sector, even though emissions generated by households through heating, cooling, and energy use represent an important component of total emissions. Since public support for environmental policies depends critically on their distributional consequences European Environmental Bureau (2022); European Commission (2021); Marcu et al. (2023), understanding how carbon pricing affects different households is essential for shaping socially acceptable transition strategies.

This paper contributes to this discussion by analysing the transition toward a net zero emissions economy within an Environmental Dynamic General Equilibrium framework that features two productive sectors, endogenous housing production, household heterogeneity, and a distinction between residential and non residential emissions. We incorporate limited household heterogeneity through the structure of Two Agent New Keynesian models, which separates financially unconstrained households from credit constrained households whose consumption and borrowing decisions depend on collateral and income.[2] This environment allows us to study the effects of carbon pricing through several channels, including changes in consumer spending patterns, factor prices, and borrowing capacity, and connects to the broader Environmental Dynamic General Equilibrium literature that examines the interaction between climate policies and economic frictions Fischer & Springborn (2011); Heutel (2012); Annicchiarico & Di Dio (2015); Coenen et al. (2018); Ferrari & Landi (2022).

Carbon pricing generates distinct macroeconomic adjustments depending on the sector where emissions occur. When applied to the non residential sector, carbon pricing primarily raises production costs, reduces labor demand, and lowers household income, which depresses consumption and aggregate activity. When applied to the residential sector, carbon pricing also affects the price of housing services, which modifies the value of the housing stock. The decline in housing prices reduces collateral values for credit constrained households and restricts their borrowing capacity, amplifying the fall in their consumption. These mechanisms highlight the importance of distinguishing across emission sources when evaluating the macroeconomic effects of climate mitigation policies.

Borrowers and lenders experience different welfare consequences during the transition.

[2]For a discussion on the aggregate implications of Two Agent New Keynesian models compared with Heterogeneous Agent New Keynesian models, see Debortoli & Galí (2024).

Borrowers are more sensitive to fluctuations in labor income and aggregate economic activity, while lenders are more affected by movements in housing prices and housing investment. The direction and magnitude of these welfare differences vary depending on whether carbon pricing targets residential or non residential emissions. This finding reinforces earlier evidence that household heterogeneity has important consequences for the incidence of carbon taxation and that analyses that omit heterogeneity may misrepresent the distributional effects of climate policy Dinan & Rogers (2002); Metcalf et al. (2008); Hassett et al. (2009); Fullerton & Heutel (2007); Araar et al. (2011); Rausch et al. (2011); Mathur & Morris (2014); Rausch & Schwarz (2016).

The composition of household types shapes the effectiveness of carbon pricing. A larger share of credit constrained households increases their sensitivity to fluctuations in housing prices and therefore strengthens the response of residential emissions to carbon taxation. In the non residential sector, lenders respond more strongly to changes in profits induced by the carbon tax, so a reduction in their population share weakens this mechanism. The degree of indebtedness in the economy therefore influences the cost effectiveness of sector specific mitigation policies.

The distribution of carbon tax revenues also plays a crucial role in shaping the transition path. Equal per capita transfers help stabilise the consumption of credit constrained households during the early phases of the transition, when carbon revenues are relatively large. As emissions decline and revenues shrink, however, the insurance provided by these transfers weakens and inequality increases. Distributional outcomes improve substantially when carbon dividends are allocated entirely to borrowers. In this case, consumption inequality decreases from the outset, with the consumption Gini coefficient showing the most pronounced improvement, and this redistribution does not diminish the effectiveness of carbon pricing in reducing emissions. These dynamics reveal periods of heightened distributional tension and suggest that targeted support may be needed as the economy approaches net zero emissions.

The remainder of the paper is organised as follows. Section 2 presents the model. Section 3 describes the calibration strategy. Section 4 reports the results, including macroeconomic effects, heterogeneity in household responses, robustness exercises, and alternative revenue recycling schemes. Section 5 concludes with policy implications and avenues for future research.

2 The Model

We develop a Dynamic General Equilibrium model with environmental externalities, nominal rigidities, and two representative household types to assess the macroeconomic and distributional effects of carbon pricing. The model captures the EU as a single-region economy with two production sectors—non-residential (consumer and investment goods) and residential (housing construction)—and a population composed of two household types: patient (lenders) and impatient (borrowers). This structure follows the Two-Agent New Keynesian (TANK) framework, as in Iacoviello (2005). Carbon emissions arise from two sources: the production of goods in the non-residential sector and the use of housing services by households, such as heating, cooling, and cooking. These emissions accumulate into a carbon stock that impairs productivity over time. Full model equations are provided in the online Appendix.

Households differ in their discount rates. Patient households (with higher β^l) lend to impatient households ($\beta^b < \beta^l$), who borrow to smooth consumption. Both types consume goods and housing services, supply labor to both sectors, and generate residential emissions. Utility is derived from consumption, housing services (proportional to housing stock), and disutility from labor. Labor is allocated across sectors via a CES aggregator with elasticity ε_n.

Residential emissions for each household type are given by:

$$e^i_{ht} = (1 - u^i_{ht})\varphi_{1h}(h^i_t)^{1-\varphi_{2h}}, \quad i \in \{l, b\} \tag{1}$$

where $u^i_{ht} \in [0, 1]$ is the household's abatement effort, φ_{1h} reflects emissions intensity, and φ_{2h} captures building energy efficiency. These emissions represent fossil fuel use for essential residential services. Emissions can be reduced through abatement, which is costly:

$$z^i_{ht} = \theta_{1h}(u^i_{ht})^{\theta_{2h}} h^i_t, \quad i \in \{l, b\} \tag{2}$$

The optimal abatement level is given by:

$$u^i_{ht} = \left(\frac{\tau_{ht}\varphi_{1h}}{\theta_{1h}\theta_{2h}(h^i_t)^{\varphi_{2h}}}\right)^{\frac{1}{\theta_{2h}-1}}, \quad i \in \{l, b\} \tag{3}$$

showing that abatement increases with the carbon tax and decreases with housing size and the curvature of the abatement cost function. Abatement effort differs between households according to housing tenure h^i_t

Impatient households face a collateral constraint that limits borrowing:

$$b^b_t \leq \kappa \mathbb{E}_t \left[\frac{q_{t+1}\pi_{t+1}h^b_t}{r_t}\right] \tag{4}$$

linking their credit access to the expected resale value of housing. This constraint affects both intertemporal consumption and housing investment. When binding, it weakens their ability to smooth consumption or invest in energy-efficient housing improvements.

Carbon pricing interacts with this constraint. A residential carbon tax τ_{ht} increases the effective user cost of housing—via both emissions charges $\tau_{ht}e^b_{ht}$ and abatement costs

z_{ht}^{b}—which depresses housing values and tightens borrowing limits. As a result, the policy has asymmetric effects: borrowers reduce consumption and housing investment more sharply than lenders, whose behavior remains unconstrained.

The non-residential sector is composed of monopolistically competitive firms that produce output using capital and labor. Production generates emissions:

$$e_{ct} = (1 - u_{ct})\varphi_{1c} y_t^{1-\varphi_{2c}} \tag{5}$$

where u_{ct} is firm-level abatement effort. Firms reduce emissions only when facing a production carbon tax τ_{ct}, which increases marginal cost and reduces demand for inputs.

Housing is produced by perfectly competitive construction firms using capital and labor. These firms productivity is negatively affected by the economy-wide carbon stock x_t. The government imposes sector-specific carbon taxes τ_{ct} and τ_{ht}, recycles revenues via lump-sum transfers, and maintains constant public consumption. Emissions accumulate as:

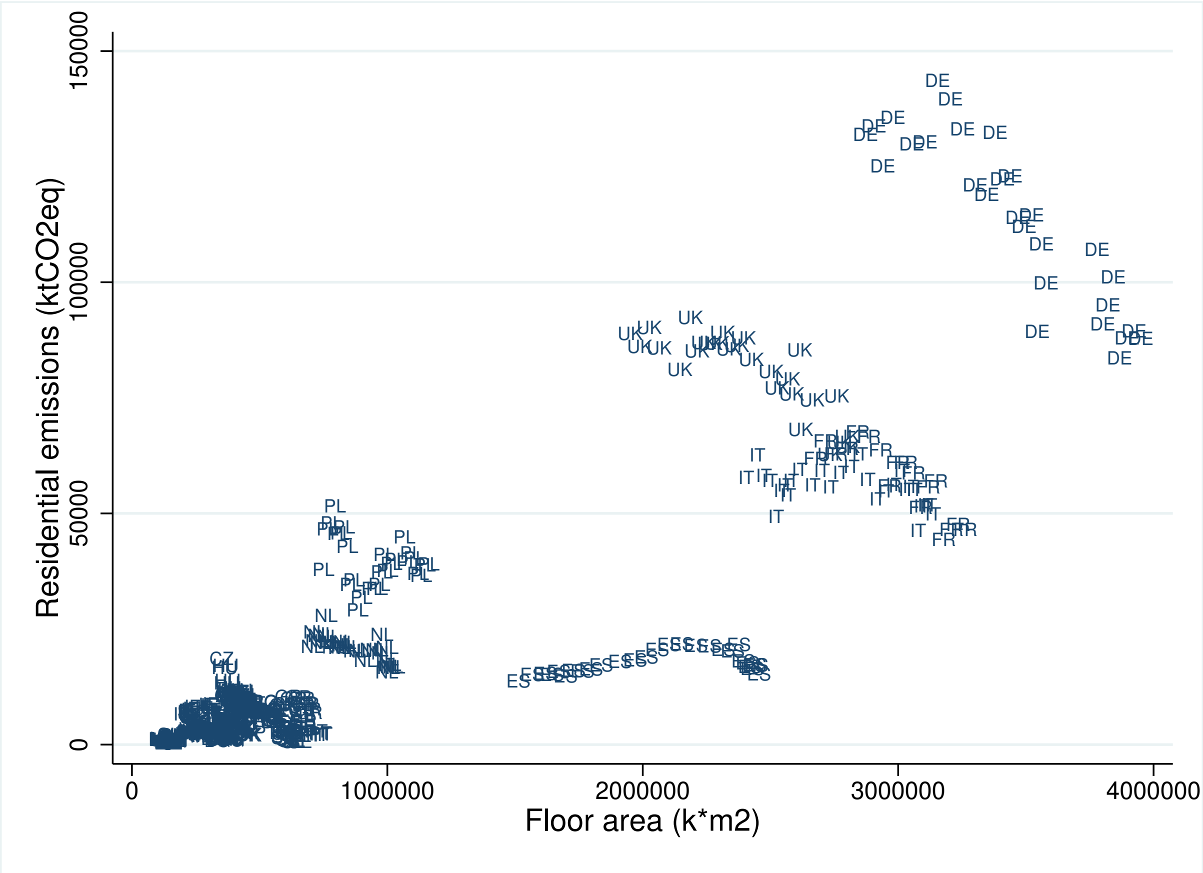
$$x_t = (1 - \delta^x)x_{t-1} + e_{ct} + e_{ht} + e^{\text{row}}, \tag{6}$$

and reduce productivity through a quadratic damage function, following Heutel (2012). Monetary policy follows a standard Taylor rule responding to deviations of inflation from target. The resource constraint reflects the use of output for consumption, investment, housing production, abatement, and price adjustment costs.

3 Calibration

We calibrate the model to replicate certain macroeconomic and environmental ratios of the EU-27 economy for 2019, the year preceding the COVID-19 crisis. While most parameters utilized in the calibration are drawn from previous literature or derived from the model's steady-state equations, we uniquely estimate the elasticity of residential emissions to housing stock using data specific to the EU-27. Our model incorporates the emission functions proposed by Heutel (2012) and Annicchiarico & Di Dio (2015). However, what distinguishes our work in the literature on E-DGE models is our consideration of how residential emissions are contingent upon the housing stock owned by households. Estimating this parameter specifically for the EU-27 ensures a more accurate representation of the dynamics of residential emissions and their relationship with housing characteristics. Below, we provide detailed information on the strategy followed to calibrate the parameters in the model.

Figure 1: Residential emissions generated in EU-27 + UK



Annual emissions over the period 1990:2017 and total floor area for housing

3.1 Estimating the Elasticity of Residential Emissions

To set the elasticity parameter φ_{1h} in the emission function, we study how carbon dioxide emissions from dwellings vary with the housing stock. Specifically, we estimate the following regression model:

$$\log(e_{hit}) = \beta_0 + \beta_1 \log(h_{it}) + \gamma_i + \gamma_t + u_{it} \tag{7}$$

where γ_i and γ_t represent country and time fixed effects, respectively, and β_1 is our parameter of interest.

We use annual country-level data on residential emissions and house floor area for the 27 countries of the European Union plus the United Kingdom over the period 1990–2017. More particularly, our dependent variable is direct emissions produced by the combustion of fossil fuels in homes, principally by heating and cooling systems. Emissions data are taken from national residential emissions reported to the UNFCCC and the EU Greenhouse Gas Monitoring Mechanism[3], and residential floor area by country is sourced

[3] https://www.eea.europa.eu/en/datahub/datahubitem-view/3b7fe76c-524a-439a-bfd2-a6e4046302a2

Table 1: Estimation of emission elasticity with respect to floor area

	(1) $log(e_h)$	(2) $log(e_{hi})$	(3) $log(e_{hit})$
	1.0656***	-0.4296***	0.4688**
	(0.03826)	(0.12833)	(0.19248)
Standard errors	Newey-West	Newey-West	Newey-West
Observations	655	655	655
Country fixed effects	No	Yes	Yes
Year fixed effects	No	No	Yes

Standard errors in parenthesis. $* \ p < 0.1$; $** \ p < 0.05$; $*** \ p < 0.01$.

from the EEA[4] and the EU Building Stock Observatory of the European Commission[5].

Figure 1 presents the relationship between annual CO_2 emissions from dwellings and total residential floor area across all countries during the sample period. It can be observed that, in the pooled scenario, residential emissions increase with floor area over the period considered. However, it is evident from the figure that this relationship varies significantly by country.

Table 1 displays the estimations. The Newey-West estimator addresses potential autocorrelation and heteroskedasticity in the error terms. We present the estimation without fixed effects, followed by models with country and time fixed effects, respectively. The no-fixed-effects estimation produces a coefficient substantially larger than those obtained with fixed effects, highlighting the influence of cross-country differences in residential floor area on carbon emissions. In column (2), after introducing country fixed effects, the coefficient becomes negative, indicating that once we control for time-invariant, country-specific characteristics, emissions per unit of floor area have decreased in Europe over the past 30 years. When time fixed effects are added to control for common trends affecting carbon emissions over time, the coefficient remains significant, suggesting that emissions increase by 0.4688% for every 1% increase in residential floor area[6]. Overall, our findings emphasize the importance of technological developments that reduce emissions per unit of dwelling floor area (column 2). Including time effects partially controls for technological progress, reversing the sign of the elasticity. However, given the importance of this parameter, we will include it in our sensitivity analysis in the simulations results later.

In a study by Cong et al. (2015), the authors find that, depending on the region, the elasticity of carbon emissions from buildings per square meter in China varies from 0.3382 to 0.6175, an interval that includes the value obtained in our preferred estimation.

[4] `https://www.eea.europa.eu/data-and-maps/figures/trends-in-heating-energy-consumption-2`

[5] `https://energy.ec.europa.eu/topics/energy-efficiency/energy-efficient-buildings/eu-building-stock-observatory_en`

[6] An alternative specification with robust standard errors yields similar results

When the elasticity is measured in emissions per square meter on a per capita basis, it varies from 0.418 to 0.8098. Brounen et al. (2012) find that the elasticity of residential gas consumption per square meter in the Netherlands is 0.295, accounting for housing construction period and thermal characteristics. This value is slightly lower than ours and falls outside the range reported by Cong et al. (2015), but it exclusively accounts for residential gas emissions[7]. Also focusing exclusively on gas consumption, Alberini et al. (2011) report that the elasticity of residential heating gas consumption per square foot in the US is 0.22. In view of the previous results in the literature, we set the value of the elasticity φ_{2h} to 0.4688, according to our estimation in column (3).

3.2 Parameters and Targeted Ratios

This section describes the values of the parameters used to obtain the numerical solution. The model is calibrated to the EU-27 setting by using parameters from previous studies and by fixing target values for the steady-state values of selected model variables to determine the remaining parameters.

General Parameters and Target Steady-State Values. We take some of the parameters from the European Central Bank's New Area-Wide Model II Coenen et al. (2018) used for policy analysis. Specifically, we set the inverse elasticity of labor to wages to $\eta = 2$, the elasticity of substitution among final goods to $\varepsilon = 3.85$, price adjustment costs in consumption goods to $\Phi^p = 71.56$, capital adjustment costs in manufacturing and construction to $\Phi^c = 10.78$ and $\Phi^h = 10.78$, the capital depreciation rate in consumer goods production to $\delta^{kc} = 0.025$, and the parameters of the Taylor rule ρ and ψ to 0.93 and 2.74, respectively. Following Iacoviello & Neri (2010), we set the housing stock depreciation rate to $\delta^h = 0.04$ per year.

The steady-state rate of time preference for patient households is set at 1% per quarter, corresponding to an annual interest rate of 4%. For impatient households, the discount rate $\frac{1}{\beta^b}$ is set roughly at 3 percentage points per quarter. Following Bielecki & Stähler (2022), we set the loan-to-value ratio to the standard value $\kappa = 0.75$ and the share of constrained households to $\tau^b = 0.40$. For the elasticity of substitution between labor types ε_n and the elasticity of housing production to capital α^h, we use the estimated values for the EU-27 from Hinterlang et al. (2022), fixing them at 1 and 0.255, respectively.

To calibrate the model, we match the steady-state ratios of economic aggregates to GDP with the corresponding values for the EU-27 in the year 2019. The ratios

[7] As indicated in the Guidelines for National Greenhouse Gas Inventories Intergovernmental Panel on Climate Change (IPCC) (2006), emissions of each greenhouse gas from stationary sources are calculated by multiplying fuel consumption by the corresponding emission factor. Therefore, the emissions of a fossil fuel are directly proportional to its consumption, making the elasticity of emissions per area straightforwardly comparable to the elasticity of consumption per area.

Table 2: Macroeconomic parameters

	Description	Value	
β^l	Lenders' discount rate	0.99009	Annual interest 4%
β^b	Borrowers' discount rate	0.97009	8pp less anually
γ_h	Preference for housing	0.1984	Calibration
θ	Weight parameter in CES	0.0648	Calibration
η	Inverse elasticity of labor supply	2	Coenen et al. (2018)
ε_n	Elasticity of labor between sectors	1	Hinterlang et al. (2022)
κ	Loan-to-value	0.75	Bielecki & Stähler (2022)
τ^b	Fraction of borrowers	0.4	Bielecki & Stähler (2022)
ε	Elasticity of final goods	3.8571	Coenen et al. (2018)
Φ^c	Manufacturing capital adjustment cost	10.78	Coenen et al. (2018)
Φ^h	Housing capital adjustment cost	10.78	Coenen et al. (2018)
δ^{kc}	Manufacturing capital depreciation rate	0.025	Coenen et al. (2018)
δ^{kh}	Construction capital depreciation rate	0.0084	Calibration
Φ^p	Pricing adjustment cost	71.56	Coenen et al. (2018)
α^c	Elasticity of output to capital	0.3648	Calibration
α^h	Elasticity of housing to capital	0.255	Hinterlang et al. (2022)
δ^h	Housing depreciation rate	0.01	Iacoviello & Neri (2010)
ρ	Interest rate smoothing	0.93	Coenen et al. (2018)
ψ	Interest rate reaction to inflation	2.74	Coenen et al. (2018)
a_t^c	TFP consumption goods sector	0.2001	Calibration
a_t^h	TFP housing sector	0.1433	Calibration

are obtained from Eurostat. The steady-state ratios of residential investment and non-residential investment to GDP are 5.2% and 18.9%, respectively. In the residential sector, corporate gross fixed capital formation is equivalent to 0.604% of GDP. Government spending (which is determined exogenously in our model) represents 21.4% of GDP in steady state. Additionally, the weight of the labor force working in the residential sector is targeted at 7.2%. For simplicity, we assume that lump-sum taxes are initially distributed equally on a per capita basis between lenders and borrowers, such that $tr_t^l = tr_t^b = tr_t$. Table 2 shows the general parameters and exogenous variables that are set and calibrated.

Environmental Parameters. Most of the environmental parameters are taken from Gibson & Heutel (2023) and adjusted to the units of the model, as total GDP is normalized to 1 (in millions of euros), and therefore, most of the variables are interpreted in terms of million euros of total production. Moreover, we impose values for emissions in both sectors e_c, e_h, and the atmospheric carbon dioxide stock x. According to data from the Global Monitoring Laboratory, the carbon stock in the atmosphere in 2019 is set at $x_{2019} = 876.84$ GtC. The steady-state ratios of the EU-27 annual emissions in 2019 for consumption goods firms and the residential sector are obtained from the UNFCCC and the EU Greenhouse Gas Monitoring Mechanism. The steady-state values of annual

Table 3: Environmental parameters

	Description	Value	
$1-\delta^x$	Pollution decay rate	0.9965	Gibson & Heutel (2023)
d_2	Damage quadratic coefficient	1.17E-07	Gibson & Heutel (2023)
d_1	Damage linear coefficient	2.70E-05	Gibson & Heutel (2023)
d_0	Damage constant	-0.0076	Gibson & Heutel (2023)
φ_{1h}	Emission intensity housing	0.0034	Calibration
$1-\varphi_{2h}$	Emissions elasticity housing	0.4689	Authors' calculations
φ_{1c}	Emission intensity in production	0.0692	Calibration
$1-\varphi_{2c}$	Emissions elasticity in production	0.6	Gibson & Heutel (2023)
θ_{1h}	Housing abatement cost coefficient	0.074	Gibson & Heutel (2023)
θ_{2h}	Housing abatement cost exponent	2.6	Nordhaus (2018)
θ_{1c}	Production abatement cost coefficient	0.074	Gibson & Heutel (2023)
θ_{2c}	Production abatement cost exponent	2.6	Nordhaus (2018)
e^{row}	rest-of-the-world emissions	0.85 ktC/mill€	Calibration

emissions in the manufacturing and residential sectors are 3284.548 and 317.035 MtCO_2 equivalent, respectively.

From Gibson & Heutel (2023), we take the elasticity of emissions to output in manufacturing, $1-\varphi_{2c}=0.6$. We also use their value for the pollution stock decay rate $1-\delta^x$, which assumes a half-life of atmospheric carbon dioxide of 50 years, as well as their coefficients of the damage and abatement cost functions. From Equation (6) we obtain e^{row}. The atmospheric carbon stock is measured in gigatons (GtC) per million euros of GDP (in constant 2015 euros). After scaling the damage function coefficients with respect to the EU-27 GDP in 2019 to match the production losses reported by Gibson & Heutel (2023), we impose $d_2 = 1.17 \times 10^{-7}$, $d_1 = 2.70 \times 10^{-5}$, and $d_0 = -0.0076$. In the abatement cost function, we choose $\theta_{1c} = 0.074$, corresponding to a total share of abatement costs over production of 7.4% when the emission reduction share is 100%. Similarly, θ_{1h} is calibrated to ensure that the cost of abating 100% of residential emissions in the steady state is equivalent to 7.4% of housing production. Specifically, $z_h = 0.074 \times IH$, which in the steady state simplifies to $z_h = 0.074 \times h \times \delta^h$. According to equation (2) under full abatement, $z_h = \theta_{1h} \times h$, meaning that $\theta_{1h} = \delta^h \times 0.074 = 0.0074$.

The cost elasticities to abatement, θ_{2c} and θ_{2h}, are set to 2.6 from Nordhaus (2018). Finally, the calibration of the elasticity of carbon emissions to the housing stock $1-\varphi_{2h}$ is based on the estimation of the log of residential emissions on the log of dwelling floor area, as shown in Section 3.1.

Table 3 summarizes the environmental parameters recovered from previous studies and those that result from the calibration strategy.

4 Results

The adoption of the European Climate Act (European Commission, 2021) transformed the political commitment of the 2019 Green Deal (European Commission, 2019) into a legal obligation. EU countries are thus required to reduce greenhouse gas emissions by at least 55% below 1990 levels by 2030, with the goal of achieving climate neutrality by 2050. The building sector is a critical area for addressing climate and environmental challenges; therefore, the EU will extend the existing EU Emissions Trading Scheme to include emissions from the residential sector. The objective is to achieve a 43% reduction in emissions from buildings by 2030 compared to 2005 levels. While the introduction of a carbon price on residential emissions rightly encourages cost-effective emissions reductions and incentivizes behavioral change, it also imposes a burden on consumers.

In this section, we present simulation results on the macroeconomic effects of policies that require households to pay for their housing emissions while also pricing carbon emissions from the production of other consumption goods and services. After analyzing these macroeconomic impacts, we focus on the differential effects between household types to uncover the inequality effects on key components of household welfare: consumption, housing holdings, and leisure time. Furthermore, we explore government strategies to alleviate the effects of carbon pricing policies during the transition to climate neutrality. Specifically, we evaluate measures to lessen the burden on households by utilizing carbon revenues from environmental policies to address redistributive issues.

4.1 Carbon Pricing

The transition to zero emissions is simulated by imposing a linear increase in the carbon price in both the residential and consumer goods production sectors. Starting from an initial steady state where no environmental policy is anticipated, an emissions pricing path is suddenly announced, and agents respond with perfect foresight until the new steady state is reached. We designate period 1 as 2019Q4, when the government introduces emission prices. The carbon price for residential emissions (τ_{ht}) and production emissions (τ_{ct}) increases linearly over 120 quarters, with a discontinuity before the halfway point, ensuring that emission reduction targets are met by 2030 and climate neutrality is achieved by 2050. Our intermediate targets include a 43% reduction in housing emissions by 2030 relative to 2005 levels and a 55% reduction in carbon emissions from the non-residential sector by 2030 compared to 1990 levels. Considering that the absorption rate used in the calibration implies that terrestrial ecosystems are able to absorb 25% of global emissions,[8] achieving NZE by 2050 requires a 75% reduction in both residential and non-residential emissions by 2050 relative to 2019 levels.

[8]This figure is consistent with observations over the past 50 years (see Brienen et al. (2020)

4.1.1 Macroeconomic impacts

Our simulations isolate the gross impacts of carbon pricing policies on greening the economy, without accounting for the effects of technological improvements on energy use or carbon emissions per unit of energy. In our model, agents adjust their allocation of resources for emissions reduction in response to increases in carbon prices. Consequently, the carbon prices depicted in Figure 2 represent an upper bound of those required to meet emissions reduction targets.

In the absence of technological change, the carbon price would need to increase by approximately €460 per ton of CO_2 in the non-residential sector, and by €260 per ton of CO_2 in the residential sector. This increase would be accompanied by a more pronounced acceleration in the carbon price in the residential sector after 2030.

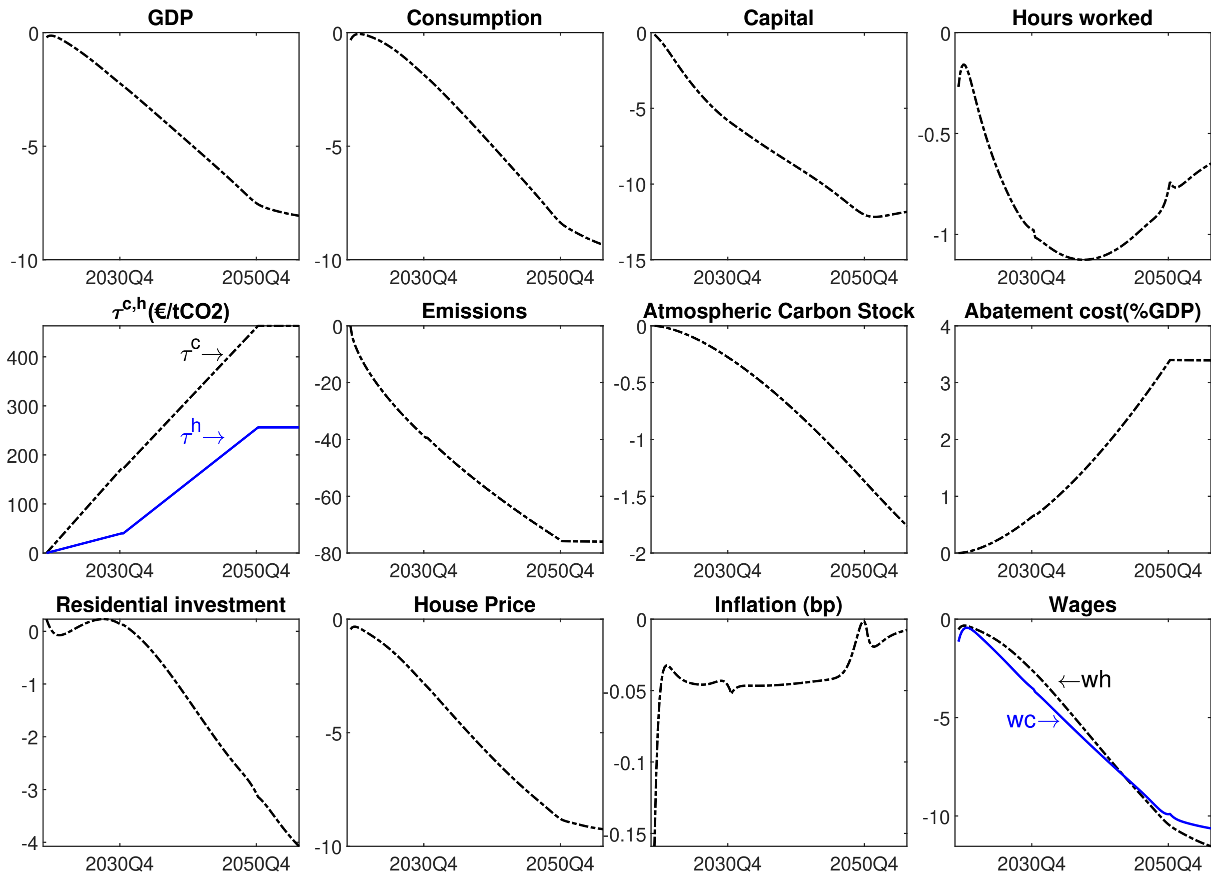
The environmental policy reduces atmospheric carbon stock by 1.36% by 2050. [9] However, the additional tax burden on households and firms during the transition period, leads to an approximate 7.45% decrease in aggregate GDP. This decline in GDP is accompanied by substantially lower levels of investment and consumption.

Firms in the consumer sector, faced with the additional costs of the carbon price, τ^c, reduce their demand for labor and capital, thereby decreasing production. Capital in the non-residential sector declines by over 10%. Similarly, housing production is adversely affected, primarily due to the demand-side effects of carbon pricing. The carbon price targeting emissions from housing deters housing demand by increasing the costs associated with emissions pricing and abatement, leading to a 3% reduction in residential investment and a significant 8.7% decrease in housing prices. The adjustment in housing prices reflects not only weakened demand but also an oversupply of capital and labor resulting from reduced overall production. Idle factors of production experience a decrease in their prices, which firms pass on as lower product prices, particularly in the construction sector. Consequently, price behavior differs notably between the two production sectors. While inflation in consumer goods production remains relatively stable—since the cost factors pushing prices upward are offset by weaker aggregate demand—housing prices decline, primarily due to the negative demand impact on this sector.

Aggregate consumption is adversely affected by declining incomes, abatement costs incurred by households during the transition, and deteriorating credit conditions. In our baseline calibration, consumption decreases by 8% by 2050, with abatement costs representing 3.4% of GDP upon achieving NZE. Since changes in housing prices are not incorporated into the Central Bank's policy framework, monetary policy does not respond to declines in housing prices as it does to consumer goods inflation. Consequently, the monetary policy tool is less effective in mitigating the adverse aggregate effects on GDP resulting from reduced consumption by borrowers, which is further exacerbated by

[9] This figure reflects only Europe achieving NZE, without contributions from other countries.

Figure 2: Macroeconomic Impacts of Carbon Pricing Policies



The results are presented as percentage deviations from their expected trajectories in the absence of environmental policies, except for emission prices, which are denominated in euros, inflation, measured in basis points, and abatement costs, expressed as percentage points of GDP.

declining housing prices and the resulting deterioration in credit conditions.

To further investigate the impact of carbon pricing on household consumption and other economic decisions, the next subsection disaggregates the effects by household type, analyzing the responses of borrowers and lenders to the emissions reduction policy.

4.1.2 Impact by household type

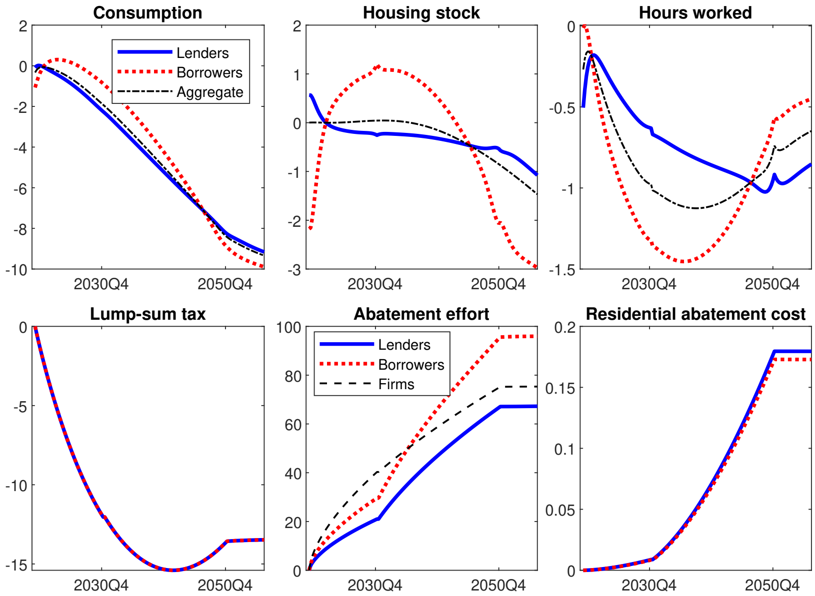
Figure 3 displays the percentage deviations of household-related variables relative to the no-policy scenario, highlighting the differential impacts of the carbon pricing policy across household types. In our benchmark scenario, depicted in Figure 3a, government carbon revenues are distributed equally among household types as a carbon dividend that offsets their tax burden.

Credit-constrained households exhibit the highest abatement effort, measured as the percentage of residential emissions reduced each period. This behavior results from the optimal abatement path, as detailed in equation (3), which indicate that abatement effort is inversely related to housing stock. The intuition behind this outcome is that households invest in abatement to mitigate the negative impact of increased carbon prices on their housing stock, as reducing housing serves as an alternative to offsetting higher carbon costs. Since borrowers own fewer houses than lenders, reducing housing would result in a greater utility loss for borrowers than for lenders due to the decreasing marginal utility of housing. Consequently, borrowers find it optimal to exert greater abatement effort. In contrast, despite lower abatement efforts by lenders, they incur slightly higher abatement costs because they own more houses.

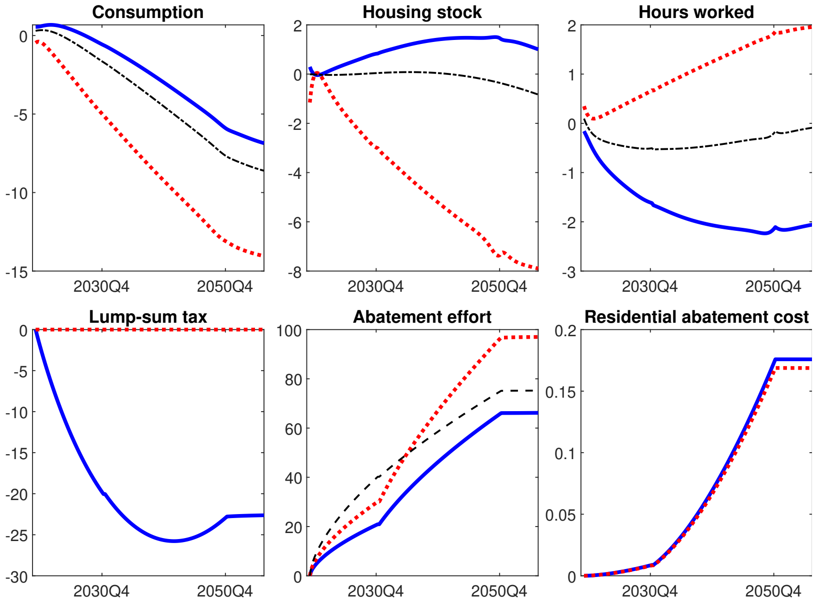
The first row of plots in Figure 3a illustrates the evolution of key variables influencing household welfare: consumption, housing property, and leisure (working hours). Significant disparities emerge in household responses concerning these variables. Borrowers, because they have a higher preference rate for the present against the future, are more responsive to government transfers (tax reductions) than lenders. As shown in the figure, lump-sum taxes decrease in a convex manner due to two opposing factors. First, the carbon tax increases over time, raising government revenues that, when rebated to households, result in lower taxes. Second, emissions decline over time, reducing the tax base. Consequently, the carbon dividend reaches an upper limit before the end of the transition period.

In the very first periods, borrowers' consumption falls compared to lenders', primarily due to the reduction in credit caused by the decline in the value of collateral housing through decreasing house prices (see Figure 4). Borrowers demand houses not only for the housing services they provide but also because they can be used as collateral to obtain credit. As a result, borrowers' housing demand decreases in the initial periods. Over time, the dynamic choices of both household types diverge significantly.

Figure 3: Impact of carbon pricing policy by household type



(a) Equitable distribution of carbon dividends across the population



(b) Allocation of all carbon dividend to patient households

Variables are presented as percentage deviations from their expected trajectories in the absence of environmental policy, except for abatement and abatement costs. Abatement is expressed as a percentage of emissions, and residential abatement costs are expressed as a percentage of GDP.

Patient households bear a greater tax burden from residential carbon taxes because they own more houses per capita. Additionally, they experience a reduction in profits from firm ownership as emissions prices increase, which adversely affects their consumption prospects. In contrast, impatient households face a relatively lower environmental tax burden and thus benefit relatively more from the carbon dividend, which is distributed evenly among households. After the initial negative impact of the policy announcement, this dividend offsets their residential emissions payments and increases their disposable income. Consequently, borrowers respond by expanding their housing stock, helping to sustain their consumption.

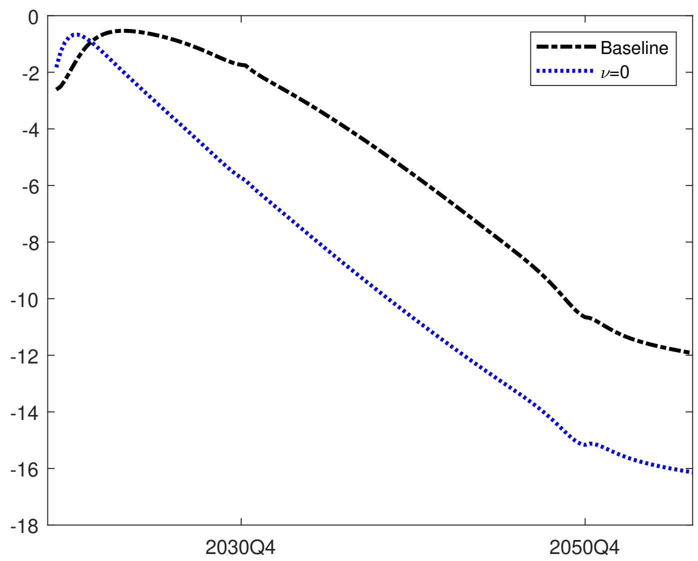
However, as emissions are progressively reduced, the diminishing carbon dividend leads to reduced government transfers, and the significant decline in house prices weakens borrowers' ability to use their properties as collateral, explaining the hump-shaped trajectory of houses owned by borrowers. This reduction in collateral limits both consumption and housing demand. Labor demand decreases with economic activity, driving down wages and hours. As consumption continues to deteriorate, borrowers are compelled to increase their labor supply over time in an effort to restore income (and consumption) stability. Eventually, the increase in labor supply compensates for the decline in labor demand, as reflected in the U-shape of hours worked.

Despite the differing transition dynamics, both borrowers and lenders conclude the transition period with substantial declines in consumption, decreasing by 9% and 8%, respectively. Housing demand decreases by 2% for borrowers compared to 0.5% for lenders, while working hours decline by 0.6% for borrowers and 0.9% for lenders.

Under an alternative carbon dividend distribution, household responses exhibit significant changes. For illustrative purposes, Figure 3b demonstrates the impact of the carbon pricing policy on consumption, housing stock, and hours worked when credit-constrained households do not receive carbon revenues, thereby maintaining their initial lump-sum taxes at a constant level. In this scenario, lenders fully receive the carbon dividend. They allocate a portion of their additional income to sustain consumption and private bonds during the initial periods. The rebound of debt in lenders' hands observed in Figure 4 helps support borrowers' consumption in the early stages of the transition to NZE. However, as the transition progresses, lenders benefit from an increasing share of their income from government transfers, while the income of impatient households deteriorates without the support of those transfers. Consequently, borrowers' consumption and credit demand shrinks, and lenders take advantage of declining house prices to redirect their savings toward housing investments.

Overall, this results in a markedly different trajectory for consumption, housing holdings, and working hours throughout the transition period for borrowers compared to the benchmark carbon dividend distribution illustrated in Figure 3a. Specifically, the previously observed hump-shaped patterns in borrowers' consumption and housing stock, as well as the U-shaped pattern in working hours, disappear. Despite the significant distributional effects of allocating government revenues from carbon pricing, the overall consumption dynamics remain largely unaffected. Therefore, focusing solely on aggregate outcomes can obscure substantial distributional impacts.

Figure 4: One-period debt held by borrowers



Two scenarios are displayed: (a) equitable distribution of carbon dividends, and (b) allocation of all carbon dividends to patient households ($\nu = 0$)

From the above discussion, we identify two key insights. First, the allocation method of carbon dividends significantly affects household consumption, housing, and labor responses, with borrowers being more sensitive to the specific dividend distribution scheme. Second, despite owning fewer houses, borrowers undertake higher abatement efforts because reducing their housing stock would result in a greater utility loss compared to lenders.

4.2 Sector-Specific and Combined Carbon Pricing Policies

So far, we have examined the effects of simultaneously increasing carbon prices in both the residential and non-residential sectors. This section investigates the distinct economic, distributional, and welfare implications of implementing sector-specific policies within the broader framework of achieving NZE targets.

4.2.1 Long-run effects

Table 4 compares scenarios where carbon prices are simultaneously increased in both residential and non-residential sectors with those where only the non-residential or only the residential carbon price is elevated.[10]

The table is organized into three sections: first, it presents the effects on a set of aggregate macroeconomic variables; second, it analyzes household-type-specific variables;

[10]In all cases, the carbon price trajectories follow those depicted in Figure 2.

Table 4: Impact of carbon pricing under three scenarios: combined, production-only, and residential-only policies

	No policy	$\tau_c + \tau_h$	Variation (%)	τ_c	Variation (%)	τ_h	Variation (%)
Macroeconomic							
GDP	1.00	0.918	(-8.23%)	0.919	(-8.05%)	0.998	(-0.18%)
c	0.545	0.490	(-10.00%)	0.491	(-9.88%)	0.544	(-0.13%)
x	262.60	246.62	(-6.09%)	248.02	(-5.55%)	261.21	(-0.53%)
z	0.00	3.39%	-	3.21%	-	0.18%	-
IH	0.0520	0.0501	(-3.75%)	0.0508	(-2.24%)	0.0513	(-1.40%)
q	1.00	0.913	(-8.70%)	0.922	(-7.79%)	0.991	(-0.90%)
b^b	1.990	1.731	(-13.02%)	1.785	(-10.32%)	1.936	(-2.74%)
Households							
c^l	0.687	0.619	(-9.91%)	0.620	(-9.74%)	0.686	(-0.18%)
c^b	0.332	0.298	(-10.30%)	0.298	(-10.32%)	0.332	(0.02%)
h^l	6.880	6.639	(-3.49%)	6.735	(-2.11%)	6.791	(-1.29%)
h^b	2.680	2.554	(-4.73%)	2.607	(-2.75%)	2.631	(-1.85%)
n^l	0.844	0.839	(-0.59%)	0.838	(-0.65%)	0.844	(0.05%)
n^b	1.213	1.208	(-0.38%)	1.209	(-0.33%)	1.212	(-0.05%)
Welfare							
W^s	-0.716	-0.826	(-15.26%)	-0.821	(-14.68%)	-0.720	(-0.54%)
CE^l	0.00	10.23%	-	9.77%	-	0.47%	-
CE^b	0.00	10.55%	-	10.3%	-	0.27%	-

Note: Abatement costs z are expressed as percentage of GDP. Equivalent consumption CE^l and CE^b represent percentage of no-policy consumption that a representative consumer would forgo to achieve the same welfare level under carbon emissions policies.

and third, it provides welfare indicators. This table focuses on the long run, with the columns representing steady-state levels with and without the policy. The differences in levels are shown in brackets as percentage deviations from the no-policy values.

The analysis reveals that, unsurprisingly given the volume of emissions in both sectors, carbon pricing in the non-residential production sector is the primary driver of economic changes. In contrast, residential carbon pricing generally has relatively modest effects on overall economic indicators. However, significant long-term effects are observed in residential investment, housing stock, and new credit, specifically due to residential emissions pricing, both in absolute and relative terms.

Residential emissions account for 8.3% of total carbon emissions in the model. However, as shown in the upper section of Table 4, the impact of a residential-specific policy on GDP or consumption is significantly lower than this share when compared to a combined policy aimed at achieving general NZE across all emission sources. This indicates that pricing residential emissions adversely affects aggregate activity much less per unit of emissions reduction than pricing non-residential emissions.[11] Consumption exhibits

[11] For example, using the figures from table row GDP, considering that residential emissions represent 8.3% of total emissions, and that the long-run rate of reduction in emissions is the same in both sectors

a similar pattern, decreasing by 10% under dual pricing and 9.88% under single non-residential pricing.

The impact of τ^h becomes more pronounced when examining variables directly related to the residential sector and credit. Residential investment declines by 3.75% due to the combined environmental policies aimed at achieving climate neutrality. While general equilibrium effects from taxing carbon emissions in the non-residential sector account for a 2.24% reduction in residential investment, Table 4 indicates that 1.40% decline can be attributed to residential carbon pricing policies in isolation. Additionally, housing prices fall by approximately 0.9% under τ^h, representing over 10% of the decline caused by the combined policy of $\tau^h + \tau^c$.

In our model, debt (the private asset issued by borrowers) expires each period and can be interpreted as fresh credit. This credit is adversely affected by the residential carbon tax, decreasing by 2.74% in the long run. Compared to the non-residential carbon policy, taxing residential emissions reduces credit by a ratio of approximately 3 to 1 per unit of emissions reduced. Thus, the impact on housing investment, housing prices, and credit per unit of carbon reduction is significantly more pronounced for τ^h than for τ^c.

In the middle section of the table, we assess the impact of carbon pricing on different household types by focusing on key determinants of utility: consumption, housing, and leisure. The findings are as follows: first, in the long term, the combined policy adversely affects borrowers' consumption and housing more than lenders', and borrowers experience a smaller increase in leisure. Overall, this indicates a relative long-term deterioration of borrowers' welfare compared to lenders'. Second, the residential emissions pricing policy has minimal long-term effects on consumption and working hours (leisure) across all household types. Third, the residential emissions policy exerts a relatively greater negative impact on borrowers' housing compared to lenders', likely further exacerbating the long-term decline in borrowers' welfare.

Next, we will examine the welfare implications of the policies in more detail. To this end, we will not only compare static solutions but also consider the full dynamics implied by the policies.

4.2.2 Welfare effects

In the bottom section of Table 4, we translate the previous results into welfare changes, this time incorporating all transition dynamics. The first row, denoted as W^s, represents a direct measure of social welfare in terms of utility, defined as the weighted sum of

(equal to 75%), we can calculate $\left(\frac{-0.18}{-8.05}\right)\left(\frac{1-0.083}{0.083}\right) \approx \frac{1}{4}$, indicating that the GDP cost of reducing emissions in the residential sector is about 1/4 of that in the non-residential sector.

individual utility across household types. It is given by

$$W^s = (1-\tau^b)(1-\beta^l)W^l + \tau^b(1-\beta^b)W^b,$$

where W^l and W^b correspond to the present expected value of lifetime welfare of patient and impatient households. The weights in the functions have been chosen so that for the same constant flow of final consumption, patients and inpatients would receive the same level of utility (see Lambertini et al. (2013) or Notarpietro & Siviero (2015))

The following two rows present household-specific changes in welfare, measured in terms of consumption equivalent variation, CE^l and CE^b. More generally, the post-policy welfare of household a (where $a = l, b$)—measured as the expected present value—is given by

$$\widetilde{W}^a = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln \tilde{c}_t^a + \gamma_h \ln \tilde{h}_t^a - \frac{(\tilde{n}_t^a)^{1+\eta}}{1+\eta} \right), \tag{8}$$

The discounted sum of utility for a representative household a, assuming the economy is at the steady state in period 0 and remains constant throughout is

$$\overline{W}^a = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln (\bar{c}_t^a) + \gamma_h \ln \bar{h}_t^a - \frac{(\bar{n}_t^a)^{1+\eta}}{1+\eta} \right), \tag{9}$$

The consumption equivalent measures the percentage of consumption in the no-policy scenario that a representative consumer would be willing to forgo to achieve the same level of welfare under the carbon emissions policies. A positive value indicates a welfare loss, suggesting that the carbon pricing policy is less desirable from a welfare perspective. To determine CE^a, we solve

$$\widetilde{W}^a = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^a)^t \left(\ln (\bar{c}_t^a (1 - CE^a)) + \gamma_h \ln \bar{h}_t^a - \frac{(\bar{n}_t^a)^{1+\eta}}{1+\eta} \right), \tag{10}$$

from which we obtain

$$CE^a = 1 - \exp\left[(1-\beta^a)(\widetilde{W}^a - \overline{W}^a) \right]. \tag{11}$$

Achieving NZE targets through a combined increase in carbon pricing across both sectors results in a generalized welfare loss of -15.3% relative to the no-policy scenario in utility units. In contrast, the welfare simulation for carbon pricing in the residential sector alone yields a modest welfare effect of -0.54%. Considering the residential sector's share of emissions, this result indicates that reducing a unit of carbon emissions in the residential sector is less than half as costly in welfare terms as reducing emissions in the non-residential sector.

In terms of consumption equivalent, the long-run welfare cost in the benchmark model represents a decline of more than 10% in consumption. This cost is relatively evenly distributed across households under the combined policy. However, significant differences emerge when comparing the impacts of taxing carbon emissions in the residential versus non-residential sectors. Specifically, the welfare cost in terms of consumption is 0.5 percentage points higher for borrowers than for lenders when the carbon price increase applies only to the non-residential sector. Conversely, when only the residential sector is subject to the carbon price increase, the welfare cost for lenders, though modest, is nearly twice that of borrowers. Therefore, a more in-depth welfare analysis contradicts the steady-state intuition that housing carbon pricing would further harm borrowers through a reduction in housing properties. When all transitional dynamics are considered from the outset of the policy, and with an equitable distribution of the carbon dividend, borrowers benefit from increased consumption, housing, and leisure during certain periods of the transition (see Figure 3a).

Overall, our findings indicate that while carbon pricing in the non-residential sector is the primary driver of economic changes and welfare losses, residential carbon pricing—despite its modest impact on aggregate indicators—significantly affects the residential sector by reducing investment, housing stock, and credit availability. Notably, reducing emissions in the residential sector is less costly in welfare terms per unit of emissions reduced compared to the non-residential sector. Additionally, the differential impacts on borrowers and lenders highlight important distributional effects: borrowers experience greater declines in consumption and housing under combined policies, whereas lenders bear a relatively higher welfare cost when only residential emissions are priced. These insights suggest that carbon pricing policies can have significant implications for inequality, emphasizing the importance of accounting for these differential impacts during the transition to NZE. This objective is addressed in the next subsection.

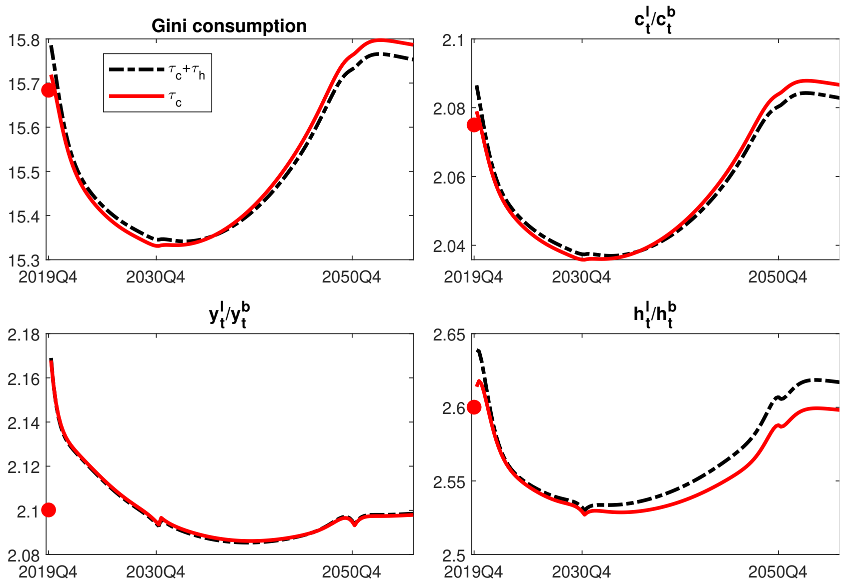
4.3 Carbon Pricing and Inequality

The burden of carbon pricing is not distributed equally across households. If it disproportionately affects less advantaged households, it may lead to social resistance against the policies. Additionally, distributional impacts may shift throughout the transition. This section examines how these evolving distributional effects influence inequality between household types.[12]

We present in Figure 5 two policy scenarios: a combined increase in both residential and non-residential carbon prices, and a single increase in the non-residential carbon price. The difference between these scenarios approximates the inequality impact of the

[12]While carbon pricing policies introduce economic adjustments, their long-term benefits—ranging from reduced climate damages and health improvements to economic growth through innovation—can outweigh the short-term costs. These long-run effects are beyond the scope of this paper, which focuses primarily on the realized costs during the transition

Figure 5: Inequality in the transition to NZE



Dashed line: Combined carbon pricing scenario; Solid line: Only carbon pricing on the non-residential sector

residential carbon policy. We maintain the benchmark of an equal per capita distribution of the carbon dividend, with the pre-policy value shown as a large solid dot at 2019.

In the first subplot of Figure 5, we assess inequality based on consumption using the Gini coefficient. Given that $(1-\tau^b)$ represents the share of high-income households who consume a fraction $(1-\tau^b)c_t^l/c_t$ of total consumption, the Gini coefficient is calculated as the difference between their implied percentage of total consumption and their percentage in population, that is, $(1-\tau^b)(\frac{c_t^l}{c_t}-1)$.[13]

The figure shows an initial increase in inequality that soon starts to decrease during the transition period as lump-sum tax reductions, which primarily benefit borrowers' consumption, are implemented (discussed in Section 4.1.2). From 2019 until 2030, coinciding with the achievement of intermediate climate targets, the Gini coefficient decreases by 0.45 percentage points. This reduction is approximately half of the decline in the Gini coefficient of disposable income observed in the EU between 2013 and 2024, according to Eurostat[14]. However, after reaching a minimum around 2030, the Gini coefficient begins to rise steadily, reversing the gains in consumption distribution and resulting in the highest level of inequality when the economy reaches NZE by 2050. As previously discussed, the reduction in the carbon dividend, as carbon target gaps close, and the decline in the value of borrowers' collateral contribute to this outcome.

Using only the ratio between lenders' and borrowers' consumption in the second subplot of the first row provides a similar picture to that obtained with the Gini coefficient.

[13]The same statistics is used by Rubio & Unsal (2017)
[14]Eurostat (2024)

Similarly, in the second row of Figure 5, we present the ratios of incomes—excluding financial incomes—and housing between the two household types. Unlike the U-shaped transition dynamics observed for consumption and housing inequality, income inequality displays an L-shaped pattern, decreasing and remaining lower throughout the entire transition period. We interpret the differences between the dynamics of consumption inequality and income inequality as indicative of the significance of wealth and financial channels, which adversely affect borrowers after 2030.

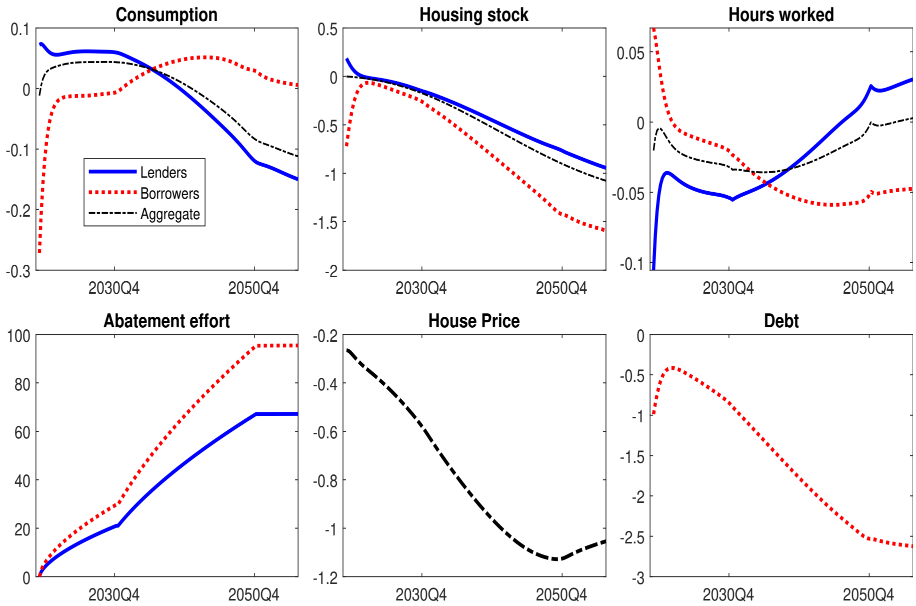
The comparison of the two policy scenarios reveals that residential carbon pricing, despite its relatively moderate impact, further increases consumption inequality for approximately half of the transition period up to 2030, before helping to mitigate the rise in inequality in the later stages. The dynamics differ when considering the ratio of housing, as the residential carbon tax induces a shift in the distribution of houses owned by households, becoming more skewed toward lenders, especially as the tax accelerates after 2030. To explain this outcome, we show in Figure 6 the response of several variables.

Borrowers demand houses not only for housing services, as lenders do, but also because they use them as collateral for obtaining credit. The increasing carbon price leads to a decline in house prices, reducing the value of the collateralized asset, which in turn negatively impacts credit availability and consumption. Regarding housing demand, borrowers must balance the marginal disutility of reducing their housing stock to cope with higher housing costs—which is greater for them than for lenders—against the loss of capacity to convert houses into consumption through credit, a constraint that lenders do not face. The results indicate that the latter effect dominates, causing borrowers to reduce their housing stock more than lenders. However, the abatement effort is much higher for borrowers than for lenders, suggesting that, without the disutility effect mentioned before, the reduction in borrowers' housing demand would have been even more pronounced.

Because borrowers are more impatient than lenders, they tend to convert changes in income into contemporaneous changes in consumption at a higher rate than lenders. Consequently, as the carbon dividend increases, borrowers are more likely to immediately allocate it to consumption. Over time, this leads to a substitution effect where borrowers shift from housing to consumption, a dynamic that is not observed for lenders.

Our results suggest that consumption Gini dynamics vary depending on whether taxes are applied to the residential or non-residential sector. In any case, the time-dependent response of the Gini coefficient for consumption highlights periods when inequality mitigation policies are most needed. In a realistic scenario that combines both residential and non-residential taxes, such policies are particularly crucial in both the early and later stages of the transition, especially after reaching the intermediate 2030 targets and advancing toward NZE. When the policy on residential emissions is considered in isolation, inequality relief policies are primarily needed during the initial years.

Figure 6: Residential carbon price effects



Variables are presented as percentage deviations from their expected trajectories in the absence of environmental policy, except for abatement, which is expressed as a percentage of emissions.

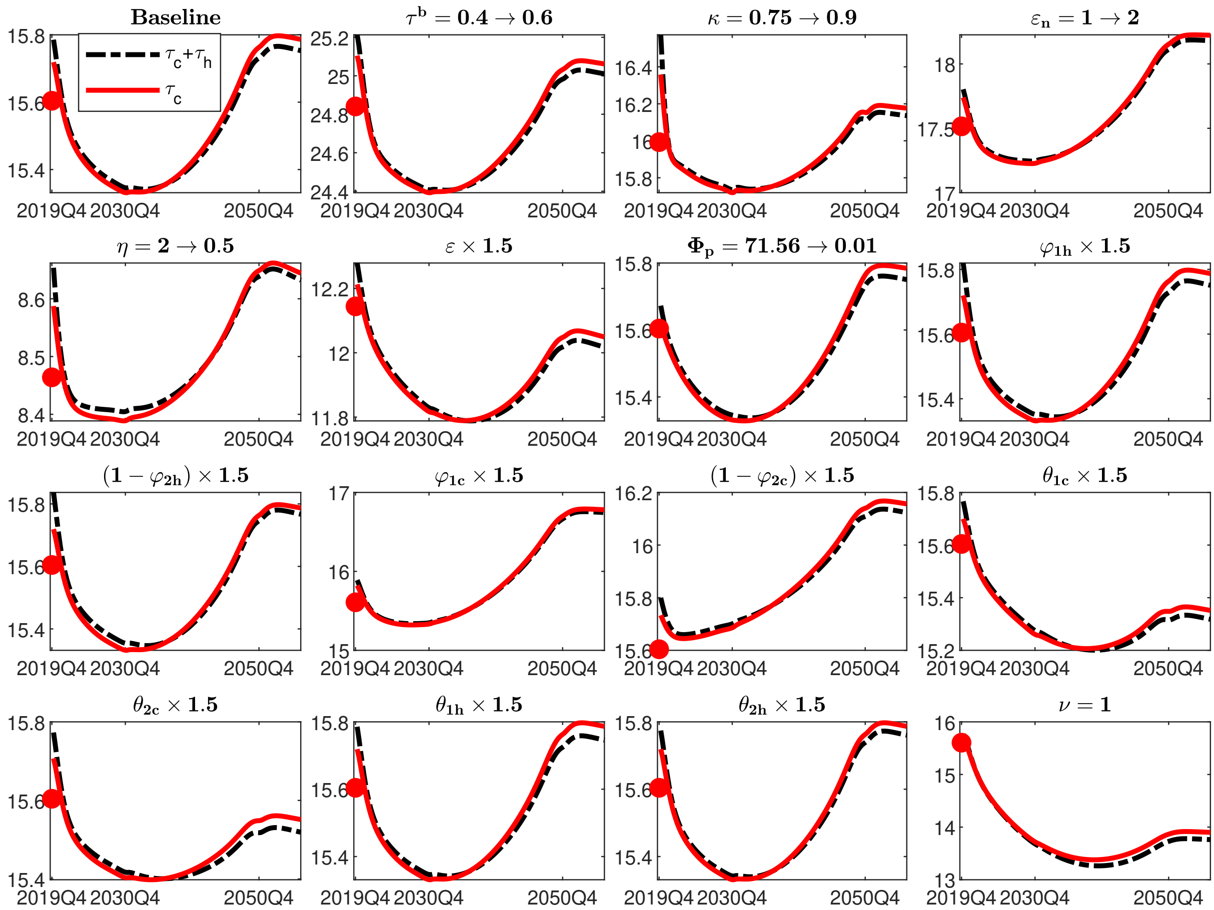
4.3.1 Robustness of inequality dynamics

We investigate the impact of several factors on inequality dynamics, including the proportion of borrowers in the economy, the intensive margin of borrowing, the distribution of the carbon dividend, labor mobility flexibility between sectors, labor supply elasticity with respect to wages, the intensity of emissions response to housing demand and to goods and services production, and the cost of abatement in both sectors.

Figure 7 presents the Gini coefficients across different parameter settings, with the benchmark result included for comparison. In each subplot, only one model parameter is altered, while the growth rate of carbon emissions prices remains constant across all simulations, ensuring that any changes in inequality dynamics throughout the transition are directly attributable to specific parameter variations. However, adjustments in parameters may alter the initial equilibrium. Additionally, Figure 8 illustrates how carbon emissions dynamics respond to changes in economic conditions. Together, Figures 7 and 8 offer a comparative assessment of the impacts on inequality (via Gini coefficients) and efficiency (via emissions reduction) of carbon pricing policies under different settings.

The overall picture that emerges is that, in general, the U-shaped pattern in inequality dynamics observed in the benchmark calibration holds across a wide range of different settings. This means that consumption inequality increases immediately after the policy announcement, then begins to decline, eventually recovering at some point. The isolated effect of residential carbon pricing also appears to remain relatively unaffected across the various subplots. However, a closer examination reveals that certain factors within the

Figure 7: Consumption Gini dynamics under different settings



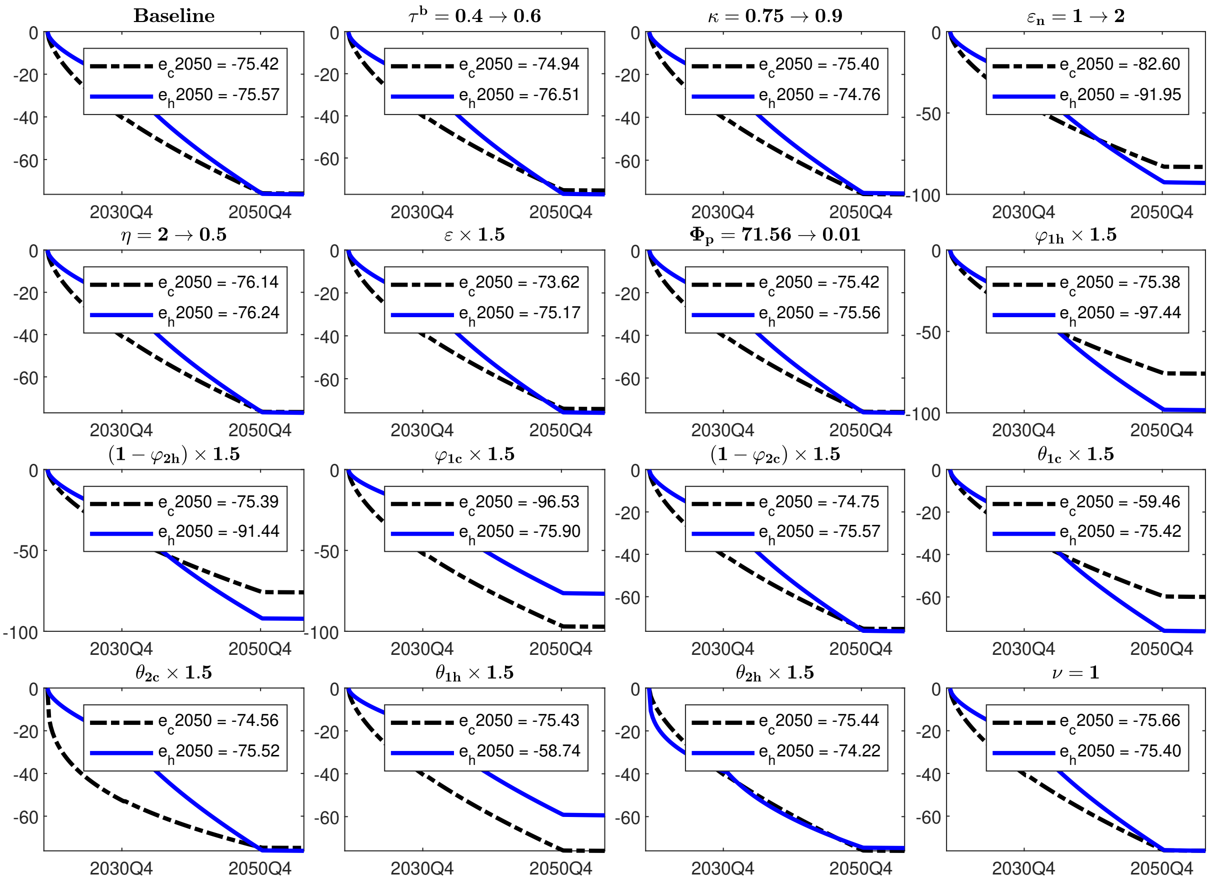
Carbon pricing is implemented under two scenarios: (a) simultaneously in both the non-residential and residential sectors, and (b) exclusively in the non-residential sector.

economy induce notable differences.

Parameters related to carbon emissions in the production sector significantly influence the trajectory of inequality over time. Economies with industries characterized by higher carbon intensity (φ_{1c}) experience a marked increase in inequality during the later stages of the transition, missing out on the benefits of early-period inequality reductions. The increased production costs from carbon pricing in more carbon-intensive economies result in higher abatement effort (as Figure 8 shows, emissions fall by more for a given carbon price) reduced economic activity, lower wages, and decreased government transfers to households, disproportionately affecting borrowers—whose consumption is more dependent on current income—and widening the consumption gap between households. Consequently, technological advances that reduce φ_{1c} are crucial not only for bridging the gap with NZE, but also for mitigating the inequality impacts of carbon emissions pricing.

Labor mobility between sectors, represented by the parameter ε_n, also significantly

Figure 8: Emissions reduction dynamics under different settings



Carbon pricing is implemented simultaneously in both the non-residential and residential sectors. Dash lines represent the evolution of emission s in the non-residential sector, and the solid line reflects the evolution of emissions in the residential sector. Values in the legend indicate emissions reduction by 2050.

impacts inequality. We represent the case of an increase in ε_n. In this case, workers move from the non-residential sector, where wages experience a greater decline, to the residential sector, amplifying the drop in activity within the non-residential sector. Reduced activity in this sector lowers emissions by more, subsequently decreasing the carbon dividend, which disproportionately harms the consumption of borrower households. The positive aspect is that the accelerated decline in non-residential activity facilitates achieving the NZE target, as shown in the corresponding subplot in Figure 8. Thus, higher labor mobility between sectors creates a trade-off between efficiency in terms of emissions and consumption inequality.

An increase in the number of credit-constrained households (τ^b) makes the economy more unequal but does not significantly alter the dynamics of the Gini index during the transition. However, increasing the capacity to take credit (a rise in κ) initially worsens inequality more than in the benchmark when environmental measures are first announced. The expected negative impact of the policy on housing prices has a greater effect on borrowers' consumption when the loan-to-value ratio is higher.

Increasing the cost of abatement in the non-housing sector (θ_{1c}) makes it more difficult to reduce emissions for a given carbon price. Hence, the reduction in the tax base is mitigated as we move along the transition period, keeping carbon dividends higher and benefiting borrowers relative to lenders. Consequently, this scenario improves consumption inequality at the cost of worsening emissions reduction. In the same vein, a higher cost of abatement in the housing sector (θ_{1h}) reduces the inequality impact of the carbon pricing policy compared to the benchmark when targeted at this sector. This effect is evident in the widening gap between the solid and dashed lines in the final years of the period.

Flexible prices ($\Phi_p = 0.01$) mitigate the initial increase in consumption inequality, though they do not alter the subsequent dynamics of the consumption Gini coefficient compared to the baseline scenario. In contrast, a lower initial firm markup (higher ε) contributes to a more pronounced reduction in consumption inequality.

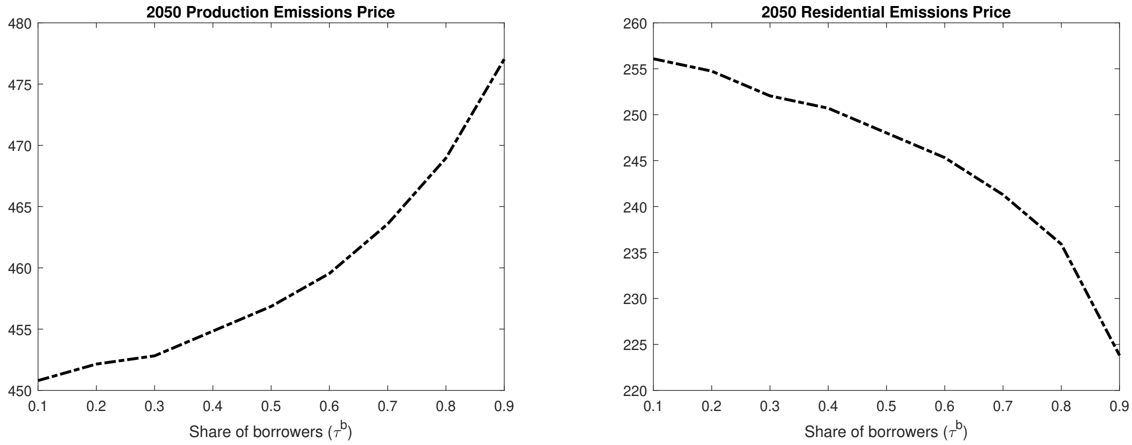
In a scenario where $\nu = 1$, meaning borrowers receive the entire carbon dividend, consumption inequality decreases from the outset, with the Gini coefficient showing the most pronounced change. Furthermore, this positive impact on inequality is achieved without compromising the effectiveness of the taxes in reducing emissions. The importance of ν will be explored in greater depth in Section 4.4.

Overall, these findings underscore the significant role that specific economic characteristics play in shaping inequality under carbon pricing policies. Some parameters that influence the behavior of the model economy can have a sizable effect on inequality, whether through increased lump-sum transfers or wage changes. While, in some cases, changes in the economic environment improve both the efficiency of the carbon pricing policies and their effects on consumption inequality, in other cases, the alternative settings can alter emissions reduction targets, necessitating increases in carbon pricing. The destination of the carbon dividend emerges as the most effective tool for managing inequality, highlighting the critical importance of carefully designing carbon dividend distribution and emissions pricing policies to mitigate short and long-term adverse distributional impacts.

4.3.2 Population Composition and Carbon Pricing

In this subsection, we explore whether changes in population composition can affect the carbon pricing levels needed to achieve climate goals. In the subplot corresponding to τ^b in Figure 8, we do not detect significant differences in the effect on emissions when the share of borrowers increases, provided that the pricing pace for both residential and non-residential carbon emissions is maintained at the baseline. However, this result may obscure differences in the tax required to achieve NZE separately in each of the two sectors.

Figure 9: Carbon emission price projections to meet NZE by 2050



(a) Variations in τ_c, with τ_h set to the baseline. (b) Variations in τ_h, with τ_c set to the baseline.

Figure 9 illustrates how, under a linear policy scheme, the carbon prices needed in 2050 to satisfy the NZE for both the residential and non-residential sectors vary in different directions as the share of borrowers in the population, τ^b, changes. As the share of borrowers increases, the effectiveness of the emissions policy in the residential sector improves, allowing for a reduction in carbon prices. As explained earlier in this paper, borrowers exert more effort than lenders in abating residential emissions due to the greater disutility associated with reducing residential demand as an alternative. Conversely, as the number of borrowers in the population increases, a higher carbon price is needed in the non-residential sector to achieve the NZE target. Non-residential carbon emissions prices directly impact lenders' income through the effect on firm profits, which provides an incentive to abate emissions. As the share of lenders shrinks, this incentive diminishes, and carbon prices need to increase further to achieve the same effect.

4.4 Equitable Compensation Mechanism

The preceding analysis highlights the negative consequences that carbon pricing policies can have on variables crucial for household welfare, emphasizing the need for well-designed compensation schemes, such as carbon dividends, to alleviate these adverse impacts. While offsetting undesired welfare effects is a vital objective, it is not the only consideration for effective climate policy. Achieving a balance among environmental goals, economic efficiency, and social equity is crucial. In this section, we explore the use of carbon dividends—lump-sum transfer rebates distributed to households—as a mechanism to compensate those affected by carbon pricing, ensuring that the financial burden of the transition is distributed more equitably.

Recall that the per capita lump-sum taxes paid by borrowers and lenders are denoted as tr_t^b and tr_t^l, respectively. The aggregate (weighted average) of these taxes is represented by tr_t. After the implementation of the carbon dividend policy, the lump-sum taxes for each household type equal the uniform lump-sum tax minus the household-specific carbon dividend rebate. The amount of carbon revenue that the government can allocate for tax reductions, referred to as the carbon dividend, is given by s_t and depends on the environmental policy:

$$s_t = \tau_{ht} e_{ht} + \tau_{ct} e_{ct}.$$

The adjusted lump-sum taxes for each household type, following the distribution of the carbon dividend, are expressed as:

$$tr_t^l = \overline{tr} - (1 - \nu) \frac{s_t}{1 - \tau^b}$$

$$tr_t^b = \overline{tr} - \nu \frac{s_t}{\tau^b}$$

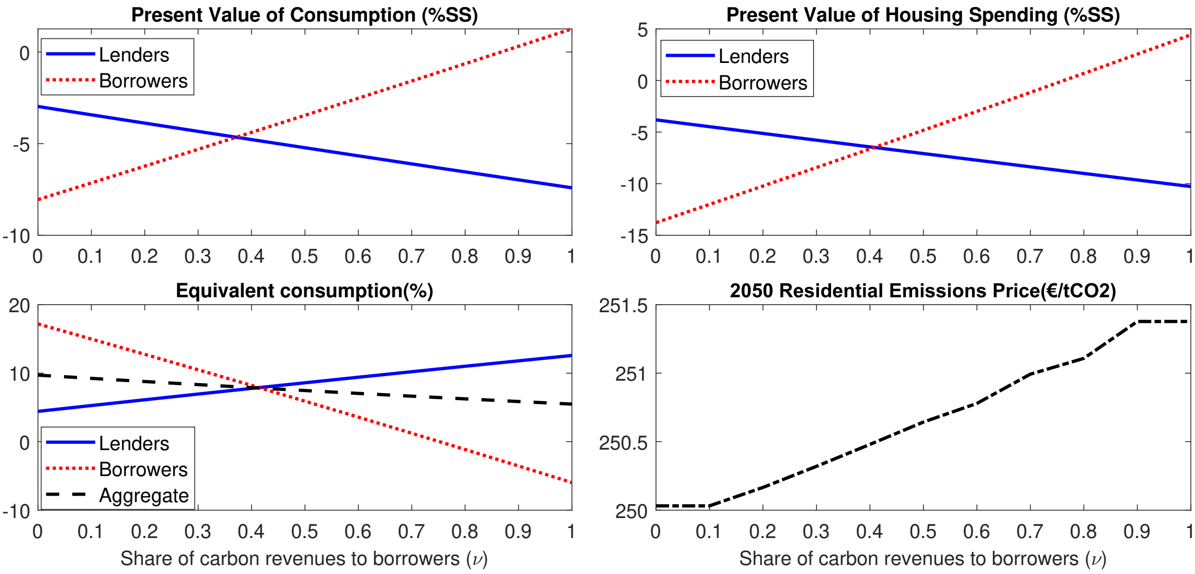
where $\overline{tr}$ is the steady-state value of lump-sum taxes required to uniformly fund government spending $\bar{g}$ in the pre-policy scenario. Hence, ν represents the proportion of the carbon dividend allocated to borrowers, while $1 - \nu$ denotes the share allocated to lenders. When $\nu = \tau^b$, the carbon dividend is distributed equally on a per capita basis between the two groups, which is our benchmark assumption. The government's budget constraint, incorporating the post-policy lump-sum taxes tr_t^l and tr_t^b, is given by:

$$\bar{g} = \tau^b tr_t^b + (1 - \tau^b) tr_t^l + \tau_{ht} e_{ht} + \tau_{ct} e_{ct}.$$

Figure 10 illustrates how different schemes in the distribution of carbon revenues among households impact both equity and efficiency during the transition to a NZE economy. Specifically, we analyze the present value of consumption and housing demand, along with welfare effects and the projected residential carbon price for 2050 required to achieve NZE. By adjusting the parameter ν, the government shifts the allocation of tax relief between lenders and borrowers, thereby altering their disposable income.

The first row in Figure 10 shows the present value of consumption and housing spending as a percentage deviation from the path they would have followed without any environmental policy, where consumption and housing demand are constant and equal to their steady-state values. The present value analysis demonstrates that the carbon dividend has a greater impact on borrowers than on lenders and affects housing demand more significantly than consumption spending. This asymmetry poses a challenge for policymakers in designing compensation mechanisms, as no Pareto optimal policy exists. Redirecting revenues to lenders leads to substantial losses for credit-constrained households, with housing spending falling by as much as -15% and consumption decreasing by up to -8% compared to a scenario without carbon pricing. Conversely, lenders experience

Figure 10: Impact on consumption, housing, welfare, and carbon emission prices when carbon revenues are redirected to reduce household taxes



Carbon pricing is implemented simultaneously in both the non-residential and residential sectors. Present values in percentage deviation from the path they would have followed without any environmental policy.

maximum losses of -10% in housing and -8% in consumption when all carbon revenues are directed to borrowers. Additionally, allocating more of the carbon dividend to borrowers raises the residential carbon emissions price projection for 2050, slightly reducing the policy's efficiency, though this effect is relatively minor.

The left subplot at the bottom captures the welfare effect, including variations in consumption, housing, and working hours, expressed in terms of equivalent variations. Interestingly, the most equitable welfare impact of the environmental policy on households occurs when the carbon dividend is distributed fairly evenly. This is indicated by the intersection of the dotted and solid lines at a value for ν close to τ^b. However, the steeper slope of the line representing the welfare effects for borrowers, compared to that for lenders, suggests that starting from a homogeneous distribution of the carbon dividend, further biasing the dividend towards borrowers more than compensates for the welfare loss experienced by lenders. Under certain reasonable welfare aggregations, this adjustment could lead to an improvement in overall social welfare.

5 Conclusion

This paper provides a comprehensive analysis of the macroeconomic and distributional effects of carbon pricing policies, with a focus on the differential impacts when these policies target residential versus non-residential emissions. Using an Environmental Dynamic General Equilibrium (E-DGE) model, we account for critical transmission mechanisms, including nominal price rigidities and mark-ups, sectoral labor adjustments, and financial frictions tied to housing collateral. A key innovation of our approach lies in the integration of borrower-lender heterogeneity within a dynamic general equilibrium framework that features two distinct production sectors. This modeling approach distinguishes our work from much of the existing literature. Unlike previous studies that often rely on static or partial equilibrium models to evaluate distributional impacts, our framework captures the dynamic interplay between climate policies and the broader economy over time.

Our findings reveal sector-specific welfare differences. Although the overall long-term welfare cost of combined carbon pricing policies is significant - exceeding 10% in terms of consumption and distributed relatively evenly between borrowers and lenders - the impacts vary notably when emissions from each sector are taxed separately. Carbon pricing in the non-residential sector imposes a greater welfare burden on borrowers due to their increased vulnerability to declines in labor demand and economic output. In contrast, residential carbon pricing disproportionately affects lenders, resulting in nearly twice the welfare loss compared to borrowers, despite lenders' lower abatement efforts. This outcome is driven by a more concentrated reduction in housing investment and property values, which are predominantly owned by lenders. Moreover, the efficiency of emissions reduction is higher in the residential sector, making it more cost-effective in welfare terms per unit of emissions reduced compared to the non-residential sector.

The paper also highlights the role of debt distribution in shaping the efficiency of carbon pricing policies. The effectiveness of carbon pricing in the residential sector increases when there is a higher proportion of credit-constrained borrowers, as these households are more inclined to invest in abatement to avoid severe welfare losses. However, the opposite occurs in the non-residential sector: a larger share of borrowers necessitates higher carbon prices to achieve emissions reductions because the income-driven incentives for lenders to abate diminish. These findings emphasize the need for sector-specific and well-calibrated carbon pricing strategies.

Our analysis of inequality dynamics, including robustness tests, shows that carbon pricing policies can have varying impacts depending on key economic parameters. The consumption Gini coefficient generally follows a U-shaped pattern during the transition, initially decreasing due to carbon dividends but then rising as these revenues dwindle. Sensitivity analysis reveals that factors such as labor mobility and the share of credit-constrained households significantly affect the inequality trajectory. For instance, higher labor mobility between sectors can exacerbate inequality, while increasing the proportion of borrowers makes the economy more unequal but does not alter the general pattern of inequality over time. These insights underline the importance of adaptive policy measures that can address distributional challenges, especially after intermediate climate targets are reached.

This study sets the stage for future research in several compelling areas. One promising direction is to delve deeper into the role of the labor market in shaping the effectiveness of environmental policies. This includes exploring within a dynamic general equilibrium framework labor mobility between sectors and wage dynamics, building on previous work like Fullerton & Monti (2013) and Aubert & Chiroleu-Assouline (2019). Gaining a better understanding of how shifts in employment across sectors influence the outcomes of carbon pricing policies could provide valuable insights into labor market dynamics that may either amplify or buffer the economic impacts of these policies. Another crucial avenue for research involves examining financial frictions, specifically the behavior of endogenous loan-to-value ratios under the influence of macroprudential tools. Investigating how these financial mechanisms interact with carbon pricing policies could shed light on the trade-offs between environmental goals and financial stability.

Additionally, expanding the model to incorporate a more diverse range of heterogeneous households, similar to the work of Andres et al. (2022), would enable a more granular analysis of distributional effects. Such a framework would better capture the varied economic realities across different household groups, helping policymakers design strategies that more effectively balance economic efficiency, equity, and environmental sustainability. and paving the way for more robust and inclusive strategies in the global transition to a low-carbon economy.

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Appendix A The complete Model

The model portrays the European Union as a unified economic region with some degree of limited heterogeneity in both the population and the production side. The population consists of two distinct household types differentiated by their subjective discount rates, reflecting varying levels of impatience. Households with lower discount rates are more patient and tend to save, while those with higher rates are more impatient and typically become borrowers. Both classes consume goods and services, engage in the real estate market, and contribute labor to the production of consumer goods and construction.

The economy encompasses two primary production sectors. The first is responsible for producing consumer and investment goods (also named "consumer production goods" or "non-residential sector" through the paper), and the second focuses on housing construction. Firms in the consumer goods sector emit greenhouse gases during production, with emissions proportionally increasing with output levels. Residential activities, such as heating and cooking, also generate carbon emissions, varying by household due to differences in real estate ownership and the associated services houses provide.

Accumulated emissions adversely affect the economy's productivity but do not inherently drive households or firms to reduce their carbon footprint due to the associated costs. In the absence of regulation, minimal effort is made to mitigate emissions.

To counteract this, the government implements carbon pricing policies to internalize the social costs of emissions, thereby encouraging investment in emissions reduction. These policies apply to both production and residential emissions, prompting firms and households to adjust their behaviors. The government redistributes any additional revenue from these policies back to households and can offset the financial burden on households by adjusting the lump-sum taxation scheme to manage distributional impacts. The financing for government expenditures is sourced from general taxation and the revenues generated from carbon taxes.

Below, we outline the various components of the model. For a complete list of the model's equations, please refer to Appendix A.

A.1 Households

As in Iacoviello(2005), there are two representative households types; N_t^l of them are patient (lenders) and N_t^b are impatient (borrowers). Total population is given by $N_t = N_t^l + N_t^b$. We call $1 - \tau^b = \frac{N_t^l}{N_t}$ and $\tau^b = \frac{N_t^b}{N_t}$ the exogenous proportion of lenders and borrowers in the population.

A.1.1 Patient households

Patient households, or lenders, choose consumption, housing services[15] and labor to maximize the following schedule (real variables are expressed in per capita terms of the specific type of household).

$$\max_{c_t^l, h_t^l, n_{ct}^l, n_{ht}^l, b_t^l, k_t^l, j_t^l, u_{ht}^l} \quad \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta^l)^i \left(\ln c_{t+i}^l + \gamma_h \ln h_{t+i}^l - \frac{(n_{t+i}^l)^{1+\eta}}{1+\eta} \right) \tag{A.1}$$

subject to the real flow of funds expressed in terms of consumption goods price, P_t, (the *numeraire*),

$$\begin{aligned} c_t^l + q_t(h_t^l - (1-\delta^h)h_{t-1}^l) - b_t^l + j_{ct}^l + j_{ht}^l + \tau_{ht} e_{ht}^l + z_{ht}^l = \\ w_{ct} n_{ct}^l + w_{ht} n_{ht}^l - \frac{r_{t-1}}{\pi_t} b_{t-1}^l + d_t - tr_t^l + r_{kt} k_{ct-1}^l + r_{ht} k_{ht-1}^l \end{aligned} \tag{A.2}$$

On the utility side, $\beta^l \in (0,1)$ represents the inverse of the patient discount factor, c_t^l is real consumption per capita, h_t^l is lenders' housing units, and n_t^l is labor time per person. The parameter γ_h captures housing preference and the inverse of η relates to the elasticity of labor supply to wages.

We assume imperfect substitutability, as in Iacoviello & Neri (2010), of labor across sectors, so that n_t^l is a CES composite of lenders' labor supply in the consumer goods industry n_{ct}^l and in the housing sector n_{ht}^l,

$$n_t^l = [\theta^{\frac{1}{\varepsilon_n}} (n_{ct}^l)^{\frac{1+\varepsilon_n}{\varepsilon_n}} + (1-\theta)^{\frac{1}{\varepsilon_n}} (n_{ht}^l)^{\frac{1+\varepsilon_n}{\varepsilon_n}}]^{\frac{\varepsilon_n}{1+\varepsilon_n}} \tag{A.3}$$

where $\theta > 0$ is a weight parameter that relates with the disutility of working in the consumption sector relative to the housing sector, and affects the proportion of patient households working hours in each sector. $\varepsilon_n \geq 0$ is the elasticity of substitution between labor supply in the two sectors, capturing the easiness of movement between sectors [16].

As for the budget constraint, patient households lend in real terms $-b_t^l$ to impatient households (where b_t^l is negative). They receive back $-\frac{r_{t-1}}{\pi_t} b_{t-1}^l$ from the previous period's loans, where $\frac{r_{t-1}}{\pi_t^c}$ is the gross real interest rate factor. They invest in new productive capital in both sectors j_{ht}^l and j_{ct}^l and receive rents from capital $(r_{kt} k_{ct-1}^l + r_{ht} k_{ht-1}^l)$ and labor $(w_{ct} n_{ct}^l + w_{ht} n_{ht}^l)$. Since they are the owners they receive profits, d_t, from firms operating in non-competitive markets, and pay lump-sum taxes, tr_t^l, in terms of consumption goods.

Households incur investment adjustment costs $\frac{\Phi^s}{2}\big(\frac{j_{st}^l}{j_{st-1}^l} - 1\big)^2$ where $s = c,\ h$ denote the consumer goods production sector and housing sector. The law of motion of private

[15] We make the standard assumption that the flow of residential services is proportional to the stock of housing.

[16] For a full discussion of this CES function see Cardi & Restout (2015).

physical capital in both sectors is given by,

$$k_{ct}^{l} = (1-\delta^{kc})k_{ct-1}^{l} + \left[1 - \frac{\Phi^{c}}{2}\left(\frac{j_{ct}^{l}}{j_{ct-1}^{l}} - 1\right)^{2}\right] j_{ct}^{l} \tag{A.4}$$

$$k_{ht}^{l} = (1-\delta^{kh})k_{ht-1}^{l} + \left[1 - \frac{\Phi^{h}}{2}\left(\frac{j_{ht}^{l}}{j_{ht-1}^{l}} - 1\right)^{2}\right] j_{ht}^{l} \tag{A.5}$$

These households incur a cost of $q_t(h_t^l - (1-\delta^h)h_{t-1}^l)$ for the acquisition of new housing services in period t and for the depreciation of their housing stock. Living in house produces carbon emissions which may be subject to taxation by the government at a rate denoted as τ_{ht}. Houses release carbon emissions into the atmosphere which are proportional to the housing stock, but abatement effort, denoted as u_{ht}^l, reduces emissions,

$$e_{ht}^{l} = (1-u_{ht}^{l})\varphi_{1h}(h_t^l)^{1-\varphi_{2h}} \tag{A.6}$$

The parameter φ_{1h} represents the carbon emissions intensity per unit of dwelling services, indicating the fossil fuel usage by housing services. The elasticity, denoted as $1-\varphi_{2h}$, relates to the energy efficiency of buildings. A higher exponent indicates lower efficiency, as the rate of growth of carbon emissions increases more significantly with the growth in residential services.

Each period, households have the option to invest in their houses to reduce a fraction of their emissions. However, this action entails a cost proportional to the amount and quality of their residential holdings,

$$z_{ht}^{l} = \theta_{1h}(u_{ht}^{l})^{\theta_{2h}} h_t^l \tag{A.7}$$

where θ_{1h} and θ_{2h} are technological parameters of the abatement cost function.

The solution to this optimization problem yields the following first-order conditions:

$$\lambda_t^l = \frac{1}{c_t^l} \tag{A.8}$$

$$\begin{aligned} \lambda_t^l q_t = \frac{\gamma_h}{h_t^l} - \lambda_t^l(\tau_{ht}(1-u_{ht}^l)\varphi_{1h}(1-\varphi_{2h})(h_t^l)^{-\varphi_{2h}} + \theta_{1h}(u_{ht}^l)^{\theta_{2h}} + \\ + \beta^l \mathbb{E}_t(\lambda_{t+1}^l q_{t+1}(1-\delta^h)) \end{aligned} \tag{A.9}$$

$$\frac{w_{ct}}{c_t^l} = (n_t^l)^{\eta}(\theta \frac{n_{ct}^l}{n_t^l})^{\frac{1}{\varepsilon_n}} \tag{A.10}$$

$$\frac{w_{ht}}{c_t^l} = (n_t^l)^{\eta}((1-\theta) \frac{n_{ht}^l}{n_t^l})^{\frac{1}{\varepsilon_n}} \tag{A.11}$$

$$\lambda_t^l = \beta^l \mathbb{E}_t \lambda_{t+1}^l \frac{r_t}{\pi_{t+1}} \tag{A.12}$$

$$\frac{\lambda_{ct}}{\lambda_t^l} = \beta^l \mathbb{E}_t \frac{\lambda_{t+1}^l}{\lambda_t^l} \left[r_{t+1}^k + \frac{\lambda_{ct+1}}{\lambda_{t+1}^l} (1 - \delta^{kc}) \right] \tag{A.13}$$

$$\frac{\lambda_{ht}}{\lambda_t^l} = \beta^l \mathbb{E}_t \frac{\lambda_{t+1}^l}{\lambda_t^l} \left[r_{t+1}^h + \frac{\lambda_{ht+1}}{\lambda_{t+1}^l} (1 - \delta^{kh}) \right] \tag{A.14}$$

$$\begin{aligned} 1 = \frac{\lambda_{ct}}{\lambda_t^l} \left[1 - \Phi^c \left(\frac{j_{ct}^l}{j_{ct-1}^l} \right) \left(\frac{j_{ct}^l}{j_{ct-1}^l} - 1 \right) - \frac{\Phi^c}{2} \left(\frac{j_{ct}^l}{j_{ct-1}^l} - 1 \right)^2 \right] + \\ + \beta^l \Phi^c \mathbb{E}_t \frac{\lambda_{ct+1}}{\lambda_{t+1}^l} \frac{\lambda_{t+1}^l}{\lambda_t^l} \left[\left(\frac{j_{ct+1}^l}{j_{ct}^l} - 1 \right) \left(\frac{j_{ct+1}^l}{j_{ct}^l} \right)^2 \right] \end{aligned} \tag{A.15}$$

$$\begin{aligned} 1 = \frac{\lambda_{ht}}{\lambda_t^l} \left[1 - \Phi^h \left(\frac{j_{ht}^l}{j_{ht-1}^l} \right) \left(\frac{j_{ht}^l}{j_{ht-1}^l} - 1 \right) - \frac{\Phi^h}{2} \left(\frac{j_{ht}^l}{j_{ht-1}^l} - 1 \right)^2 \right] + \\ + \beta^l \Phi^h \mathbb{E}_t \frac{\lambda_{ht+1}}{\lambda_{t+1}^l} \frac{\lambda_{t+1}^l}{\lambda_t^l} \left[\left(\frac{j_{ht+1}^l}{j_{ht}^l} - 1 \right) \left(\frac{j_{ht+1}^l}{j_{ht}^l} \right)^2 \right] \end{aligned} \tag{A.16}$$

$$u_{ht}^l = \left(\frac{\tau_{ht} \varphi_{1h}}{\theta_{1h} \theta_{2h} (h_t^l)^{\varphi_{2h}}} \right)^{\frac{1}{\theta_{2h}-1}} \tag{A.17}$$

Equation (A.9) represents the intertemporal condition for housing demand, requiring the lender to balance the marginal utility of current consumption against the marginal benefit of housing, offset by the marginal costs resulting from carbon taxation. These costs arise as long as the government imposes a carbon price on housing emissions, negatively affecting housing demand. From the equation, it is clear that the marginal cost of an additional unit of housing consists of two environmental components: the costs associated with emissions $\lambda_t^l \tau_{ht} (1 - u_{ht}^l) \varphi_{1h} (1 - \varphi_{2h}) (h_t^l)^{-\varphi_{2h}}$, which depend on the carbon tax on residential emissions, and the costs associated with abatement in housing emissions $\lambda_t^l \theta_{1h} (u_{ht}^l)^{\theta_{2h}}$. The marginal benefit of housing, typical in models à la Iacoviello, stems from both the direct utility gain from an additional unit of housing and the expected utility from the possibility of increasing future consumption through the resale value of real estate holdings.

Equations (A.10) and (A.11) represent the labor supply of patient workers to the manufacturing and construction sectors, respectively. Equation (A.17) delineates the optimal abatement path and is influenced by the residential carbon tax and the housing stock. The optimal household abatement equates the cost savings from reduced emissions (left-hand side) to its marginal cost (right-hand side).

A.1.2 Impatient households

Impatient households are characterized by having a relatively high discount rate, so the inverse of the discount factor $\beta^b < \beta^l$. Similarly to their patient counterparts, impatient household maximizes the following utility function,

$$\max_{c_t^b, h_t^b, n_{ct}^b, n_{ht}^b, b_t^b, u_{ht}^b} \quad \mathbb{E}_0 \sum_{i=0}^{\infty} (\beta^b)^i \left(\ln c_{t+i}^b + \gamma_h \ln h_{t+i}^b - \frac{(n_{t+i}^b)^{1+\eta}}{1+\eta} \right) \tag{A.18}$$

where c_t^b is borrowers' real consumption, h_t^b denotes borrower's holding of housing units, and n_t^b is a composite of labor supply to the consumption goods sector n_{ct}^b and to the housing sector n_{ht}^b represented by the following CES function,

$$n_t^b = [\theta^{\frac{1}{\varepsilon_n}} (n_{ct}^b)^{\frac{1+\varepsilon_n}{\varepsilon_n}} + (1-\theta)^{\frac{1}{\varepsilon_n}} (n_{ht}^b)^{\frac{1+\varepsilon_n}{\varepsilon_n}}]^{\frac{\varepsilon_n}{1+\varepsilon_n}} \tag{A.19}$$

Impatient households become borrowers, so in addition to the budget constraint (A.20), they face a financial constraint on the maximum amount of credit they can obtain. In particular, expression (A.21) limits the amount of borrowing, b_t^b, to a fraction κ of the expected resale value of housing held by household.

$$c_t^b + q_t(h_t^b - (1-\delta^h)h_{t-1}^b) - b_t^b + \tau_{ht} e_{ht}^b + z_{ht}^b = \\ w_{ct} n_{ct}^b + w_{ht} n_{ht}^b - \frac{r_{t-1}}{\pi_t} b_{t-1}^b - tr_t^b \tag{A.20}$$

$$b_t^b \leq \kappa \mathbb{E}_t \frac{q_{t+1} \pi_{t+1} h_t^b}{r_t} \tag{A.21}$$

Similar to lenders, homes owned by borrowers emit carbon emissions e_{ht}^b through the combustion of fossil fuels used in cooking and heating and cooling systems. These emissions depends on total housing services h_t^b and are subject to the same price τ_{ht}, making them costly. To mitigate the expenses incurred by residential carbon pricing policy, impatient households can opt for abate emissions, u_{ht}^b. The investment in emissions reduction incurs a cost z_{ht}^b, which is contingent upon their dwelling stock. The emission and abatement cost functions for borrowers are as follows,

$$e_{ht}^b = (1 - u_{ht}^b) \varphi_{1h} (h_t^b)^{1-\varphi_{2h}} \tag{A.22}$$

$$z_{ht}^b = \theta_{1h} (u_{ht}^b)^{\theta_{2h}} h_t^b \tag{A.23}$$

The first order conditions derived from maximizing the utility function provide the equations governing consumption, housing investment, labor supply, borrowing, and emissions abatement for impatient households,

$$\lambda_t^b = \frac{1}{c_t^b} \tag{A.24}$$

$$\lambda_t^b q_t = \frac{\gamma_h}{h_t^b} - \lambda_t^b(\tau_{ht}(1-u_{ht}^b)\varphi_{1h}(1-\varphi_{2h})(h_t^b)^{-\varphi_{2h}} + \theta_{1h}(u_{ht}^b)^{\theta_{2h}} + \\ + \beta^b \mathbb{E}_t(\lambda_{t+1}^b q_{t+1}(1-\delta^h) + \kappa\mu_t^b q_{t+1}\pi_{t+1}) \tag{A.25}$$

$$\frac{w_{ct}}{c_t^b} = (n_t^b)^\eta \left(\theta \frac{n_{ct}^b}{n_t^b}\right)^{\frac{1}{\varepsilon_n}} \tag{A.26}$$

$$\frac{w_{ht}}{c_t^b} = (n_t^b)^\eta \left((1-\theta) \frac{n_{ht}^b}{n_t^b}\right)^{\frac{1}{\varepsilon_n}} \tag{A.27}$$

$$\lambda_t^b = \beta^b \mathbb{E}_t \lambda_{t+1}^b \frac{r_t}{\pi_{t+1}} + \mu_t^b r_t \tag{A.28}$$

$$u_{ht}^b = \left(\frac{\tau_{ht}\varphi_{1h}}{\theta_{1h}\theta_{2h}(h_t^b)^{\varphi_{2h}}}\right)^{\frac{1}{\theta_{2h}-1}} \tag{A.29}$$

Equation A.25 is the inter-temporal condition for borrowers' housing demand, indicating that, similar to lenders, housing demand is negatively affected by emissions and abatement costs when environmental policies come into play. Housing demand for borrowers differs from that of lenders in the term $\beta^b \mathbb{E}_t \kappa \mu_t^b q_{t+1}\pi_{t+1}$, which represents the marginal utility of relaxing the borrowing constraint. This can be influenced by environmental policies through various means, such as the impact on the housing price q_{t+1} or the tightness of the credit constraint μ_t^b, which introduces a differential in the consumption and housing demand dynamics between impatient and patient households.

A.2 Aggregation

Since our economy is composed of two representative households, the value in per capita terms of the aggregate variables may be affected by the weight of each type of household, τ_b. Aggregate per capita values are calculated as follows.

Debt issued by borrowers is fully purchased by patient households

$$\tau^b b_t^b = -(1-\tau^b) b_t^l \tag{A.30}$$

Private physical capital inn both sectors involve exclusively patient households

$$k_{ct} = (1 - \tau^b) k_{ct}^l \tag{A.31}$$

$$k_{ht} = (1 - \tau^b) k_{ht}^l \tag{A.32}$$

Net per capita investment is defined for each sector as

$$j_{ct} = (1 - \tau^b) j_{ct}^l \tag{A.33}$$

$$j_{ht} = (1 - \tau^b) j_{ht}^l \tag{A.34}$$

Aggregate per capita consumption and housing stock depends on the borrowers and lenders mix

$$c_t = \tau^b c_t^b + (1 - \tau^b) c_t^l \tag{A.35}$$

$$h_t = \tau^b h_t^b + (1 - \tau^b) h_t^l \tag{A.36}$$

Likewise, aggregate per capita residential emissions and total households abatement cost are defined as,

$$e_{ht} = \tau^b e_{ht}^b + (1 - \tau^b) e_{ht}^l \tag{A.37}$$

$$z_{ht} = \tau^b z_{ht}^b + (1 - \tau^b) z_{ht}^l \tag{A.38}$$

Total lump-sum transfers paid by households becomes,

$$tr_t = \tau^b tr_t^b + (1 - \tau^b) tr_t^l \tag{A.39}$$

We also consider different measures of working hours. Aggregate hours per worker in the consumption goods sector

$$n_{ct} = \tau^b n_{ct}^b + (1 - \tau^b) n_{ct}^l \tag{A.40}$$

Aggregate hours per worker in the housing sector

$$n_{ht} = \tau^b n_{ht}^b + (1 - \tau^b) n_{ht}^l \tag{A.41}$$

Aggregate hours per worker in the economy

$$n_t = \tau^b (n_{ct}^b + n_{ht}^b) + (1 - \tau^b)(n_{ct}^l + n_{ht}^l) \tag{A.42}$$

A.3 Producers of Consumer Goods

In our model, we distinguish between two levels of production. At the top level, there is a competitive bundler that aggregates the output of intermediate-good firms y_{jt}, priced at P_{jt}, into a single composite product y_t. Although we use the term 'consumer goods' for simplicity, this composite product is sold to households for both consumption and investment purposes, as well as to the government.

Aggregation Technology We assume a constant returns to scale technology à la Dixit & Stiglitz (1977) for the aggregation of the output, described by the following expressions for the aggregate production, the aggregate production price index (PPI), and the total demand for each variety:

$$y_t = \left(\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.43}$$

$$P_t = \left(\int_0^1 P_{jt}^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{A.44}$$

$$y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\varepsilon} y_t \tag{A.45}$$

Production at the Firm Level At the bottom level, a continuum of firms indexed by $j \in [0, 1]$ produces a differentiated good y_{jt} in a monopolistically competitive market. Each firm employs a uniform production technology, specifically a Cobb-Douglas production function, where physical capital k_{jt-1} and labor n_{jct} are the inputs:

$$y_{jt} = (1 - D(x_t)) a_t^c k_{jt-1}^{\alpha_c} n_{jct}^{1-\alpha_c} \tag{A.46}$$

where a_t^c represents a technological factor common across all firms. Following Heutel(2012), we introduce a negative externality where the stock of pollutants x_t adversely affects productivity via the damage function:

$$D(x_t) = d_0 + d_1 x_t + d_2 x_t^2 \tag{A.47}$$

Emissions and Abatement Following Annicchiarico & Di Dio (2015), firms emit greenhouse gases (GHGs) as a byproduct of their production processes. The volume of emissions depends on the production level and the firm's abatement efforts. Although emissions can be reduced, the associated cost is a function of the firm's abatement effort relative to its output. The emissions and abatement cost functions for firm j at time t are modeled as follows:

$$e_{jct} = (1 - u_{jct})\varphi_{1c}(y_{jt})^{1-\varphi_{2c}} \tag{A.48}$$

$$z_{jct} = \theta_{1c}(u_{jct})^{\theta_{2c}} y_{jt} \tag{A.49}$$

Similarly to the case of housing services, in these equations, the parameter φ_{1c} denotes the carbon intensity of production, quantifying emissions per unit of output. The exponent $1 - \varphi_{2c}$ reflects the elasticity of emissions with respect to changes in output, indicating how emissions increase with a rise in production. For the abatement costs, θ_{1c} represents the baseline cost of reducing emissions, while θ_{2c} captures the degree of cost increase as the effort in abatement intensifies. These parameters are uniformly applied across all firms, consistent with the model's assumption of homogeneous abatement technologies within the production sector.

A.3.1 Firm's Optimization Problem

Intermediate-good firms, operating under monopolistic competition, adjust their pricing in response to the demand dynamics dictated by the final goods firm. Following Rotemberg (1982), firms incur a nominal adjustment cost AC_{jt} when they deviate from the target inflation rate $\bar{\pi}$. This adjustment cost is influenced by the nominal production in the sector, represented by $P_t y_t$ (with P_t serving as the *numeraire*),

$$AC_{jt} = \frac{\Phi_p}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \bar{\pi}\right)^2 P_t y_t$$

Firms aim to set optimal prices at period t that maximize their discounted expected profits, taking into account both the demand for their goods (A.45) and their technological capabilities (A.46). The optimization problem in real terms is formulated as:

$$\max_{P_{jt}, n_{jct}, k_{jct-1}, u_{jct}} \mathbb{E}_t \sum_{t=0}^{\infty} (\beta^l)^t \frac{\lambda_t^l}{\lambda_0} \left(\frac{P_{jt}}{P_t} y_{jt} - w_{ct+i} n_{jct+i} - r_{t+i}^k k_{jct-1+i} - \right.$$
$$\left. \tau_{ct}(1 - u_{jct})\varphi_{1c}(y_{jt})^{1-\varphi_{2c}} - \theta_{1c}(u_{jct})^{\theta_{2c}} y_{jt} - \frac{\Phi_p}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \bar{\pi}\right)^2 y_t \right)$$

The resulting first-order conditions dictate the optimal choices for capital, labor, abatement efforts, and pricing for each firm. In a symmetric equilibrium, where firms choose identical prices, inputs, and outputs, the conditions simplify as follows:

$$w_{ct} = mc_t(1 - \alpha_c)\frac{y_t}{n_{ct}} \tag{A.50}$$

$$r_{kt} = mc_t \alpha_c \frac{y_t}{k_{ct-1}} \tag{A.51}$$

$$u_{ct} = \left(\frac{\tau_{ct} \varphi_{1c}}{\theta_{1c} \theta_{2c} (y_t)^{\varphi_{2c}}} \right)^{\frac{1}{\theta_{2c}-1}} \tag{A.52}$$

$$\begin{aligned} \pi_t(\pi_t - \bar{\pi}) = \beta^l \frac{\lambda_{t+1}^l}{\lambda_t^l} \pi_{t+1}(\pi_{t+1} - \bar{\pi}) \frac{y_{t+1}}{y_t} + \\ + \frac{\varepsilon}{\Phi_p} \left[\frac{1-\varepsilon}{\varepsilon} + (1 - \varphi_{2c}) \tau_{ct} \varphi_{1c} (1 - u_{ct}) y_t^{-\varphi_{2c}} + \theta_{1c} (u_{ct})^{\theta_{2c}} + mc_t \right] \end{aligned} \tag{A.53}$$

The variable mc_t denotes the Lagrange multiplier related to the production constraint common to all firms, and it reflects the incremental production costs associated with a unit increase in production (marginal cost).

Given a carbon tax τ_{ct} on emissions from consumer goods production, equation (A.52) equates the value of the marginal product of abatement, $\tau_{ct} \varphi_{1c} (y_t)^{1-\varphi_{2c}}$, to its marginal cost, $\theta_{1c} \theta_{2c} (u_{ct})^{\theta_{2c}-1} y_t$. In the absence of τ_{ct}, firms have no incentive to abate emissions, and $u_{ct} = 0$. This equation implies that the abatement effort is uniform across all firms; hence, emissions and abatement costs are also uniform and can be aggregated as follows:

$$e_{ct} = (1 - u_{ct}) \varphi_{1c} (y_t)^{1-\varphi_{2c}} \tag{A.54}$$

$$z_{ct} = \theta_{1c} (u_{ct})^{\theta_{2c}} y_t \tag{A.55}$$

With all firms using identical technology, aggregate production can be expressed as:

$$y_t = (1 - D(x_t)) a_t^c (k_{ct-1})^{\alpha_c} (n_{ct})^{1-\alpha_c} \tag{A.56}$$

Equation (A.53) represents the nonlinear New Phillips Curve, which links inflation to both current and future real marginal costs, with $\frac{\varepsilon}{\varepsilon-1}$ denoting the desired markup in equilibrium. In our environmental model, the marginal costs of producing an additional unit of output include mc_t, as well as costs related to pollution. In the absence of a carbon tax, the New Phillips Curve simplifies to the standard formulation.

Real profits for intermediate firms in a symmetric equilibrium are obtained using Equations (A.50) and (A.51), expressed as follows:

$$(1 - \tau^b) d_t = y_t \left[1 - mc_t - \tau_{ct} (1 - u_{ct}) \varphi_{1c} (y_t)^{-\varphi_{2c}} - \theta_{1c} (u_{ct})^{\theta_{2c}} - \frac{\Phi_p}{2} (\pi_t - \bar{\pi})^2 \right] \tag{A.57}$$

A.4 Housing Construction Firms

There are many small firms owned by patient households that build identical houses in a perfectly competitive environment. The price of housing, q_t, is perfectly flexible, and firms cannot influence it (see Monacelli et al. (2006) and Iacoviello & Neri (2010)). Construction firms produce houses using labor and capital according to a Cobb-Douglas function,

$$IH_t = (1 - D(x_t))a_t^h(k_{ht-1})^{\alpha_h}(n_{ht})^{1-\alpha_h} \tag{A.58}$$

where a_t^h is the steady-state technology factor common to all firms, and $D(x_t)$ is the damage function that positively depends on the emissions stock, as defined in (A.47). The economic objective of the representative firm is to maximize profits subject to the production function

$$\max_{n_{ht},k_{ht-1}} \sum_{i=0}^{\infty} \left(q_{t+i}IH_{t+i} - w_{ht+i}n_{ht+i} - r_{t+i}^h k_{ht-1+i}\right)$$

which yields the labor and capital demand in the construction sector,

$$w_{ht} = q_t(1 - \alpha_h)\frac{IH_t}{n_{ht}} \tag{A.59}$$

$$r_{ht} = q_t\alpha_h\frac{IH_t}{k_{ht-1}} \tag{A.60}$$

Considering a constant depreciation rate δ^h for the stock of houses, the total supply of houses h_t evolves according to,

$$h_t = IH_t + (1 - \delta^h)h_{t-1} \tag{A.61}$$

A.5 Policy

The European authority sets carbon pricing for both the residential and production sectors, denoted as τ_{ht} and τ_{ct} respectively, in line with EU carbon emissions reduction targets. It collects carbon revenues and decides whether to allocate them to lump-sum tax reductions. We assume that the government maintains a constant and exogenous per capita level of consumption $\bar{g}$. In the absence of any environmental policy, public consumption is entirely financed through lump-sum taxes tr_t, which are initially distributed equally on a per capita basis between lenders and borrowers. When carbon taxes are introduced, government consumption are financed by lump-sum taxes and carbon revenues.

$$\bar{g} = tr_t + \tau_{ht}e_{ht} + \tau_{ct}e_{ct} \tag{A.62}$$

Monetary policy is managed by the European Central Bank following a Taylor's interest rate rule that respond to deviations of euro-zone inflation π_t from its long-run target

$(\bar{\pi} = 1)$. The rule takes the form:

$$r_t = \bar{r}^{1-\rho}(r_{t-1})^{\rho}(\pi_t)^{\psi(1-\rho)} \tag{A.63}$$

where $\bar{r}$ is the steady state level of the gross interest rate, ρ is a parameter that controls the persistence of the interest rate and ψ represents the weight given by the ECB to inflation targeting.

A.6 Pollution Dynamics

The economic losses resulting from climate change are incorporated through the damage function (A.47), which reduces productivity in both sectors and thus dampens total production. The damage function captures the relationship between the level of carbon in the atmosphere and economic damages, measured as a percentage of final-good output. In addition, to include the reverse effects, i.e., the impact of economic activity on emissions and carbon accumulation in the atmosphere, we adopt a simplification based on models such as those of Heutel(2012) and Annicchiarico & Di Dio (2015), which assume that the concentration of carbon in the atmosphere x_t follows the following process:

$$x_t = (1 - \delta^x)x_{t-1} + e_{ct} + e_{ht} + e^{row} \tag{A.64}$$

where, in each period, emissions from production of consumer goods, e_{ct}, and from combustion of fossil fuels in houses, e_{ht}, feed the stock of carbon. Additionally, we assume that emissions from the rest of the world, e^{row}, also impact productivity in the European region and remain constant. The parameter δ^x measures the fraction of carbon that naturally decays in each time period.

A.7 Total Resource Constraint

The aggregation of household budget constraints leads to

$$c_t + tr_t + q_t(h_t - (1 - \delta^h)h_{t-1}) + j_{ct} + j_{ht} + \tau_{ht}e_{ht} + \tau^b z^b_{ht} +$$
$$(1 - \tau^b)z^l_{ht} = w_{ct}n_{ct} + w_{ht}n_{ht} + (1 - \tau^b)d_t + r_{kt}k_{ct-1} + r_{ht}k_{ht-1}$$

Using the equality between production and income in both sectors[17] and considering the fiscal budget condition (A.62) while defining total aggregate production ($\tilde{y}^A_t$) as the sum of consumer goods production and housing production, $y_t + q_t IH_t$, the resource

[17] The equality between production and income in consumption goods sector is given by $y_t = w_{ct}n_{ct} + (1 - \tau^b)d_t + r_{kt}k_{ct-1} + \tau_{ct}e_{ct} + z_{ct} + \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2 y_t$ and in house construction sector by $q_t IH_t = w_{ht}n_{ht} + r_{ht}k_{ht-1}$

constraint of the economy can be expressed as:

$$\tilde{y}_t^A = c_t + \bar{g} + q_t(h_t - (1-\delta^h)h_{t-1}) + j_{ct} + j_{ht} + z_{ct} + z_{ht} + \frac{\Phi_p}{2}(\pi_t - \bar{\pi})^2 y_t \qquad \text{(A.65)}$$

GDP can be expressed as:

$$GDP_t = c_t + \bar{g} + q_t(h_t - (1-\delta^h)h_{t-1}) + j_{ct} + j_{ht} \qquad \text{(A.66)}$$

These equalities highlight the inherent limitations within the economy, indicating that all available resources are allocated to production, which in turn is used for consumption, investment, housing production, and to cover the costs associated with emissions abatement and price adjustments. Therefore, our model explores how environmental policies, particularly those aimed at reducing carbon emissions, affect resource allocation across the economy. Specifically, the resources allocated to mitigate the environmental impacts of carbon emissions are represented by abatement costs in both production (z_{ct}) and housing (z_{ht}). As the government implements carbon pricing policies to curb emissions, a significant portion of the economy's resources is redirected towards investments in emissions reduction. This shift within the resource constraints leads to a decrease in both investment and consumption and, hence, on GDP.

Appendix B Model Variables and Parameters Tables

Table B.1: Endogenous variables

Variable	Description
c_t^l	Lenders' consumption
h_t^l	Lenders' housing holding
n_t^l	Lenders' supply of labor
n_{ct}^l	Lenders' supply of labor in consumption goods sector
n_{ht}^l	Lenders' supply of labor in housing sector
λ_t^l	Lagrange multiplier on lenders' budget constrait
b_t^l	Domestic real debt held by lenders
e_{ht}^l	Lenders'houses emissions
u_{ht}^l	Lenders' emissions abatement
z_{ht}^l	Lenders' abatement costs
k_{ct}^l	Lenders' physical capital in manufacturing
k_{ht}^l	Lenders' physical capital in construction
j_{ct}^l	Lenders' investment in manufacturing
j_{ht}^l	Lenders' investment in construction
λ_{ct}	Lagrange multiplier on capital in manufacturing
λ_{ht}	Lagrange multiplier on capital in construction
tr_t^l	Lenders' lump-sum transfers
c_t^b	Borrowers' consumption
h_t^b	Borrowers' housing holding
n_t^b	Borrowers' supply of labor
n_{ct}^b	Borrower's supply of labor in consumption goods sector
n_{ht}^b	Borrower's supply of labor in housing sector
λ_t^b	Lagrange multiplier on borrowers' budget constrain
b_t^b	Borrower's domestic real debt
μ_t^b	Lagrange multiplier on borrowers' collateral constraint
e_{ht}^b	Borrowers' houses emissions
u_{ht}^b	Borrowers' emissions abatement
z_{ht}^b	Borrowers' abatement costs
tr_t^b	Borrowers' lump-sum transfers
e_{ht}	Aggregate household emissions
z_{ht}	Aggregate household abatement cost
tr_t	Aggregate lump-sum taxes

w_{ct}	Real wage in consumption goods sector
w_{ht}	Real wage in housing sector
n_{ct}	Labor supply to the consumption goods sector
n_{ht}	Labor supply to the housing sector
n_t	Labor supply
c_t	Consumption
h_t	Total supply of houses
k_{ct}	Aggregate physical capital in manufacturing
k_{ht}	Aggregate physical capital in construction
j_{ct}	Aggregate investment in manufacturing
j_{ct}	Aggregate investment in construction
y_t	Consumption goods production
e_{ct}	Emissions in production
u_{ct}	Abatement in manufacturing
z_{ct}	Abatement cost in manufacturing
GDP_t	Gross domestic product
$\tilde{y}_t^A$	Total aggregate production
d_t	Real profits
mc_t	Marginal cost
IH_t	Residential investment
q_t	Real housing price
π_t	Domestic CPI inflation
r_t	Nominal interest rate
r_{kt}	Rental rate of physical capital in production
r_{ht}	Rental rate of physical capital in construction
x_t	Atmospheric carbon stock

Table B.2: Exogenous variables

Variable	Description
a_t^c	Productivity shock in manufacturing
a_t^h	Productivity shock in construction
g_t	Government spending
τ_{ct}	Manufacturing carbon tax
τ_{ht}	Residential carbon tax
e_t^{row}	Rest of the world emissions

Table B.3: General Parameters

Parameter	Description
β^l	Lenders' discount rate
β^b	Borrowers' discount rate
η	Inverse elasticity of labor supply
γ_h	Weight of utility from housing
θ	Weight parameter in labor services aggregator
ε_n	Elasticity of substitution between labor types
κ	Loan-to-value ratio
ε	Elasticity of substitution among final goods
τ^b	Fraction of borrowers
ρ	Coeffcient on lagged nominal interest rate in Taylor rule
ψ	Taylor rule reaction to inflation
δ^h	Depreciation rate of housing stock
δ^{kc}	Capital depreciation rate in manufacturing
δ^{kh}	Housing capital depreciation rate
α^c	Elasticity of production output to capital
α^h	Elasticity of construction output to capital
Φ^c	Capital adjustment-cost parameter in production
Φ^h	Construction capital adjustment-cost parameter
Φ^p	Production pricing adjustment-cost parameter
ν	Share of carbon revenues

Table B.4: Environmental parameters

Parameter	Description
$1-\delta^x$	Pollution decay rate
d_2	Damage function quadratic coefficient
d_1	Damage function linear coefficient
d_0	Damage function constant
φ_{1h}	Emission intensity in housing
$1-\varphi_{2h}$	Emissions elasticity in housing
φ_{1c}	Emission intensity in manufacturing
$1-\varphi_{2c}$	Emissions elasticity in manufacturing
θ_{1h}	Housing emissions abatement coefficient
θ_{2h}	Housing emissions abatement exponent
θ_{1c}	Manufacturing emissions abatement coefficient
θ_{2c}	Manufacturing emissions abatement exponent