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**Vicente Meneu
Hipòlit Torró**

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Asymmetric Covariance in Spot-Future Markets^{*}

Vicente Meneu^{**} and Hipòlit Torró^{**}

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Corresponding Author:

Hipòlit Torró
Dep. Economia Financera
Universitat de València
Av. dels Tarongers s/n,
46071 València (Spain)
Tel.: 34-6-382 83 92; Fax: 34-6-382 83 70
Email: hipolit.torro@uv.es

Abstract

In modelling conditional variance-covariance matrix it is important to choose a model that filters volatility asymmetries. The Asymmetric Dynamic Model proposed by Kroner and Ng (1998) encompasses the existing multivariate GARCH models, and it is very successful in filtering the conditional covariance matrix asymmetries. We corroborate both findings empirically in the case of a stock index and its future contract. The Volatility Impulse Response Function for asymmetric multivariate GARCH structures is obtained, extending Lin (1997) findings for symmetric GARCH models. The empirical results indicate that the spot-future variance system is more sensitive to negative than to positive shocks and spot volatility shocks are much more important producing volatility than future volatility shocks. These results confirm that future markets acts as a stabilising factor on their underlying stock markets. Additionally, we obtain that dynamic hedging strategies with futures contracts are not sensitive to the well known asymmetric volatility behaviour in stock markets.

JEL: C22, C32, G10, G19.

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^{**} Dep. Economia Financera at the Universitat de València.

1. INTRODUCTION

The importance of correctly modelling the covariance in finance comes from its presence in many financial models. Theory suggests that the price of an asset depends on its risk, usually measured by its volatility. Furthermore, risk compensation can be measured through the relationship or covariance with the risk factors driving the economy. Consequently, an understanding of how volatility evolves over time, its transmission across markets and the covariance dynamics between assets is relevant for pricing and portfolio management purposes.

The asymmetric behaviour in the stock market volatility refers to the empirical evidence that stocks yields use to be more volatile in bear than in bull markets. This feature first appeared in the finance literature with Black (1976) and it has motivate many lines of work¹. Modelling the asymmetric behaviour of the covariance matrix in a multivariate setting, and studying its consequences in a stock index spot-future system is the main object of this paper.

The case of stock market indexes and its future contracts is interesting for several reasons. First, in modelling conditional covariance it is important to choose a model that filters volatility asymmetries. The Asymmetric Dynamic Model (ADC) proposed by Kroner and Ng (1998) encompasses the existing multivariate GARCH models, and it is very successful filtering the conditional covariance matrix asymmetries. This framework has not been yet applied to represent the spot-future covariance dynamics, to identify its sources of asymmetry in the covariance matrix, and to compare how these asymmetries are filtered through several conditional covariance representations.

A second important feature of the spot-future volatility dynamics is the perennial discussion about whether if the introduction of derivative contracts increase the volatility of the underlying derivative asset. Comparing derivatives markets with its underlying asset markets reveals that the first ones easily allow to take short and high leveraged positions, to have lower transaction costs, and they are more liquid in most cases. These differential features make very attractive the derivative trading for speculators. Many authors maintain that

¹ As a small sample of the different hypothesis explaining this volatility feature see Schwert (1989), Campbell and Hentschel (1992), Braun *et al.* (1995) and Bekaert and Wu (2000).

speculators in the derivative markets transfer uncertainty to the underlying asset market to maintain non-arbitrage relationships. But a contraire consequence is also argued, derivative markets allow risk allocation between investors willing to transfer it to investors willing to bear it, and they have a positive effect on the underlying asset liquidity. Hence, the net effect of derivative trading on the underlying asset is unclear and has to be empirically tested. To make clear this point, it is useful to obtain the impulse response functions proposed by Sims (1980), but adapting them to multivariate second moment representations. Lin (1997) is the first and most important contribution in this field where the Volatility Impulse Response Function (VIRF) is obtained for multivariate GARCH structures. This tool will clear up the sensitivity and dynamics of the spot or future market volatilities to any unexpected news coming from one of these markets. In addition, as we are dealing with asymmetries, an important objective of this paper is to adapt the VIRF to asymmetric GARCH structures. This natural extension will be very useful in identifying the main sources of volatility in the spot-future system. Furthermore, it will allow to explore if the shock sign (positive or negative) implies a different behaviour in the spot-future volatility system.

Finally, the last question to investigate in the spot-future volatility system is how important asymmetries in volatility are to portfolio strategies. Specifically, we wonder if hedging strategies performance and hedging ratios are sensitive to volatility asymmetries. As most hedging ratios are obtained as a quotient between second order moments, it is not evident whether asymmetries in the original second order moment will remain in the hedging ratio.

Three are the most relevant contributions of the paper. First, we found that the general model proposed by Kroner and Ng (1998) fits better with data than the remaining GARCH models, and it achieves to clean up the system of asymmetries in the conditional covariance matrix. These results can be relevant to many areas. For example, volatility modelling is important for non-linear causality studies. Brooks and Henry (2000) point out that “use of non-linear Granger causality test on data which have been prefiltered using a multivariate GARCH model enables one to determine whether the posited model is sufficient to describe the relationship between the series” (see also Cheung and Ng (1996) for a causality-in-variance test). This line of research appears also in Karoly (1995) who studies the information flow across international stock markets using shocks prefiltered by a multivariate conditional covariance model.

Another contribution of the paper is to extend the VIRF proposed by Lin (1997) to asymmetric multivariate GARCH structures. This new VIRF can be used to further investigate the spillover volatility in any financial system. Specifically, we have distinguished between volatility impulse response functions when the shock is positive and when the shock is negative. Our empirical findings show that the main source of uncertainty comes from the spot market, specially with bad news. There exist volatility spillover across both markets but future volatility incorporates in a more efficient way volatility news, leading the volatility system. In this point, we can recover one important theoretical result obtained by Ross (1989). He proofs that “in an arbitrage free economy, the volatility of prices is directly related to the rate of flow of information to the market” and when prices are more volatile then more information must be released and *vice versa*. In our context, the results can be regarded as the future market been the first market incorporating information. The flexibility offering future markets to take leveraged positions can produce an important asymmetry behaviour in their volatility that is also transferred to the spot market (see Schwert (1989) and Stoll and Whalley (1990)).

Finally, we have found that hedging ratios and hedging strategies performance are insensitive to volatility asymmetries. The hedging performance of the multivariate GARCH model proposed by Kroner and Ng (1998) has not been tested before in any future-spot pair and its hedging performance was previously unknown. We find that the general model proposed by Kroner and Ng (1998) does not significantly beat simpler GARCH structures in hedging performance, although it fits better with data and has the desirable properties. An empirical test allows us to maintain that the above results are obtained because hedging ratios are insensitive to the well know asymmetric volatility behaviour in stock markets.

The paper is structured as follows: the second section introduces the multivariate GARCH framework, the third section is devoted to obtain the volatility impulse response function for multivariate asymmetric GARCH structures. In section four, the future hedging performance of estimated models is compared. Finally, the main conclusions are compiled.

2. MULTIVARIATE GARCH MODELLING

2.1. The Econometric Framework

As this paper is mostly concerned on modelling volatility and not on return forecasting, a two step estimation procedure is followed. First, a model in means is estimated and then the residuals of this model are taken in the second step as an input to model the conditional variance. The model for the means that we take is a vector error correction model (VECM) to clean up for any autocorrelation behaviour. Furthermore, as one important point in GARCH modelling is to consider asymmetries in the variance structure, a threshold term is added to the VECM to ensure that any asymmetry obtained in the GARCH modelling cannot be managed in the mean equations. The model for the means is (with $i = 1$ for the spot and $i = 2$ for the future overall the paper)

$$R_{it} = \delta_{i0} + \gamma_i z_{t-1} + \sum_{j=1,2} \sum_{\tau=1,p} [\delta_{j\tau} R_{jt-\tau} + d_{j\tau} \max(-R_{jt-\tau}, 0)] + \varepsilon_{it} \quad (1)$$

where z_{t-1} is the lagged error correction term of the cointegration relationship between R_{1t} and R_{2t} ; δ_{i0} , γ_i , $\delta_{j\tau}$ and $d_{j\tau}$ are the parameters to estimate, p is the lag of the VECM², we have taken $p = 10$, which eliminates any autocorrelation pattern. The VECM model is estimated by Ordinary Least Squares (see Engle and Granger (1987)). The residual series of this model, ε_{1t} and ε_{2t} , are saved and they will be used as observable data to estimate the multivariate GARCH models. This two steps procedure (see Kroner and Ng (1998) and Engle and Ng (1993)) reduces the number of parameters to estimate in the second step, decreases the estimation error and allows a faster convergence in the estimation procedure.

There is in the GARCH literature a set of models used in the multivariate analysis. Each model has its virtues and drawbacks and there is not a clear way to choose among them. For a multivariate GARCH model it is desirable to obtain a positive definite conditional-variance matrix, to produce stationary conditional variance structures and to have a low number of parameters to estimate. In this section we offer a brief review on multivariate GARCH models, how to choose across them and how volatility asymmetries are filtered through each model.

² A log-likelihood ratio test for the exogeneity of the threshold variables overall the estimated VECM (see Enders (1995)) is carried out. The results show that the exogeneity can not be accepted.

The number of published papers modelling conditional covariance is quite scarce compared to the enormous bibliography on time-varying volatility. One consequence of this lack of studies in covariance modelling is that asymmetries receipts in volatility are directly extended to the multivariate setting. The consequences of this extension are unclear because there is no evidence enough on how asymmetries behave in the covariance. The most common case of volatility asymmetry in stock markets is the negative one where unexpected falls in prices increase more the volatility than an unexpected increase in prices of the same amount. Engle and Ng (1993) analyse different asymmetric volatility models; they show that the asymmetry depends not only on the sign but also on the innovation size. That is, the asymmetry, if it exists, is clearer when unexpected shocks in prices are important. These authors propose a battery of tests to verify the importance and sense of the asymmetries. They obtain evidence for the Glosten *et al.* (1993) model where a dummy variable is included in a GARCH(1,1) taking value 1 when the previous innovation is negative.

The natural extension from a univariate to a multivariate setting in asymmetries modelling can have unexpected effects along all the elements of the covariance matrix because of the cross effects generated in each multivariate GARCH model. Multivariate asymmetric GARCH allows for spillover in volatility between future and spot markets. Furthermore, the cross relationships existing in multivariate modelling allows, for example, for the spot volatility to be sensitive to the future volatility asymmetry although no asymmetries exist in the spot market volatility. These kind of cross relationships can have several consequences in the spot-future covariance dynamics, specially in periods of high volatility. We will try to clarify this point in two ways: first, we will plot the news impact surfaces with the aim to point out the differences across models; second, we will use a robust conditional moment test to analyse asymmetries in the covariance.

The three most widely used models are: (1) the VEC model proposed by Bollerslev *et al.* (1988), (2) the constant correlation, CCORR, model proposed by Bollerslev (1990) and (3) the BEKK model of Engle and Kroner (1995)³. Each model imposes different restrictions on the conditional covariance and can lead to substantially different conclusions in any application that involves forecasting conditional covariance matrix. Recently, Kroner and Ng

³ A fourth model is not included in this review: the factor model proposed by Engle *et al.* (1990). This is suitable model to describe high dimension conditional covariance matrix whose changes are driven by a reduced number

(1998) have derived another multivariate GARCH model, the Asymmetric Dynamic Covariance Matrix model, ADC. This model encompasses the above models in the sense that under certain restrictions any particular model can be obtained. This is a good framework to compare the existing models and the significance of the restrictions imposed by each one. Kroner and Ng (1998) introduce asymmetries following the Glosten *et al.* (1993) approach. This is the most common way to introduce asymmetries in multivariate GARCH modelling (see Gagnon and Lypny (1995), Henry and Sharma (1999) and Bekaert and Wu (2000)).

At this point, we will shortly summarise the four multivariate models used in this paper in their asymmetric extended version: VEC, BEKK, CCORR and ADC. We also include the restrictions to impose to the ADC to obtain each one of the previous models. This is a clear specification test and it will allow choosing the right multivariate time-varying covariance avoiding *ad hoc* selections.

The VEC Model

The VEC provided by Bollerslev (1988) was the first extension of GARCH modelling to a multivariate setting. The diagonal form of the VEC model restricts each variance-covariance term to follow a GARCH type equation without cross-relationships between any element of the conditional covariance matrix, and guarantees definite positive covariance matrix. Another advantage of this formulation is that univariate analysis and its intuition is extended to the multivariate case without effort. The diagonal VEC model is characterised by the following equation

$$h_{ijt} = \omega_{ij} + b_{ij}h_{ijt-1} + a_{ij}\varepsilon_{it-1}\varepsilon_{jt-1} + g_{ij}\eta_{it-1}\eta_{jt-1} \quad \text{for all } i, j = 1, \dots, N \quad (2)$$

where ω_{ij} , b_{ij} , a_{ij} , and g_{ij} for all $i, j = 1, \dots, N$ are parameters, ε_{it} and ε_{jt} for all $i, j = 1, \dots, N$ are the unexpected shocks series obtained from equation (1), $\eta_{it} = \max [0, -\varepsilon_{it}]$ and $\eta_{jt} = \max [0, -\varepsilon_{jt}]$ for all $i, j = 1, \dots, N$, are the Glosten *et al* (1993) dummy series collecting a negative asymmetry from the shocks and h_{ijt} are the conditional second moment series. Myers (1991)

of factors. This factorisation reduces the number of parameters to estimate, but here it is not a sensible model because of the reduced dimension (two variables) of the model.

and Baillie and Myers (1991) have used this specification, without the asymmetric extension ($g_{ij} = 0$ for all i, j), in spot-future covariance modelling for hedging purposes.

The BEKK model

The BEKK model is deeply studied in Engle and Kroner (1995). This model has the following form

$$H_t = \Omega + B' H_{t-1} B + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + G' \eta_{t-1} \eta_{t-1}' G \quad (3)$$

where Ω , A , B and G are $N \times N$ matrices of parameters with Ω symmetric and positive definite, H_t is the $N \times N$ conditional covariance, ε_t and η_t are N dimension vector containing the shocks and the threshold terms series defined in (2). This model has been used in Baillie and Myers (1991) (without asymmetries, G as a zero matrix) and Gagnon and Lypny (1995) in modelling spot-future covariance for agricultural commodities and interest rates, respectively.

Constant correlation model (CCORR)

The CCORR model has been used many times because it is easy to compute and definite positive covariance matrices are guaranteed. When the correlation matrix is time invariant Bollerslev (1990) pointed out that maximum likelihood estimate of the correlation matrix is equal to the sample correlation matrix. This fact simplifies the estimation process and as any correlation matrix is always positive definite, the covariance matrix will be always positive definite as far as conditional variances are positive. However, as constant correlation is not a common fact in finance, it is convenient to test this hypothesis before using the model (see Tse (2000)). The model proposed by Bollerslev (1990) is the following one

$$\begin{aligned} h_{iit} &= \omega_{ii} + b_{ii} h_{iit-1} + a_{ii} \varepsilon_{it-1}^2 + g_{ii} \eta_{it-1}^2 \quad \text{for all } i = 1, \dots, N \\ h_{ijt} &= \rho_{ij} \sqrt{h_{iit}} \sqrt{h_{jjt}} \quad \text{for all } i \neq j, \end{aligned} \quad (4)$$

where ρ_{ij} is the correlation coefficient between ε_{it} and ε_{jt} and all the remaining notation has already been presented. The CCORR model has been used many times modelling spot-future

covariance dynamics. Some examples are Cecchetti *et al.* (1988) in Public Debt, Kroner and Sultan (1993) in currencies and Park y Switzer (1995) in stock indexes.

The Asymmetric Dynamic Covariance Model (ADC)

Kroner and Ng (1998) adopt a structured approach, similar to Hentschel (1995). They introduce a General Dynamic Covariance (GDC) matrix model nesting the existing models. This model can be generalised to include asymmetric effects, the ADC. Under this framework, model selection is much easier by testing restrictions on the ADC. The specification of the ADC model for the conditional covariance matrix is the following

$$H_t = D_t R D_t + \Phi \circ \Theta_t \quad (5)$$

where $\circ$ is the Hadamard product operator (element-by-element matrix multiplication) and

$$\begin{aligned} D_t &= [d_{ijt}] & d_{ijt} &= \sqrt{\theta_{ijt}} & \text{for all } i, & d_{ijt} = 0 & \text{for all } i \neq j \\ \Theta_t &= [\theta_{ijt}] \\ R &= [r_{ij}] & r_{ii} &= 1 & \text{for all } i, & r_{ij} = \rho_{ij} & \text{for all } i \neq j \\ \Phi &= [\phi_{ij}] & \phi_{ii} &= 0 & \text{for all } i \\ \theta_{ijt} &= \omega_{ij} + b_i' H_{t-1} b_j + a_i' \varepsilon_{t-1} \varepsilon_{t-1}' a_j + g_i' \eta_{t-1} \eta_{t-1}' g_j & \text{for all } i, j \end{aligned}$$

and

a_i, b_i and $g_i, i = 1, \dots, N$ are $N \times 1$ vector of parameters,
 ω_{ij}, ρ_{ij} , and $\phi_{ij}, i, j = 1, \dots, N$ are scalars with $\Omega \equiv [\omega_{ij}]$ positive definite.

The ADC model can be thought of as a weighted average between the constant correlation model and the BEKK model. In equation (5) $D_t R D_t$ is like the constant correlation model, but with the variance functions given by that of the BEKK model, where R works like a weight in the $i \neq j$ elements of the matrix. In the second term, $\Phi \circ \Theta_t$ has zero diagonal elements, but has off-diagonal elements given by the BEKK-type, so Φ can be understood as the BEKK

weight in the model. To see this interpretation easily we write down the expressions for the elements of H_t

$$\begin{aligned} h_{iit} &= \theta_{iit} \quad \text{for all } i \\ h_{ijt} &= \rho_{ij} \sqrt{\theta_{iit}} \sqrt{\theta_{jjt}} + \phi_{ij} \theta_{ijt} \quad \text{for all } i \neq j, \end{aligned}$$

where θ_{ijt} for all $i, j = 1, \dots, N$ are given by the above BEKK form, see equation (5).

Specification test

Kroner and Ng (1998) proof that under certain restrictions on the ADC model the other models discussed above can be derived. The ADC for $N = 2$ and the restrictions to impose to obtain the other models can be written as

$$\begin{aligned} \begin{bmatrix} h_{11t} & h_{12t} \\ \cdot & h_{22t} \end{bmatrix} &= \begin{bmatrix} \sqrt{\theta_{11t}} & 0 \\ 0 & \sqrt{\theta_{22t}} \end{bmatrix} \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{\theta_{11t}} & 0 \\ 0 & \sqrt{\theta_{22t}} \end{bmatrix} + \begin{bmatrix} 0 & \phi_{12} \\ \phi_{12} & 0 \end{bmatrix} \circ \begin{bmatrix} \theta_{11t} & \theta_{12t} \\ \cdot & \theta_{22t} \end{bmatrix} \\ &= \begin{bmatrix} \theta_{11t} & \phi_{12} \theta_{12t} + \rho_{12} \sqrt{\theta_{11t}} \sqrt{\theta_{22t}} \\ \cdot & \theta_{22t} \end{bmatrix} \end{aligned} \quad (6)$$

where

$$\begin{aligned} \begin{bmatrix} \theta_{11t} & \theta_{12t} \\ \cdot & \theta_{22t} \end{bmatrix} &= \begin{bmatrix} \omega_{11} & \omega_{12} \\ \cdot & \omega_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} h_{11t-1} & h_{12t-1} \\ \cdot & h_{22t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\ &+ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{11t-1}^2 & \varepsilon_{1t-1} \varepsilon_{2t-1} \\ \cdot & \varepsilon_{22t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} \eta_{11t-1}^2 & \eta_{1t-1} \varepsilon_{2t-1} \\ \cdot & \eta_{22t-1}^2 \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \end{aligned}$$

where $\eta_{it} = \max [0, -\varepsilon_{it}]$ for $i = 1, 2$.

- (1) If $\rho_{12} = b_{12} = b_{21} = a_{12} = a_{21} = g_{12} = g_{21} = 0$, a restricted asymmetric VECH is obtained with restrictions b_{12} , a_{12} and g_{12} in the VECH model defined as b_{12} b_{21} , a_{12} a_{21} and g_{12} g_{21} taken from the ADC model.
- (2) If $\phi_{12} = b_{12} = b_{21} = a_{12} = a_{21} = g_{12} = g_{21} = 0$, the asymmetric CCORR model is derived.
- (3) If $\phi_{12} = 1$ and $\rho_{12} = 0$ the asymmetric BEKK model is obtained.

Asymmetries Analysis

A three dimensional extension of the “news impact curve” from Engle and Ng (1993) is presented in Kroner and Ng (1998). A “news impact surface” is defined as the relationship between each conditional second moment and the last period pair of shocks holding past conditional variances and covariances constant at their unconditional sample mean. In order to test how the Glosten *et al.* (1993) modification to the multivariate GARCH models clean the asymmetries in the conditional covariance matrix the robust conditional moment test of Wooldridge (1990) will be applied. This test allow to identify possible sources of misspecification in the model, and is robust to distributional assumptions (see also Brenner *et al.* (1996) and Kroner and Ng (1998)). We define the generalised residual $v_{ijt} = \varepsilon_{it}\varepsilon_{jt} - h_{ijt}$, as the distance between the covariance news impact surface and the realised data. In the same way, $v_{iit} = \varepsilon_{it}^2 - h_{iit}$ and $v_{jjt} = \varepsilon_{jt}^2 - h_{jjt}$ measure the distances between variance news impact and realised data. Using the same misspecification indicators as Kroner and Ng (1998), the Wooldridge (1990) robust conditional moment test can be computed. Kroner and Ng (1998) suggest the use of three kinds of indicator variables to detect misspecification of the conditional covariance matrix. These indicators try to detect misspecification caused by shock signs ($I(\varepsilon_{1t-1} < 0)$ and $I(\varepsilon_{2t-1} < 0)$), the four quadrants sign combinations ($I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} > 0)$, $I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} > 0)$, $I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} < 0)$, $I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} < 0)$) and the misspecification induced because the cross effect of shock sings and sizes ($\varepsilon_{1t-1}^2 I(\varepsilon_{1t-1} < 0)$, $\varepsilon_{1t-1}^2 I(\varepsilon_{2t-1} < 0)$, $\varepsilon_{2t-1}^2 I(\varepsilon_{1t-1} < 0)$, $\varepsilon_{2t-1}^2 I(\varepsilon_{2t-1} < 0)$).

2.2. Data and Empirical Results

The data used are five o'clock prices⁴ for the Spanish⁵ stock index⁶ IBEX-35 and its future contract. The data period goes from January 3rd of 1994 to June 29th of 2001 and contains

⁴ To avoid non simultaneous closing prices we have taken five o'clock prices in both markets. As a daily future price, the traded price just before five is taken, usually a few seconds before.

⁵ The relationship between the Spanish stock market and its derivatives has been studied in many articles. The main conclusion in our revision is that this market has a similar behaviour to the most developed ones. The linear causality between spot and future stock index is studied in Pardo and Climent (2000) with intraday data. Their empirical results show the existence of a linear causality in a double sense, however future prices seems to lead

1865 observations. The data series for the future contract is built rolling-over on the first to delivery contract. That is, the third Friday of each natural month a contract expires, a day before expiration the following nearest to delivery contract is considered and so on. Returns series are obtained by taking first differences⁷ in the *log* prices.

Table I displays the quasi maximum likelihood estimates of the Multivariate GARCH models written in equations (2) to (5). Estimates have been computed assuming a conditional normal distribution for the innovation vector $(\varepsilon_{1t}, \varepsilon_{2t})'$. The standard errors and their associated critical significance levels are calculated using the quasi-maximum likelihood method of Bollerslev and Wooldridge (1992) which are robust to the non-normality assumption. The low critical significance level obtained for all the parameter estimates reveals that this model fit very well with the data. The encompassed model restrictions appearing below equation (6) were tested using Wald test, and all of them were rejected. This result means that the ADC can not be reduced to any of their nested models. One consequence of this is that cross relationships across all conditional moments and their shocks (symmetric and non-symmetric) cannot be skipped. Moreover, asymmetries but it self are also significant. The values obtained by ϕ_{12} (close but significantly different to one) and ρ_{12} (close but significantly below zero) reveal that the estimated ADC model have similar properties to the BEKK model although the encompassing restrictions are clearly rejected.

Figure 1 displays the news impact surfaces from each multivariate GARCH model but without asymmetry correction⁸. Following Engle and Ng (1993) and Kroner and Ng (1998) each surface is represented in the region $\varepsilon_{it} = [-5,5]$ for $i = 1,2$. From Figure 1 we can

the relationship (about five minutes). On the other hand, the IBEX-35 and its derivative markets are two integrated markets and do not offer significant arbitrage opportunities (see Balbas *et al.* (2000) and Pardo (1996)). Blanco (2000) and Pardo (1998) show that the introduction of the derivative market does not seem to have had a remarkable impact on the cash volatility. Corredor and Santamaría (2001) show that implied volatility on IBEX-35 traded options can improve volatility forecasting performance based only on GARCH methodologies on the underlying asset. Finally, Andrés-Alonso (2001) and Pardo (1998) have studied the expiration day effect of derivative contracts on volume and volatility in the cash market. These authors show that volatility and volume are significantly higher in expiration days on the stock market.

⁶The IBEX-35 is computed taking the 35 shares with most traded volume and market capitalisation in the *Mercado Continuo* (the electronic system, property of the *Sociedad de Bolsas*, connecting in real time the four Spanish stock exchanges). For the studied period, the relative stock exchange capitalisation of the IBEX-35 overall the *Mercado Continuo* capitalisation is about 82%. Data were provided by the *Sociedad de Bolsas* and the Spanish derivatives market *MEFF*

⁷ The common tests of unit roots and cointegration (Dickey and Fuller (1981) and Phillips and Perron (1988)) offered no doubt about this point.

⁸ In Figure 1-A the VEC model, in Figure 1-B the CCORR model, in Figure 1-C the BEKK model and in Figure 1-D the GDC model. The GDC model is the symmetric version of the ADC model.

appreciate that each GARCH specification produces different variance and covariance plots for the same pair, the IBEX-35 and its future contract. The structural differences across models will imply also different success in asymmetry filtering. In CCORR and VEC models it can be appreciated that the spot (future) conditional variance is not affected by unexpected shock in the future (spot). This is because no cross relationships are allowed within these models. The BEKK and GDC show more flexible structures but they seem to take explosive values when cross signed outliers appear, specially in the spot volatility impact surface. In the opposite sense, the CCORR model will be revealed unable to filter asymmetries because it is the most restricted model (Figure 1-B shows that its surfaces are almost plane). Finally, the covariance news impact surfaces are quite similar across models except for the CCORR model. Covariance is high when correlation between shocks is positive and is low for negative correlation and can take values below zero for theoretical cross signed outliers. The covariance news impact surface in the CCORR model cannot take negative values and its profile is quite plane but looking it carefully one can see that it increases in all the extremes no matter which the shock signs are. The surfaces for the Minimum Variance Hedge Ratio (the ratio between covariance and future variance news impact surfaces) will be discussed later.

Table II shows the result of the robust conditional moment calculated for the four GARCH models extended⁹ to cope with asymmetry following Glosten *et al.* (1993). For any significance levels the only model filtering the asymmetries is the ADC; the remaining models provide worse results. The asymmetric VEC offer nearly the same test results but it does not allow for cross moment relationships.

3. VOLATILITY IMPULSE RESPONSE FUNCTION

3.1. Theoretical Framework

The Volatility Impulse Response Function (VIRF) is a convenient methodology to obtain information on the second moment interaction between related markets unexpected shocks. In this section we follow Lin (1997), but extending her results to the asymmetric GARCH

⁹ The symmetric specifications offer very high significant values for this test and are not displayed to conserve space.

structures. From section 2 we have that the residuals in equation (1) have a normal distribution with zero mean and conditional covariance matrix H_t , that is $\varepsilon_t | \psi_{t-1} \sim N(0, H_t)$ where ψ_{t-1} is the set of conditioning information and H_t can be parameterised in many ways as a function of past information. The impulse response function for conditional volatility is defined as the impact of a small perturbation of the i th innovation on the future predicted volatility. That is,

$$R_{s,n}(i) = E(\text{vech}(H_{t+s}) | \varepsilon_t = \varepsilon_0 + \delta_i, \psi_{t-1}) - E(\text{vech}(H_{t+s}) | \varepsilon_t = \varepsilon_0, \psi_{t-1}) \quad (7)$$

where $R_{s,n}(i)$ is an $n \times 1$ vector, $n = N(N+1)/2$ with N the number of random variables, $s=1,2,\dots$ is the lead indicator for the conditioning expectation operator and H_t is an $N \times N$ covariance matrix. The operator ‘vech’ denotes the operator that transforms a symmetric $N \times N$ matrix into a vector by stacking each column of the matrix one underneath the other eliminating all supradiagonal elements. Finally, ε_0 can be interpreted as the initial condition before an unexpected shock occurs and δ_i is the small perturbation to the contemporaneous ε_0 . Let ε_0 be a zero vector as an initial condition without any impulse and δ_i be a zero vector with the i th element equal to 1 or -1 . Now, the $\varepsilon_t \varepsilon_t'$ matrix is equal to $\delta_i \delta_i'$, a zero matrix with the (i,i) element equal to 1, regardless of the shock sign. In this case the VIRF is defined as

$$R_{s,n} = \frac{\partial \text{vech } E[H_{t+s} | \psi_t]}{\partial \text{dg}(\varepsilon_t \varepsilon_t')} \quad (8)$$

where $R_{s,n}$ is now an $n \times N$ matrix and $\text{dg}(\varepsilon_t \varepsilon_t') = (\varepsilon_{1,t}^2, \varepsilon_{2,t}^2, \dots, \varepsilon_{N,t}^2)'$. A consistent estimator of $R_{s,n}$ covariance matrix can be showed to be obtained by a two steps procedure: (i) obtaining the covariance matrix of the quasi maximum likelihood estimator, (ii) computing the derivatives appearing in equation (8) evaluated at the QMLE, see Lin (1997) for more details.

3.2. The Asymmetric VIRF

In symmetric GARCH structures it is not necessary to distinguish between positive and negative shocks to obtain the VIRF, but with asymmetric GARCH structures the VIRF must

change with the shock sign. Lin (1997) results can be extended to this case in an easy way. As the ADC specification can be considered a mixture of CCORR and BEKK specification, we now describe how to compute the VIRF for asymmetric BEKK and CCORR structures. Without lack of generality, our analysis is developed taking the structures appearing in equations (3) to (6) to simplify the exposition.

To carry out the analysis we introduce some matrix operators in order to transform equation (3) in an easier-to-handle expression. We denote ‘vec’ as an operator that transforms an $m \times n$ A matrix into a vector by stacking the columns of the matrix one underneath the other. A duplication matrix¹⁰ D_N , an $N^2 \times N(N+1)/2$ matrix, transforms $\text{vech } A$ into $\text{vec } A$, and its Moore-Penrose inverse¹¹ D_N^+ transforms $\text{vec } A$ into $\text{vech } A$ (i.e. $D_N \text{vech } A = \text{vec } A$ and $D_N^+ \text{vec } A = \text{vech } A$). A Kronecker product of two matrices is denoted as $\otimes$. The equation (3) can now be reformulated as

$$\begin{aligned} \text{vech } H_{t+1} = & \text{vech } \Omega + D_N^+ (B' \otimes B') D_N \text{vech } H_t \\ & + D_N^+ (A' \otimes A') D_N \text{vech } (\varepsilon_t \varepsilon_t') \\ & + D_N^+ (G' \otimes G') D_N \text{vech } (\eta_t \eta_t') \end{aligned} \quad (9)$$

The conditional variance of the asymmetric BEKK in equation (3) can be transformed with (9) to be written in the following form

$$h_{t+1} = c + b h_t + (a + I(\varepsilon_t < 0)g) u_t \quad (10)$$

where $h_t = \text{vech } H_t$ and $u_t = \text{vech } (\varepsilon_t \varepsilon_t')$, c is a $n \times 1$ parameter vector, a , b and g are $n \times n$ parameter matrices and the indicator function $I(\varepsilon_t < 0)$, takes value 1 if the condition inside brackets is true and zero otherwise. As $\varepsilon_{t+1} | \psi_t \sim N(0, H_{t+1})$, we have that $E[u_{t+1} | \psi_t] = h_{t+1}$, so we obtain that $E[(a + I(\varepsilon_{t+1} < 0)g)u_{t+1} | \psi_t]$ is equal to $(a + g) h_{t+1}$ when t shocks are all

¹⁰ In this case, $D_N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

¹¹ In this case, $D_N^+ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

negatives and is equal to $a h_{t+1}$ for positive shocks¹². Thus, it is not obvious how negatives and positive shocks will influence on the future conditional variance. We need to know the probability distribution of shock signs to obtain the conditioning information flow at any time.

As $u_t - h_t$ is a martingale difference sequence¹³, equation (10) have the following ARMA(1,1) representation

$$h_{t+1} = c + (a + b + I(\varepsilon_t < 0)g)h_t + (a + I(\varepsilon_t < 0)g)v_t \quad (11)$$

where $v_t = u_t - h_t$. To obtain the expectation s steps ahead, $E[h_{t+s} | \psi_t]$ we will suppose that $\text{prob}(\varepsilon_t < 0) = 1/2$ and $\text{prob}(\varepsilon_t \geq 0) = 1/2$ for all t , shock sign independence along time and independence between shock signs and squared unexpected shocks¹⁴. Now, we apply all the preceding assumptions to obtain the following recursive specification for $E[h_{t+s} | \psi_t]$

$$E[h_{t+1} | \psi_t] = h_{t+1} = \begin{cases} c + (a + b + g)h_t + (a + g)v_t & \text{if } \varepsilon_t < 0 \\ c + (a + b)h_t + av_t & \text{if } \varepsilon_t \geq 0 \end{cases}$$

$$E[h_{t+2} | \psi_t] = c + (a + b + 1/2g)h_{t+1} \quad (12)$$

$$E[h_{t+s} | \psi_t] = c + (a + b + 1/2g)E[h_{t+s-1} | \psi_t]$$

From equation (12) we have that once we know the initial shock sign the remaining expected values for the conditional volatility take the same structure. Finally, to obtain the VIRF of a BEKK asymmetric structure written in equation (3), the equation (8) has to be applied to equation (12). So, by taking matrix derivatives with respect to $\text{dg}(\varepsilon_i \varepsilon_i)$, the VIRF results as the following alternative (depending on the initial shock) $n \times N$ matrix recursive relation,

¹² Here, an independence assumption between unexpected innovation signs and unexpected squared innovations is needed, $E[I(\varepsilon_{t+1} < 0)u_{t+1} | \psi_t] = E[I(\varepsilon_{t+1} < 0) | \psi_t]E[u_{t+1} | \psi_t]$.

¹³ That is, $E[u_t - h_t | \psi_{t-1}] = 0$, for $t = 2, 3, \dots$, and $\psi_{t-1} = \{u_{t-j}\}_{j=1}^{\infty}$.

¹⁴ This implies that $E[I(\varepsilon_{t+s} < 0)h_{t+s} | \psi_t] = E[I(\varepsilon_{t+s} < 0) | \psi_t]E[h_{t+s} | \psi_t]$ and $E[I(\varepsilon_{t+s} < 0)u_{t+s} | \psi_t] = E[I(\varepsilon_{t+s} < 0) | \psi_t]E[u_{t+s} | \psi_t]$ for $s = 1, 2, \dots$

$$R_{s,n}^+ = \begin{cases} a & s = 1 \\ (a + b + \frac{1}{2}g)R_{s-1,n}^+ & s > 1 \end{cases} \quad (13)$$

$$R_{s,n}^- = \begin{cases} a + g & s = 1 \\ (a + b + \frac{1}{2}g)R_{s-1,n}^- & s > 1 \end{cases} \quad (14)$$

where $R_{s,n}^+$ ($R_{s,n}^-$) represents the VIRF to positive (negative) initial shocks.

As we have said many times, the ADC model is a kind of mix between a BEKK structure and a CCORR structure. To obtain the ADC' VIRFs we have to deduce CCORR's VIRF and to put all the results together. Looking again the equations (5) and (6) we have only cared about the cross moment term because if we assume that $\phi_{12} = 0$ and $\rho_{12} = 1$ in equation (6), the diagonal terms will not change and its VIRFs components neither do. Here we can recover the Lin (1997) results for the CCORR model

$$r_{12,k} = \partial h_{12,t} / \partial \varepsilon_{kt}^2 = \frac{1}{2} \rho_{12} h_{11,t}^{-\frac{1}{2}} h_{22,t}^{\frac{1}{2}} R_s(k,1) + \frac{1}{2} \rho_{12} h_{11,t}^{\frac{1}{2}} h_{22,t}^{-\frac{1}{2}} R_s(k,2) \quad (15)$$

where $R_s(k,i)$ for $i = 1, 2$ is the response of volatility of the i th variable to the k th shock for $k = 1, 2$ and $r_{12,k}$ is the covariance element in the impulse response function, $R_{s,n}$. For computing purposes, it is quite wise in our case to reduce the equation (15) to $r_{12,k} = \frac{1}{2} \rho_{12} (R_s(k,1) + R_s(k,2))$ assuming the initial condition $h_{11,t} = h_{22,t}$ for all t . So, the ADC' VIRF is obtained by adding ϕ times the BEKK's VIRF (see equations (13) and (14)) to the covariance impulse response obtained in the equation (15) in the crossed second moment term.

3.3. Empirical Findings

Figure 2 exhibits the VIRF for the ADC model estimated in Table 1, as this model cannot be reduced to any encompassed model. We have some very appealing results: (1) It can be appreciated that IBEX-35 variance is much more sensitive to any shock than the IBEX-35 future variance and their covariance, (2) all shocks take a very long time to die out, about 100 days, except for positive shocks in future returns, which have a very small impact overall

subsequent conditional covariance matrix —Figure 2-b— (3) conditional variances are far more sensitive to negative —Figures 2-c and 2-d— than positive shocks —Figures 2-a and 2-b—, and (4) the whole variance covariance matrix is much sensitive to spot than to future shocks.

As a conclusion of this graphical analysis we can say that the main source of uncertainty comes from the spot market, specially from its negative shocks. The spot market volatility has an overshooting reaction over a protracted period of time, converging to the future variance impact level in about twenty days. There exists a volatility spillover across both markets over this overshooting period, but the dropping effect from future volatility to the spot volatility is relatively smaller. It seems that the future volatility response level acts as an asymptotic stabilising and absorbing long-run equilibrium level. This suggest that future volatility incorporates in a more efficient way volatility news than its underlying market and hence have a leading role in incorporating any source of uncertainty. This leading function may be caused because some hypothesis on perfect markets fail. In an efficient market and with non-stochastic interest rates, the information must be incorporated simultaneously in the spot and its derivative market. Furthermore, both assets must have the same volatility and be perfectly correlated. Transaction costs, short selling restrictions and other imperfections can produce that the main source of uncertainty comes from negative shocks in the spot market. It is quite eloquent in this point to see how negative shocks in the future markets —Figure 2-d— have a double impact on the spot than on the future market volatility over the overshooting period.

4. DYNAMIC HEDGING WITH FUTURES

4.1 The optimal hedge ratio

From a portfolio theory point of view, hedging with futures can be considered as a portfolio allocation problem in which the investor use future contracts as one more asset to include into the financial asset set of the economy in order to maximise his utility function. In this context, the different approaches existing in the doctrine try to answer to a specific allocation problem under some specific probability distribution or utility function. We define the result from a hedged portfolio as

$$x_t = \Delta S_t - b_t \Delta F_t \quad (16)$$

where x_t is the portfolio value variation between $t - 1$ and t , x_t is the sum of the spot value variation, ΔS_t , less the future value variation, ΔF_t , multiplied by the hedging ratio, b_t . If b_t is positive (negative), short (long) positions are taken in futures. The most used definition for the optimal hedge ratio¹⁵ is the Minimum Variance Hedge Ratio (MV Hedge Ratio) which conditional expression can be written as

$$b_t = \frac{\text{cov}(\Delta S_t, \Delta F_t | \psi_{t-1})}{\text{var}(\Delta F_t | \psi_{t-1})} \quad (17)$$

where second moments are taken conditioned to the information set available one period before, ψ_{t-1} . The definition appearing in equation (17) can be obtained using normality assumption for returns (see Myers (1991) and Lence (1995)) or, alternatively, assuming mean-variance utility functions (see Gagnon and Lypny (1995)). When unconditional probability distribution is taken, the hedge ratio in equation (17) can be estimated from a linear relationship between spot and future returns. That is, estimating by ordinary least squares (OLS) the linear relationship $\Delta S_t = a + b \Delta F_t$. The OLS estimator of b will be in this case the unconditional definition of the optimal hedge ratio used in equation (17) (see Ederington (1979)). A more recent extension of this “static” method is obtained when future and spot prices are cointegrated. Then, an error correction term is added to the linear relationship before estimating b . It is interesting to note that there exist important reasons, both theoretical and practical, to use an error correction model to estimate b . As Brenner and Kroner (1995) and Lien (1996) show, OLS procedure underestimate b when cointegration exists. Although this important fact, our empirical applications show no significant differences in hedging results across both ways of estimating b , so this last fact will be skipped in the exposition to maintain space.

4.2. Measuring hedging effectiveness.

¹⁵ For an excellent revision on future hedging see Lien and Tse (2000).

In order to compare the hedging effectiveness of each strategy we compute the risk reduction and its economical significance when transaction costs are considered. Furthermore, we will distinguish between *ex post* and *ex ante* results by splitting the data sample in two parts. In the first one, the hedging strategies are compared *ex post* whereas in the second one an *ex ante* approach is used. That is, in the *ex ante* study, strategies are compared using forecasted hedge ratios and models are estimated every time a new observation is considered.

Risk reduction

The variance of a hedge strategy is calculated as the variance of the hedged portfolio. That is,

$$VAR[x_t | \psi_{t-1}] = VAR[\Delta S_t - b_t \Delta F_t | \psi_{t-1}] \quad (18)$$

The risk reduction achieved for each strategy is computed by comparing with the variance of the spot position ($b_t = 0$ for all t).

Economical significance of the risk reduction

In order to measure if the risk reduction is economically significant it is necessary to consider the transaction costs implied in each strategy. When transaction costs are included it is necessary to consider the investors preferences. So, the different strategies are comparable when the risk reduction achieved is penalised with the transaction costs implied. One way to do this is by using utility functions. Therefore, following the standard methodology (see Kroner and Sultan (1993), Park and Switzer (1995) and Gagnon and Lypny (1995)) the study is restricted to a specific case: the mean–variance utility function¹⁶. The expected utility for a hedging strategy can be written as

¹⁶ The use of other utility functions (logarithmic, exponential, power,...) can be considered as special cases of the mean-variance utility function, taking this as a second order approximation to any utility function. The theoretical framework of Pratt-Arrow for the premium risk can be used in this point. Let the investor to have an initial wealth of W_t and a Von Neumann-Morgenstern utility function, U , defined over the end of period wealth. This utility function is increasing, strictly concave and double differentiable ($U' > 0$, $U'' < 0$). This investor would pay the risk premium π for not to bear the risk of an investment whose return x_t is a random variable with zero mean and variance σ_t^2 . It is easy to show that an approximation to the risk premium is $\pi \cong (0.5)\sigma_t^2(-W_t(U''(W_t)/U'(W_t))) = (0.5)\sigma_t^2 A$, where ‘ A ’ is the infamous Pratt-Arrow relative risk aversion measure. This more general framework can be used to read the equations (19) and (20) if λ is defined to be equal to $0.5A$. An additional required assumption is $E[x_t | \psi_{t-1}] = E[\Delta S_t - b_t \Delta F_t | \psi_{t-1}] = 0$. Therefore, for any specific

$$E[U(x_t) | \psi_{t-1}] = E[x_t | \psi_{t-1}] - \lambda \text{VAR}[x_t | \psi_{t-1}] \quad (19)$$

where λ is the aversion degree parameter ($\lambda > 0$) and ψ_{t-1} the information set available at the beginning of the hedge. The most common estimated values of λ are 3 or 4 (see Kroner and Sultan (1993) and Gagnon and Lypny (1995) for American markets and Alonso *et al.* (1990) for Spanish stock market). We compute our results for λ values of 1, 4 and 10 to see how sensitive are results to the aversion degree parameter.

In the hedging literature (see Myers (1991), Kroner and Sultan (1993) and Gagnon and Lypny (1995)), it is very common to suppose that futures prices follows a martingale behaviour, $E[\Delta F_t | \psi_{t-1}] = 0$. Under this assumption, and including transaction costs (TC), it follows that $E[x_t | \psi_{t-1}] = E[\Delta S_t | \psi_{t-1}] - TC$. As $E[\Delta S_t | \psi_{t-1}]$ is independent of the hedging strategy, the relevant utility function to compare hedging strategies becomes

$$E[U(x_t) | \psi_{t-1}] = -TC(b_t) - \lambda \text{VAR}[x_t | \psi_{t-1}] \quad (20)$$

where the transaction costs are expressed in percentage and as a function of the hedging strategy adopted. This way of measuring the effectiveness is especially useful when dynamic and static strategies are compared. Dynamic strategies imply more transaction costs to keep on future optimal position. This can be, in some cases, very expensive and might be worth enough to eliminate the advantage of a greater risk reduction. The procedure followed to distinguish if it is advantageous to modify the future position in a dynamic strategy is the following: the future position is modified when the new hedging ratio to apply obtains enough risk reduction to compensate the transaction costs incurred altogether achieving a greater utility than if no modification is done. Otherwise the ratio used is the last one that was optimal¹⁷. Finally the utility attained is calculated using equation (20) where transaction costs have a proportional weight depending on the number of optimal hedge ratio modifications over all the sample period. Although the risk reduction of some strategies might be small, it

utility function and initial wealth one obtain different values for A . In this way, the analysis carried out in this section for a wide range of λ values takes a general nature.

¹⁷ More specifically, an investor using a dynamic hedging will decide to modify the future position if and only if $-TC - \lambda(\sigma_{1t}^2 - 2b_t\sigma_{12t} + b_t^2\sigma_{2t}^2) > -\lambda(\sigma_{1t'}^2 - 2b_{t'}\sigma_{12t'} + b_{t'}^2\sigma_{2t'}^2)$ where $\sigma_{1t}^2, \sigma_{12t}, \sigma_{2t}^2$ are the spot and future conditional second moments, b_t is the optimal hedge ratio in t and $b_{t'}$ is the last optimal hedge ratio applied following this strategy.

does not mean that they were not economically viable. If after considering transaction costs the strategies ranking is no modified one can say that the difference between strategies are economically viable. Transaction costs, without considering spreads, change about 1 and 10 Euros depending on volume, broker and client considerations according to the information available at the web site of the MEFF, the Spanish derivatives market¹⁸.

4.3. Empirical Results

Figure 1 and Table 2 offer some preliminary analysis on the hedging ratio features. Figure 1 collects the news impact surfaces for symmetric dynamic hedging strategies. The last graph of each model displays the news impact surface for the Minimum Variance Hedge Ratio (MV Hedge Ratio). It can be seen that when future and spot shocks are perfectly correlated, the hedge ratio is very close to 1 and it is insensitive to the size shock. Except for the CCORR model, cross-signed shocks cause hedge ratio to fall to small values and to take negative values, very wisely, for huge size cross-signed shocks.

On the other hand, we have calculated the robust conditional moment test to the Minimum Variance Hedge Ratio. Last column in Table II displays test results for asymmetric GARCH structures (similar result were obtained with symmetric GARCH). These results offer that hedge ratio are insensitive to any asymmetry. This appealing result comes from the fact that a ratio between two second moment tends to compensate and make to disappear the asymmetric effect if a proportion is maintained between both conditional second moments. This lack of sensitivity to volatility asymmetries will be also be seen later in the study of risk reduction achieved by each dynamic strategy where asymmetries consideration have not impact at all.

Table III displays the variance reduction of the different hedging methods. Panel (A) reports *in the sample* results for the period January 3rd of 1994 to December 30th of 2000 with 1740 data observations. The Panel (B) reports *out of the sample* results for the period January 3rd to June 29th of 2001 with a total of 126 observations. The spot variance reduction is calculated comparing with the unhedged spot portfolio variance in the first row. The reduction obtained is about 92% *in the sample* and 94% in the *out of the sample* part. In both parts, dynamic methods reduce the risk more than the static hedging method considered (OLS). Nevertheless

¹⁸ The percentage this money represents with respect a spot position changes as the spot position value does. If the IBEX-35 is in the level of 10,000 points the percentages would be 0.001%, 0.005% and 0.01% for 1, 5 and 10 Euros, respectively. These percentages are taken in the dynamic hedging analysis.

the differences are quite small, no more than 3% in any case. For example, in the case of *in the sample* period dynamic hedging achieve a greater risk reduction in about a 2%. These small differences are more appealing in the *out of the sample* part (about 1%) in which the better forecasting capability of the spot-future models were expected to have some effect in the variance reduction. Furthermore, the differences across dynamic hedging strategies are minimal and asymmetries inclusion have not impact at all. This result implies that the better statistical performance of the ADC model does not imply a better hedging result. Now we investigate if the better performance of dynamic hedging strategies can be exploited economically by investors if transaction costs are considered.

Table IV displays the utility of each strategy following the above selective procedure. Results are only displayed for the ADC model but the consequences are the same overall the dynamic hedging strategies. Results on Table IV agree with Table III but only in the *ex post* analysis. That is, the reduction achieved is economically significant *in the sample* period but not in the *out of the sample* period, where transaction costs in dynamic hedging absorb the small differences in risk reduction. In Table IV it can be seen that the number of economically significant modifications decrease as the transactions costs are higher and it increase with the degree aversion parameter. This is quite sensible and it produces than dynamic hedging with high transaction costs to be delayed in updating information affecting the optimal hedge ratio.

5. CONCLUSIONS

The main results of this work are three. First, in GARCH model selection it is important to consider asymmetries as not all the GARCH structures have the same capability filtering them. Our empirical findings for the spot-future covariance modelling in the Spanish stock index IBEX-35 suggest that the ADC model proposed by Kroner and Ng (1998) cleans up all the asymmetries found in a first approach overall the conditional second moments, variances and covariances. The ADC model can be used as a specification test as encompassing restrictions can be tested for the most famous multivariate GARCH structures. The encompassing test shows that the ADC structure is the most significant one and cannot be reduced to any nested model.

A second and most important result, both theoretical and empirical, comes from studying Volatility Impulse Response Functions (VIRF) for asymmetric GARCH structures. We have extended the Lin (1997) theoretical results to this case by including asymmetries as proposed by Glosten *et al.* (1993). The theoretical expressions obtained for the asymmetric BEKK model and constant correlation model with asymmetries can be directly applied to obtain the ADC's VIRF, as the ADC model is a weighted mix between the above mentioned models. The empirical results show that the spot IBEX-35 volatility is far more sensitive than future volatility to any shock (coming from the spot or the future market). The spot volatility overshoots over the first twenty days, converging then to the future volatility impact level. We also obtain that the main source of uncertainty comes from the spot market and is transmitted to the future market. The reverse volatility spillover also exists but is far less important. Looking at the sign of the shocks, it can be appreciated that negative shocks increase much more the volatility in the spot-future system than positive shocks. These results suggest that volatility news are incorporated to the future volatility in a more efficient way, it acts as leading market and has a clear stabilisation effect on the underlying market. Transaction costs, short selling restrictions and other imperfections might be the reason producing that the main source of uncertainty comes from negative shocks in the spot market.

Finally, in the hedging study we carried out we found that hedge ratios and hedging strategies are insensitive to any asymmetric consideration. Furthermore, any multivariate GARCH structure produces similar results in the dynamic hedging and scarcely improves the static strategies performance. When transaction costs are included, investors would prefer, in some cases, a static strategy because the small risk reduction achieved do not compensate the transaction costs needed to maintain a dynamic strategy.

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Table I. Multivariate GARCH model estimates and restrictions tests

Panel (A). Multivariate GARCH model estimates							
ADC		BEKK		VECH		CCORR	
ω_{11}	0.0023 (0.00)	ω_{11}	0.0017 (0.00)	ω_{11}	0.0024 (0.00)	ω_{11}	0.0018 (0.00)
ω_{22}	-0.0027 (0.00)	ω_{22}	-0.0026 (0.00)	ω_{22}	0.0029 (0.00)	ω_{22}	0.0021 (0.00)
ω_{12}	-0.0022 (0.00)	ω_{12}	-0.0018 (0.00)	ω_{12}	0.0026 (0.00)	a_{11}	-0.0377 (0.00)
a_{11}	0.2953 (0.00)	a_{11}	0.2855 (0.00)	a_{11}	0.0793 (0.00)	a_{22}	0.0167 (0.00)
a_{21}	0.1404 (0.00)	a_{21}	0.1901 (0.00)	a_{12}	0.0771 (0.00)	b_{11}	0.9692 (0.00)
a_{12}	-0.0634 (0.00)	a_{12}	-0.0249 (0.00)	a_{22}	0.0737 (0.00)	b_{22}	0.9655(0.00)
a_{22}	0.0301 (0.00)	a_{22}	0.0786 (0.00)	b_{11}	0.8884 (0.00)	g_{11}	0.0470 (0.00)
b_{11}	0.9125 (0.00)	b_{11}	-1.0844 (0.00)	b_{12}	0.8732 (0.00)	g_{22}	-0.0079 (0.00)
b_{21}	0.0521 (0.00)	b_{21}	-0.3031 (0.00)	b_{22}	0.8558 (0.00)	ρ_{12}	0.9656 (0.00)
b_{12}	0.0287 (0.00)	b_{12}	0.1276 (0.00)	g_{11}	0.0146 (0.00)		
b_{22}	0.8975 (0.00)	b_{22}	-0.6611 (0.00)	g_{12}	0.0438 (0.00)		
g_{11}	0.7480 (0.00)	g_{11}	0.6581 (0.00)	g_{22}	0.0831 (0.00)		
g_{21}	0.5065 (0.00)	g_{21}	1.0091 (0.00)				
g_{12}	-0.5151 (0.00)	g_{12}	-0.5887 (0.00)				
g_{22}	-0.1890 (0.00)	g_{22}	-1.0018 (0.00)				
ρ_{12}	-0.0232 (0.00)						
ϕ_{12}	1.0294 (0.00)						
ℓ	17010.86		16984.80		16914.21		16825.70
Panel (B). Testing restrictions for nested models							
BEKK				6.94 10^9 (0.00)			
VECH				9.13 10^{12} (0.00)			
CCORR				9.24 10^{12} (0.00)			

Panel (A) of this table displays the quasi maximum likelihood estimates of the ADC, BEKK, VECH and CCORR models (see equations (2) to (5) in the text) assuming a conditional normal distribution for the innovation vector $(\varepsilon_{1t}, \varepsilon_{2t})'$. Critical significance levels appear in brackets. Last row of Panel (A) contains the maximum log-likelihood function value, ℓ , obtained in the estimation process for each model.

Panel (B) displays Wald test for the restrictions to impose on the ADC model to obtain the encompassed models. See equation (6) in the text. Critical significance levels appear in brackets.

Table II. Robust conditional moment tests

Generalised residual tests				
	$v_{12t} = \varepsilon_{1t}\varepsilon_{2t} - h_{12t}$	$v_{11t} = \varepsilon_{1t}^2 - h_{11t}$	$v_{22t} = \varepsilon_{2t}^2 - h_{22t}$	$v_{MVHR\ t} = \varepsilon_{1t}\varepsilon_{2t}/\varepsilon_{2t}^2 - h_{12t}/h_{22t}$
Panel (A). ADC model.				
$I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} < 0)$	0.1077	0.6233	0.2796	0.0753
$I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} > 0)$	0.1802	0.5666	0.0109	0.0919
$I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} < 0)$	0.0062	0.2899	0.4372	0.0495
$I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} > 0)$	0.7340	0.8041	0.5376	0.6324
$I(\varepsilon_{1t-1} < 0)$	1.2030	0.4683	2.2341	0.7963
$I(\varepsilon_{2t-1} < 0)$	0.4648	1.0157	0.0062	0.1183
$\varepsilon_{1t-1}^2 I(\varepsilon_{1t-1} < 0)$	1.1969	0.3677	1.8576	0.3871
$\varepsilon_{1t-1}^2 I(\varepsilon_{2t-1} < 0)$	0.9146	0.3203	1.7801	0.4069
$\varepsilon_{2t-1}^2 I(\varepsilon_{1t-1} < 0)$	0.1863	0.7850	0.9550	0.4196
$\varepsilon_{2t-1}^2 I(\varepsilon_{2t-1} < 0)$	0.5310	0.3677	1.8576	0.3744
Panel (B). BEKK model.				
$I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} < 0)$	0.08647	7.6254***	2.74486*	0.0855
$I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} > 0)$	0.14218	7.4223***	3.51786*	0.1149
$I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} < 0)$	0.00172	6.6735***	1.97807	0.0635
$I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} > 0)$	0.78395	0.8507	1.09948	0.6259
$I(\varepsilon_{1t-1} < 0)$	1.16937	0.1870	1.99063	0.8027
$I(\varepsilon_{2t-1} < 0)$	0.39095	8.4243***	4.38319**	0.1368
$\varepsilon_{1t-1}^2 I(\varepsilon_{1t-1} < 0)$	0.01458	5.5461**	5.83891**	0.2393
$\varepsilon_{1t-1}^2 I(\varepsilon_{2t-1} < 0)$	0.76951	5.4084**	5.56557**	0.2526
$\varepsilon_{2t-1}^2 I(\varepsilon_{1t-1} < 0)$	0.20897	6.7128***	6.12657**	0.7524
$\varepsilon_{2t-1}^2 I(\varepsilon_{2t-1} < 0)$	0.2396	5.5461**	5.83891**	0.7581
Panel (C). CCORR model.				
$I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} < 0)$	7.7022***	8.5317***	5.4300**	$3.32 \cdot 10^{-07}$
$I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} > 0)$	8.1032***	8.4832***	6.7745***	$1.90 \cdot 10^{-07}$
$I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} < 0)$	7.0149***	7.7157***	4.5974**	$5.60 \cdot 10^{-08}$
$I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} > 0)$	0.3032	0.4072	0.8952	$6.72 \cdot 10^{-06}$
$I(\varepsilon_{1t-1} < 0)$	0.3190	0.3393	2.5446	$3.89 \cdot 10^{-06}$
$I(\varepsilon_{2t-1} < 0)$	8.8111***	9.3390***	7.6661***	$1.44 \cdot 10^{-06}$
$\varepsilon_{1t-1}^2 I(\varepsilon_{1t-1} < 0)$	6.2225**	7.2284***	0.4738	$2.71 \cdot 10^{-05}$
$\varepsilon_{1t-1}^2 I(\varepsilon_{2t-1} < 0)$	6.0991**	7.1381***	0.4633	$2.66 \cdot 10^{-05}$
$\varepsilon_{2t-1}^2 I(\varepsilon_{1t-1} < 0)$	6.1618**	1.9651	3.8434**	$9.26 \cdot 10^{-05}$
$\varepsilon_{2t-1}^2 I(\varepsilon_{2t-1} < 0)$	6.2442**	1.9346	3.9350**	$7.15 \cdot 10^{-05}$
Panel (D). VEC model.				
$I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} < 0)$	0.1691	0.2041	1.0169	0.2107
$I(\varepsilon_{1t-1} < 0; \varepsilon_{2t-1} > 0)$	0.0763	0.2207	0.7244	0.2466
$I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} < 0)$	0.2801	0.1042	1.2130	0.2212
$I(\varepsilon_{1t-1} > 0; \varepsilon_{2t-1} > 0)$	0.3992	0.2517	0.4540	0.0900
$I(\varepsilon_{1t-1} < 0)$	1.1162	0.2801	1.1389	0.4910
$I(\varepsilon_{2t-1} < 0)$	0.0225	0.3611	0.5705	0.1829
$\varepsilon_{1t-1}^2 I(\varepsilon_{1t-1} < 0)$	1.0460	1.3513	0.4662	0.0001
$\varepsilon_{1t-1}^2 I(\varepsilon_{2t-1} < 0)$	2.4299	1.0784	0.4815	0.0001
$\varepsilon_{2t-1}^2 I(\varepsilon_{1t-1} < 0)$	2.7824*	0.9388	2.2457	0.0023
$\varepsilon_{2t-1}^2 I(\varepsilon_{2t-1} < 0)$	2.7616*	0.9210	1.3274	0.0031

Each panel gives the robust conditional moment test statistic for each of the four estimated models for the IBEX-35 index and its future contract. The misspecification indicators are listed in the first column and the remaining columns in each panel give the test statistic computed for the generalised residual calculated for each variance and covariance (h_{12t} , h_{11t} and h_{22t}). ε_{1t-1} is the return shock to the spot position and ε_{2t-1} is the return shock to the future position. Last column displays the test for the Minimum Variance Hedging Ratio (MVHR). The indicator function $I()$ takes the value one if the expression inside the parentheses is satisfied and zero otherwise.

All the statistics are distributed as a $\chi^2(1)$. Test values highlighted with one (*), two (**) and three (***) asterisks are significant at 90%, 95% and 99% of confidence level, respectively.

Table III. Hedging effectiveness (I): Risk reduction

HEDGING STRATEGY	Panel (A): In the sample results	Panel (B): Out of the sample results
Spot variance (no hedged)	1.7831	1.9555
OLS	90.26 %	93.57 %
Symmetric GARCH		
Structures		
VECH	92.88 %	94.52 %
CORR	91.82 %	94.64 %
BEKK	92.98 %	94.66 %
GDC	92.14 %	94.70 %
Asymmetric GARCH		
Structures		
VECH	92.71 %	94.32 %
CORR	91.82 %	94.64 %
BEKK	92.63 %	94.62 %
ADC	91.62 %	94.67 %

This table displays the reduction achieved by each hedging strategy, both static (OLS) and dynamic (VECH, CCORR, BEKK and GDC or ADC). Panel (A) reports *in the sample* results for the period January 3rd of 1994 to December 30th of 2000 with 1740 data observations. The Panel (B) reports *out of the sample* results for the period January 3rd to June 29th of 2001 with a total of 126 observations. The spot variance reduction is calculated comparing with the unhedged spot portfolio variance in the first row of each panel. Variance is computed as $VAR[\Delta S_t b_t - \Delta F_t | \psi_{t-1}]$ where ΔS_t is the spot return, ΔF_t the future return, b_t the hedging ratio and ψ_{t-1} is the set of information available in $t-1$. In Panel (B) results are obtained using an *ex ante* approach where hedging ratios are forecasted values in $t-1$ and each time an observation is added the model is estimated again.

Table IV. Hedging effectiveness (II): Utility comparisons

HEDGING STRATEGY	Panel (A):	Panel (B):
	In the sample results	Out of the sample results
Aversion degree $\lambda = 1$		
No hedging utility	-1.7832	-1.9556
OLS	-0.1736	-0.1236
ADC (TC = 0.001%)	-0.1496 (346)	-0.1501 (22)
ADC (TC = 0.005%)	-0.1599 (113)	-0.1504 (10)
ADC (TC = 0.01%)	-0.15173 (63)	-0.1504 (5)
Aversion degree $\lambda = 4$		
No hedging utility	-7.1328	-7.8223
OLS	-0.6943	-0.5025
ADC (TC = 0.001%)	-0.5996 (660)	-0.6002 (41)
ADC (TC = 0.005%)	-0.5977 (305)	-0.6006 (20)
ADC (TC = 0.01%)	-0.5977 (201)	-0.6007 (11)
Aversion degree $\lambda = 10$		
No hedging utility	-17.8321	-19.5556
OLS	-1.7358	-1.2563
ADC (TC = 0.001%)	-1.4970 (905)	-1.5001 (59)
ADC (TC = 0.005%)	-1.4920 (491)	-1.5009 (31)
ADC (TC = 0.01%)	-1.4955 (346)	-1.5014 (22)

This Table shows the utility achieved by the static hedging strategy (OLS) and a dynamic hedging strategy (ADC) for the IBEX-35 index and its future contract. Results for the other dynamic hedging strategies are not showed to conserve space because the same consequences are derived. Panel (A) reports *in the sample* results for the period January 3rd of 1994 to December 30th of 2000 with 1740 data observations. The Panel (B) reports *out of the sample* results for the period January 3rd to June 29th of 2001 with a total of 126 observations. In Panel (B) results are obtained using an *ex ante* approach where hedging ratios takes the values forecasted values in $t-1$ and each time an observation is added the model is estimated again.

The utility achieved by each hedging strategy is computed as $U(b_t) = -TC(b_t) - \lambda \text{VAR}[\Delta S_t b_t - \Delta F_t | \psi_{t-1}]$ where ΔS_t is the spot return, ΔF_t the future return, b_t the hedging ratio and ψ_{t-1} is the set of information available in $t-1$, λ is the aversion degree parameter and TC represent the transaction costs expressed in percentage and it is a function of the hedging strategy adopted.

Between brackets appear the number of hedge ratio modifications economically significant. The optimal position in future contracts is modified only if the reduction in variance is big enough to drain the transaction costs paid. Otherwise the previous hedging ratio position is keep. More specifically, an investor using a dynamic hedging will decide to modify the future position if and only if $-TC - \lambda(\sigma_{1t}^2 - 2b_t\sigma_{12t} + b_t^2\sigma_{2t}^2) > -\lambda(\sigma_{1t}^2 - 2b_{t-1}\sigma_{12t} + b_{t-1}^2\sigma_{2t}^2)$ where $\sigma_{1t}^2, \sigma_{12t}, \sigma_{2t}^2$ are the spot and future conditional second moments. b_t is the optimal hedge ratio in t and b_{t-1} is the last optimal hedge ratio applied following this strategy.

Figure 1. News impact surfaces for each multivariate GARCH model without asymmetric correction

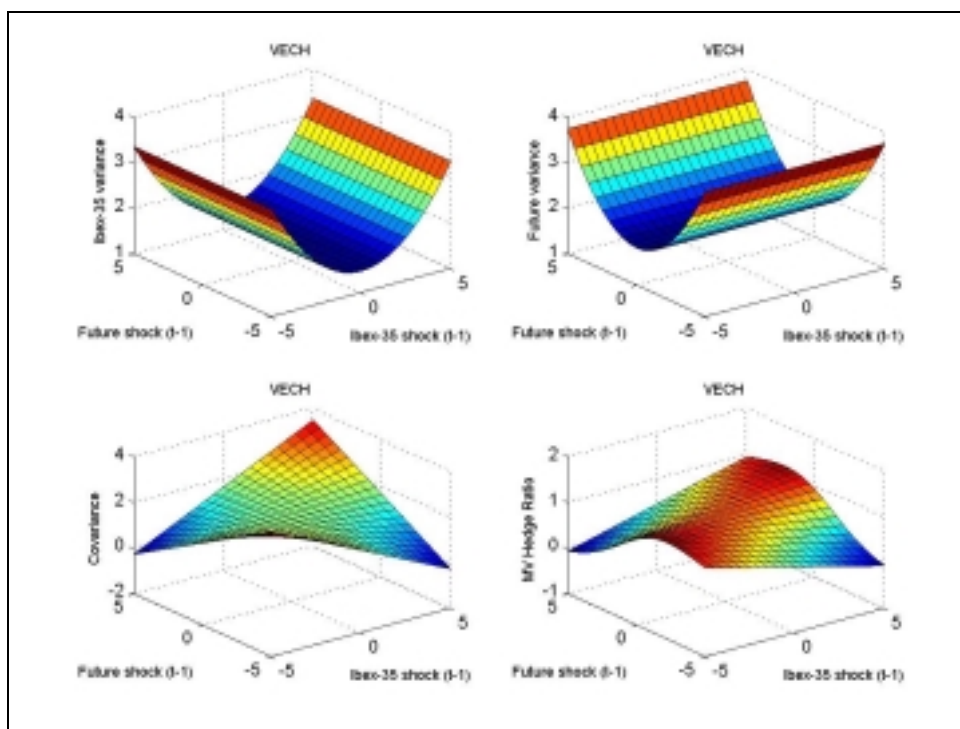


Figure 1-A: VECH model

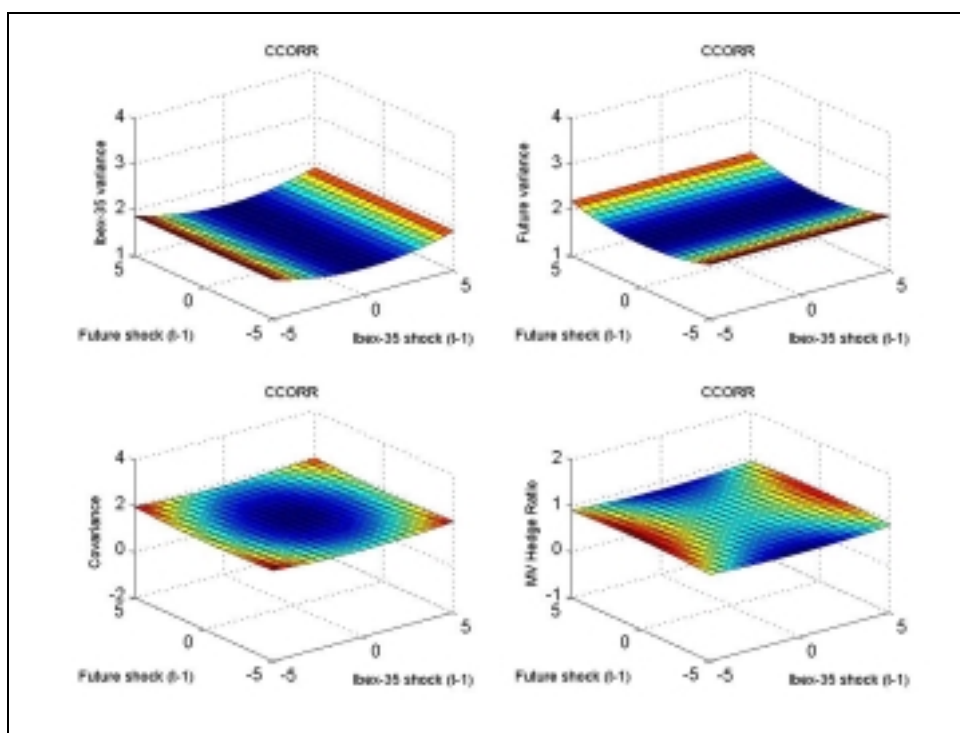


Figure 1-B: CCORR model

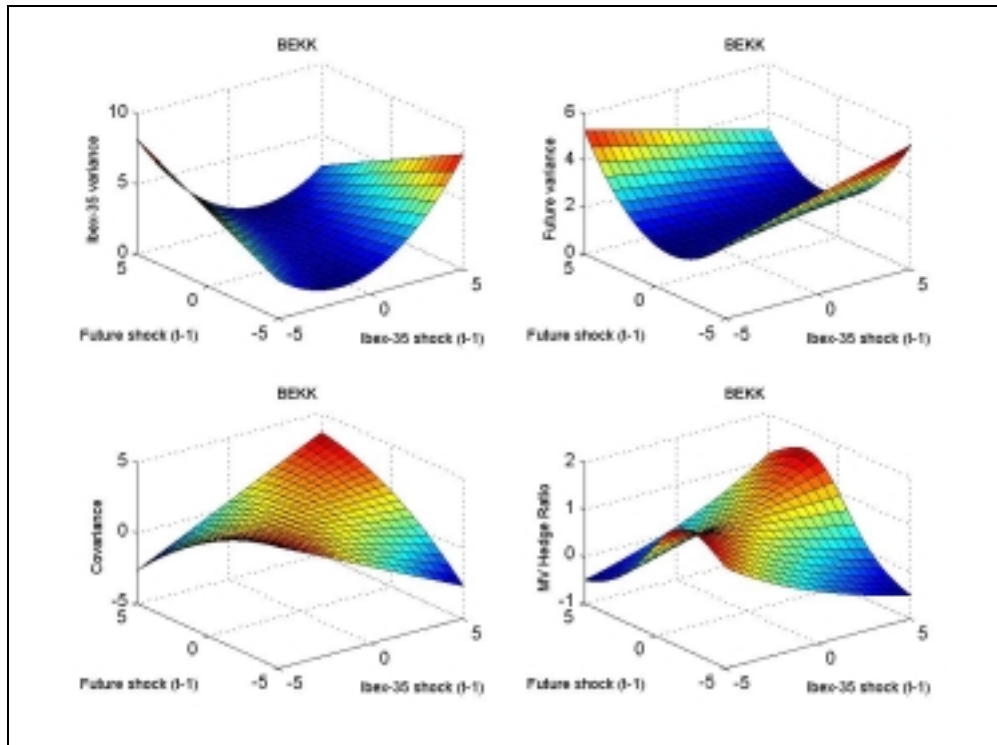


Figure 1-C: BEKK model

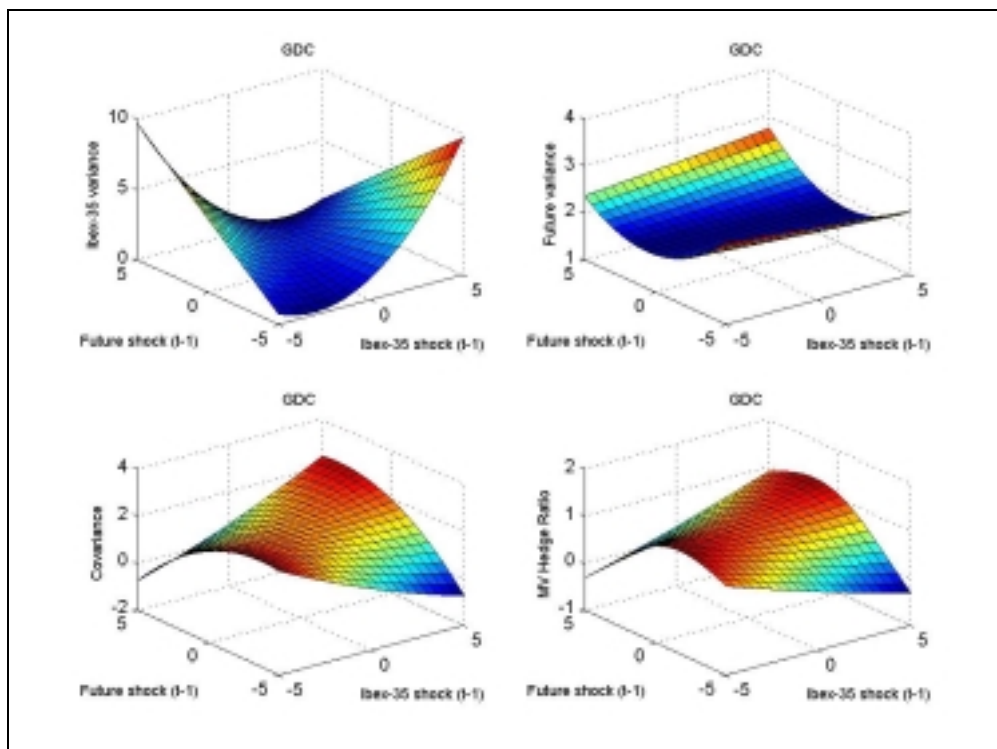


Figure 1-D: GDC model

Figure 2. Volatility impulse response functions

Figure 2-a

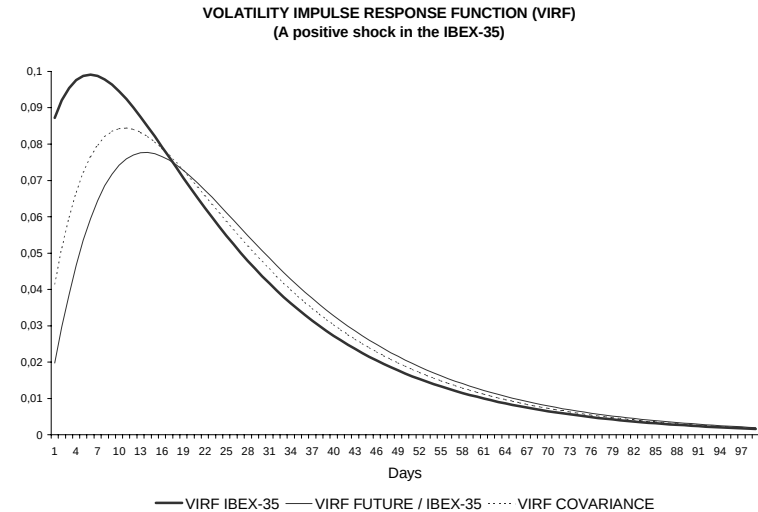


Figure 2-b

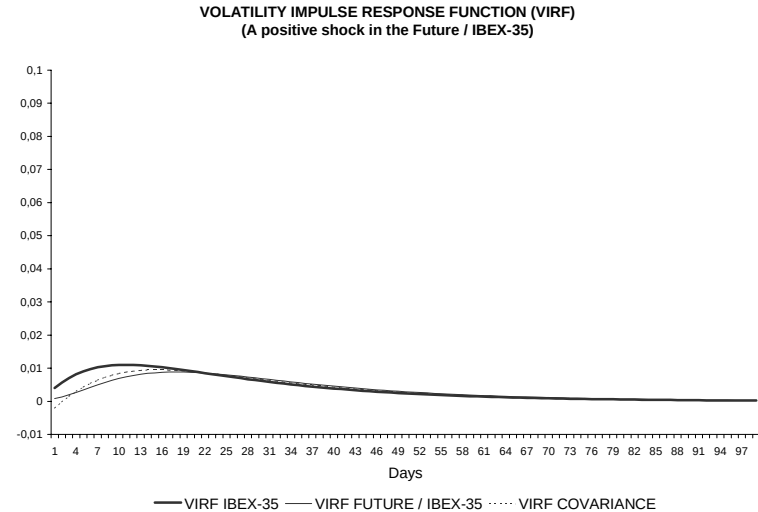


Figure 2-c

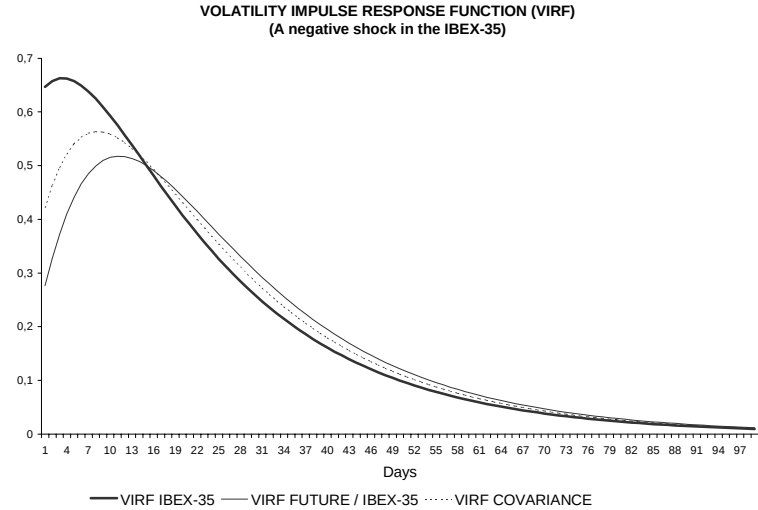


Figure 2-d

