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The supplementary table on pensions (Table 29) and Actuarial Balance Sheets: Assessing the solvency of the Spanish Pension System

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**The supplementary table on pensions (Table 29) and Actuarial Balance Sheets:
Assessing the solvency of the Spanish Pension System.**

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Abstract

The aim of this paper is to advocate for a shift in the management of pension liabilities within social security schemes. The proposed change is from an 'out of sight, out of mind' approach to a more transparent approach that accurately reflects the net worth of the system, i.e. a 'tell it like it is' approach. This involves disclosing all assets and liabilities of the scheme through the use of an actuarial balance sheet (ABS) and its associated income statement (IS). EU regulations since 2017 have required all Member States to disclose their accrued-to-date pension liabilities (ADL) using a standard actuarial cost method and some common assumptions. These pension liabilities have to be disclosed in a supplementary table referred to as Table 29. The ABS can be used to assess the solvency of SS schemes, whereas Table 29 cannot. This paper develops the detailed steps to be followed in order to obtain the ABS, the IS and the information on the solvency of social security schemes from Table 29 and updates the results of Garvey et al. (2023) for the case of the Spanish social security system according to the most recent data from Table 29. The Spanish pension system's lack of solvency and its continuous decline is very worrying; the indicators for the period 2020-2021 show values that, if they arose in a private pension institution, would lead to immediate intervention by the Spanish authorities. In short, we can say that the Spanish social security system is in a "critical and declining state".

JEL: G22, H55, H83.

KEYWORDS: Accountability, Actuarial Balance Sheet, Pension Liabilities, Procrastination, Spain, Table 29, Useful Information.

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1 INTRODUCTION

There has been a long-standing debate among academics regarding the relative merits and shortcomings of various public sector financial health indicators, and public pension schemes are an important part of this debate. The literature has highlighted that 'traditional' fiscal indicators, such as the relationship between income and expenditure each year (the fiscal deficit/surplus) and the total amount of government financial debt¹, may not provide a complete picture of government fiscal operations as they do not take into account the changes in government assets, alongside government liabilities.

The conventional frameworks are inadequate for managing government finances. They fail to ensure long-term sustainability, efficient resource management, and intergenerational fairness. Many governments do not produce full financial statements, exempting themselves from the reporting obligations placed on private or public sector bodies. Even when accounts are produced, they are often published too late to be of practical use or without important information about the value of government assets. Only a few countries, perhaps only New Zealand, place the balance sheet at the heart of government financial decision-making (Ball et al., 2024).

Several researchers, Buiters (1983), Milesi-Ferretti and Moriyama (2006), IMF (2014), Brede and Henn (2019), Cabezon and Henn (2020), Levy and Sturzenegger (2023) or Ball et al., (2024) to name just a few, have recommended using a balance sheet approach (BSA). Public sector balance sheets bring together the entirety of what the state owns and owes, offering a broader fiscal picture beyond debt and deficits. Once governments understand the size and nature of public assets, they can start managing them more effectively, raising considerable additional revenue (IMF, 2018). The BSA can improve the identification and management of risks by providing relevant information for policy decisions and revealing asset-liability mismatches. Fiscal stress tests on the basis of the balance sheet can provide a more comprehensive and accurate view of the resilience of public finances in response to adverse shocks. Finally, focusing on net worth and the balance sheet information is a better way to manage public assets and liabilities, in short, having a strong balance sheet is essential for effective government (Ball et al., 2024).

Pension entitlements are a critical issue for public finances and budgets. The focus of our interest is the Actuarial Balance Sheet (ABS), and specifically the concept of net worth. Net worth is a summary figure that indicates the difference between the assets and liabilities of a public pension system, in other words, its overall financial position. A wide range of pension statistics already exist, for example as part of social protection statistics. However, these are based on flows during the year - contributions and benefits paid - and not on a forward-looking basis considering the pensions that will be paid in the future. Until recently, public pension entitlements were treated on an "out of sight, out of mind" basis. Traditionally, governments have been very reluctant to recognise pension liabilities on the balance sheet for a number of reasons, including the perceived 'complexity of the actuarial calculations' required to measure pension obligations and the 'specialist knowledge' required to make these calculations, the conceptual debate about whether there is a present obligation for the government in respect of these defined benefit pension plans, and (most importantly) the unwillingness to disclose large liabilities that would negatively impact the net financial position of the government (PriceWaterhouseCoopers, 2014).

¹ To provide a sense of proportion, both figures are often expressed as a percentage of GDP. Governments typically offer multi-year forecasts of income, expenditure, and anticipated financing activity to demonstrate sustainability (Ball et al., 2024).

Eurostat, EU Member States and the European Free Trade Association (EFTA) countries are helping to fill this gap by publishing for the third time a snapshot of these pension scheme liabilities² (known as Table 29 of supplementary pensions) in 2021. The first Table 29 data transmission took place in December 2017 with 2015 as the reference year. Transmissions occur every three years for data relating to year t-2. Eurostat has published Table 29 statistics on its website since the end of 2018. The second Table 29 data wave was reported in December 2020, relating to pension liabilities at the end of 2018. The data from the third wave of Table 29, which reports on pension entitlements at the end of 2021, has recently been released (Eurostat, 2024).

Table 29 shows social insurance pension entitlements accrued by the end of a reporting period for the current affiliates and pensioners (not only for the retirement contingency) broken down by: Type of pension scheme: defined contribution schemes and defined benefit schemes; Institutional sector and type of pension manager: private sector and government; Type of recording: pension schemes recorded in the national accounts framework (all funded and private unfunded employment related schemes) and pay-as-you-go (PAYG) general government schemes (social security schemes and unfunded schemes for general government employees); and as will be seen in Section 2.1., by accounting category: stocks (pension entitlements at the beginning and end of a reporting period) and flows leading to changes in entitlements (payments of social contributions/social benefits to/by the pension schemes and other changes in volume, revaluations, transfers between schemes and enacted reforms).

The Table 29 pension reporting exercise is based on the accrued-to-date pension liability (ADL) method, which is consistent with the closed-group approach (CG). Thus a plan's liabilities are equal to the present value of all expected future benefits to pensioners and all accrued rights of current affiliates. Accrued pension rights derive from social contributions already paid by current workers and the remaining pension entitlements payable to existing pensioners. In conceptual terms, this is similar to a termination reserve in a private sector or occupational pension plan. This amount therefore also equals the resources that would be required to shut down a social security pension scheme while honouring all past commitments (Holzmann et al., 2001; Kaier and Müller, 2015; Drouin et al., 2018; Wiener and Stokoe, 2018).

The Table 29 methodology may provide relevant information for pension systems financed using a full advance funding method, but it does not allow for the assessment of the financial sustainability of SS schemes, which are typically financed on a pay-as-you-go/partially funded basis. The actuarial balance sheet (ABS) and its associated income statement (IS) are suitable for a wide variety of schemes (Boado-Penas et al., 2008; Billig and Menard, 2013; Ventura-Marco and Vidal-Meliá, 2014; Vidal-Meliá, 2014; Pérez-Salamero et al., 2017; Metzger, 2018; Vidal-Meliá et al., 2018; Billig and Menard, 2018; Metzger, 2019; Garvey et al., 2021; Garvey et al., 2023; TSPS, 2023). This approach addresses the main weakness of Table 29 which only includes liabilities and as a consequence does not show solvency or sustainability indicators: pensioners and contributors may not have information on the probability of receiving their future benefits.

² Liabilities are expressed in terms of “actuarial present value” (APV). APV is the sum of money needed now which, invested over the duration of the scheme's pension commitments, is expected to be sufficient to pay out all the pensions promised. Appendix 1 describes the analysis of the methodology that can be used to estimate the accrued-to-date pension entitlements included in Table 29.

The ABS is a financial statement that outlines a pension system's obligations to contributors and pensioners at a specific date, along with the amounts of the assets (financial and in particular those from contributions) that underwrite those commitments. The results of the activities of a pension plan during a given period are presented in the IS. It shows the gains, expenses and losses and the net profit or net loss for a given period.

The ABS/IS approach based on Swedish open group (SOG) methodology³ has three positive features. First, the measurement of the system's assets and liabilities is transparent and does not require complicated projections of economic, financial or demographic variables that could distort sustainability and solvency indicators. Second, this approach is suitable for introducing automatic balance mechanisms (ABM) as it avoids endless debates about the accuracy of long-term projections. And third, the inclusion of an IS as part of the published information on a country's public pension system could improve people's understanding of the information, make the system more transparent and enhance the protection of pension promises through monitoring, reporting and disclosure.

The paper contributes significantly to the field of pension management by advocating for a transparent methodology that accurately reflects the net worth of social security systems, moving away from the traditional 'out of sight, out of mind' approaches. It provides a detailed methodology to be followed in order to convert Table 29 data into an Actuarial Balance Sheet (ABS) and Income Statement (IS) and the information on the solvency of social security schemes from Table 29, specifically updating the results for the Spanish social security system. This approach aims to improve public pension information, assisting various stakeholders including policymakers and public finance economists, in understanding and implementing the ABS and IS model.

Additionally, the paper discusses policy implications, comparing the solvency of Spain's pension system with other countries and exploring measures to restore solvency. It offers a comprehensive methodology for estimating pension entitlements and presents the main formulas needed to transform Table 29 data into an ABS and compile the associated IS. Emphasizing transparency and simplicity, the ABS/IS approach aims to improve public understanding and protection of pension promises through better monitoring, reporting, and disclosure. The paper also highlights that net worth, which includes all assets and liabilities, provides a superior indicator of sustainability, solvency, and resilience compared to traditional fiscal indicators.

The structure of the paper is as follows. After this introduction, Section 2 describes the methodology and data collection. Section 2 is divided into two parts. In the first part, the method used to convert Table 29 into an ABS and to produce the related IS is briefly explained. Special attention is paid to the development of nine steps to be followed in order to obtain the ABS, its corresponding IS and the information on the solvency of social security schemes from Table 29. The data and information needed for the compilation of ABS and IS are presented in the second part of Section 2. Section 3 updates the results of Garvey et al. (2023) for the case of the Spanish social security system according to the most recent data from Table 29. Section 4 discusses some policy implications that could be drawn from the results in Section 3, which illustrate the great

³ SOG is the methodology used by the Swedish authorities since 2001 when measuring the assets and liabilities of Social security pension schemes (SSSs) (TSPS, 2023). Very briefly, SOG is based on the Close Group method but modified to make it equivalent to Open Group. It can be considered "open group" in any particular year t because, in order to value the system's assets and liabilities, it takes new entrants into account and assumes that there will be contributions to meet liabilities, although valuation formulas consider only pensioners and contributors at the valuation date (Garvey et al., 2021, 2023).

usefulness of transforming Table 29 into an ABS: In comparison to other countries that draw up actuarial balance sheets, is the pension system in Spain more or less favourable in terms of its solvency? How can solvency be restored? What measures would have to be taken if the public system were a funded defined-benefit pension plan? How would Spain's public debt be affected if the social security gap were taken into account? If the social security system were a private company, what steps would be required to initiate insolvency proceedings? and What measures have been implemented in recent years to address the sustainability issues in the Spanish pension system? The paper ends with the concluding comments, the references used and two technical appendices. The first of these describes the methodology for estimating the accrued-to-date pension entitlements included in Table 29, while the second shows the main formulas used to transform Table 29 into an ABS and compile its associated income statement (IS).

2.- METHODOLOGY AND DATA

2.1.- Methodology

The supplementary table (Table 29) records all positions and flows relating to pension obligations for all social insurance pension schemes, showing all those that correspond to the pension schemes in an economy and (as a counterpart) the pension entitlements held by households. Figure 1 summarizes through a stock and flow model the main content of Table 29. When an inset in Figure 1 has 2 arrows to the central line, each pointing in a different direction, this means that the value may be positive or negative. An outflow is represented by an arrow pointing in the direction of the inset from the central line, while an inflow is the opposite, an arrow pointing from the inset in the direction of the central line.

The methodology used in this paper was developed by Garvey et al. (2023). Their method makes it possible to transform Table 29 into an actuarial balance sheet (ABS) and produce the associated income statement (IS).

Figure 2 summarizes their method through a stock and flow model with the following items. Stock at $t-1$: pension entitlements, other liabilities and the system's assets at the beginning of the period. Stock at t : pension entitlements, other liabilities and the system's assets at the end of the period along with transactions and other flows between $t-1$ and t : All transactions and other economic flows that lead to a change in the opening and the closing stock within the given period are included given that they are necessary items to compile the income statement. In short, the system's assets and liabilities on two consecutive valuation dates along with their changes are shown in Figure 3. The main formulae for the conversion of Table 29 into an ABS and its associated IS can be found in Annex 2.

We can see the reconciliation between the opening stock of pension assets and liabilities at the beginning of a period and the closing stock of assets and pension obligations at the end of a period, taking into account all transactions and other flows during the period. Figure 2 illustrates in the shaded sections information that has been derived, to some extent or fully, from Table 29. This is regardless of whether the recalculation of specific items is required. The unshaded sections correspond to new information that is necessary to complete the ABS.

Figure 1: Table 29 summarized through a stock and flow model

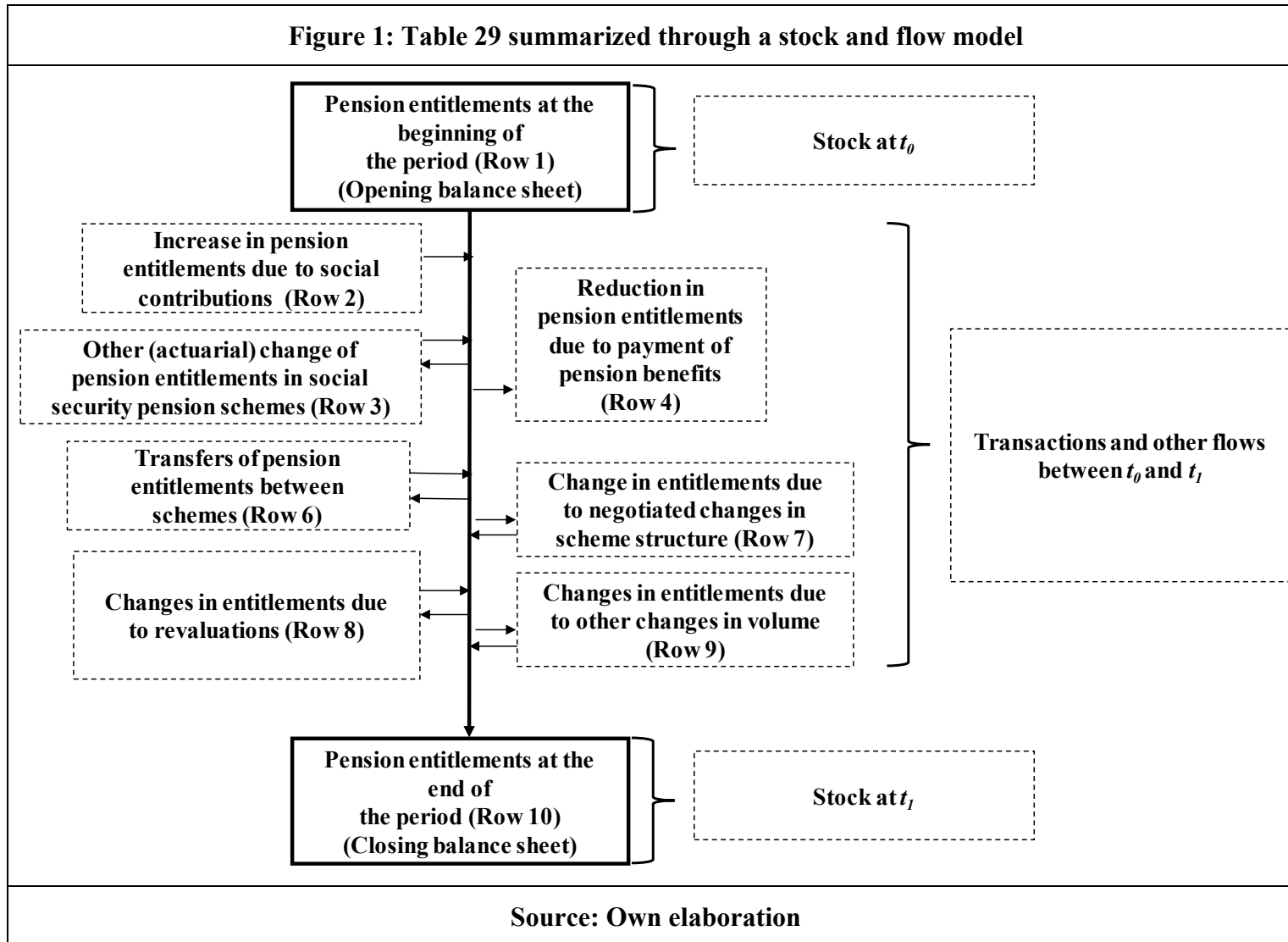
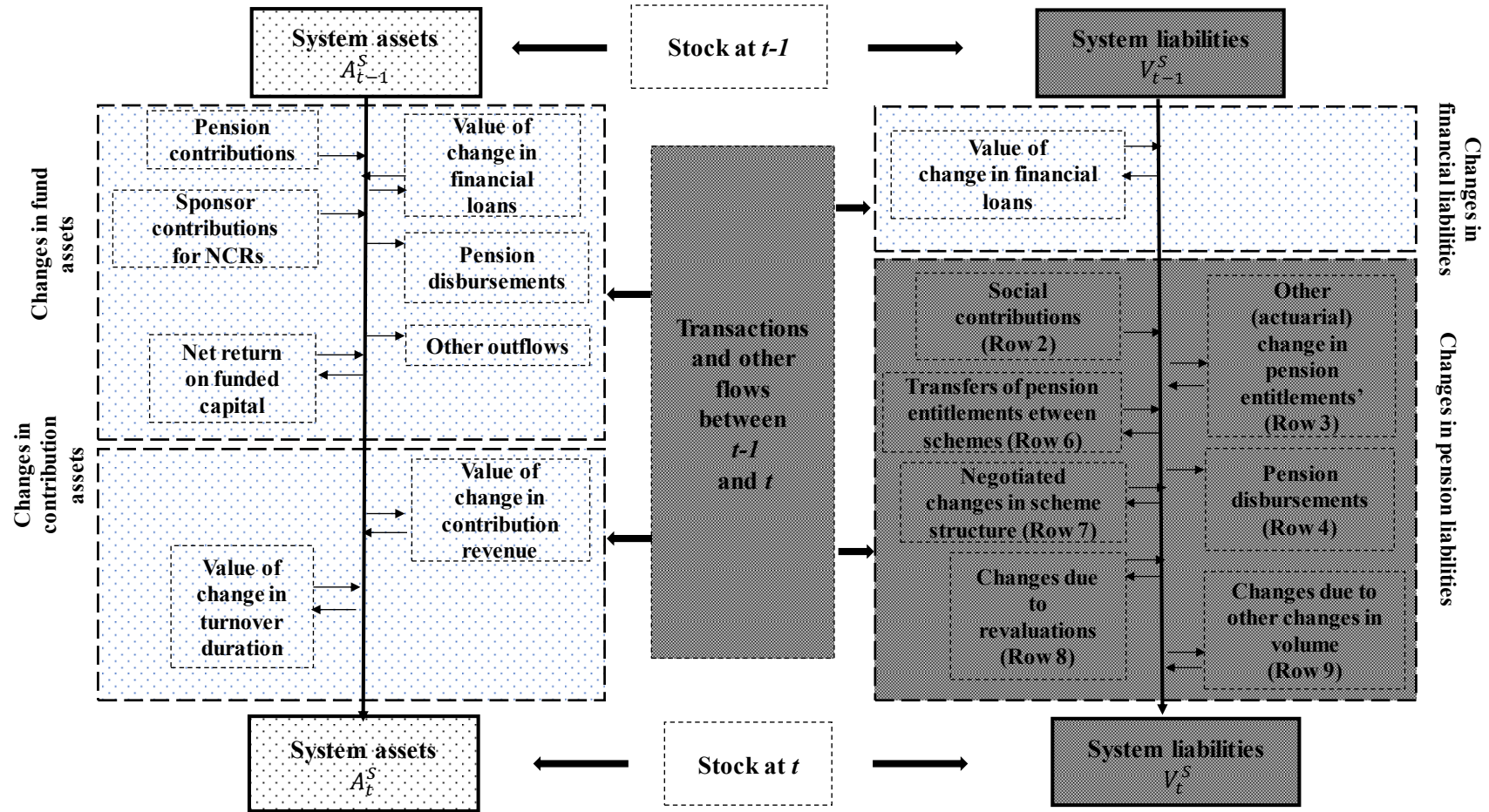
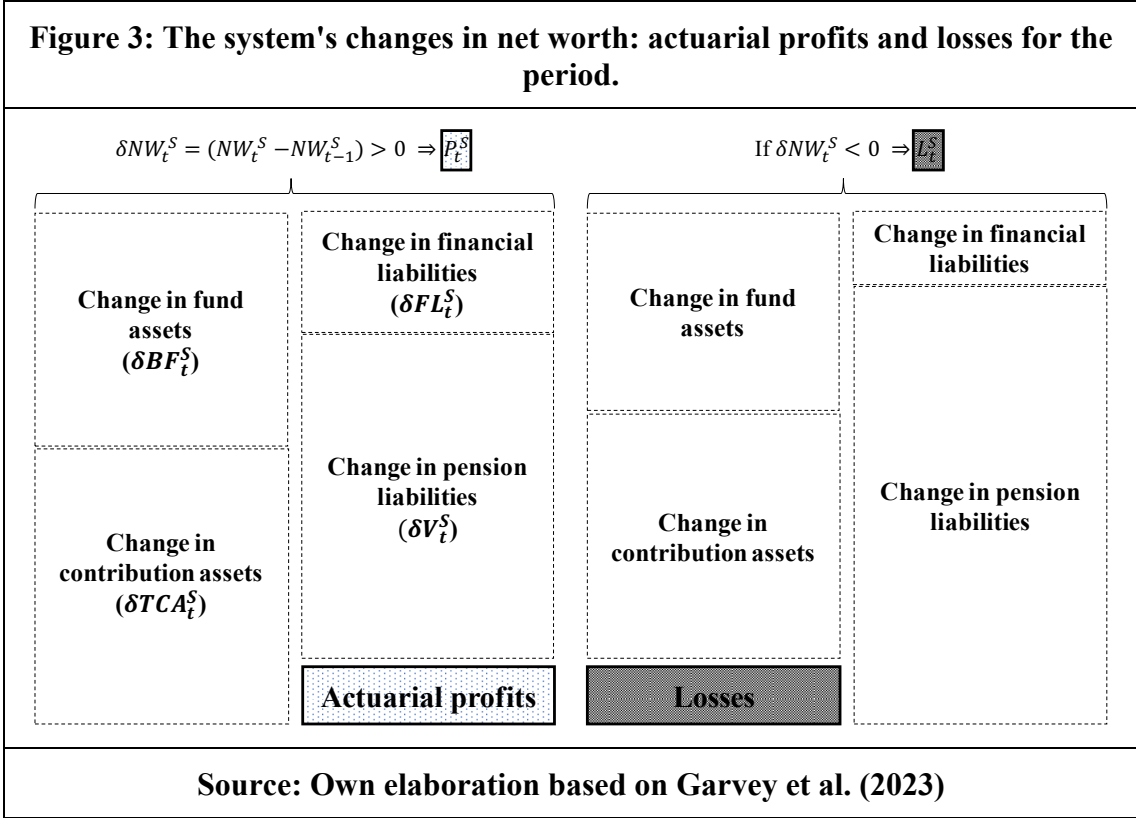


Figure 2: The system's assets and liabilities on two consecutive valuation dates and their changes.



Source: Own elaboration based on Garvey et al. (2023)



Accrual-based accounting⁴ is considered more suitable than cash-based accounting⁵ to ensure intergenerational fairness⁶ and sustainability. By adopting accrual accounting and fully understanding assets and liabilities, pension systems can measure their financial position more accurately and completely. This approach provides a clear and objective view of the financial health, which is essential for making informed decisions (Ball et al., 2024).

As shown in Figures 2 and 3, public pension accrual accounting is more complex than pension cash accounting, but it is not overly difficult. Pension obligations are considered deferred remuneration and must be accounted for through an actuarial representation discounted back to the present time. They should be recognised on the liability side of the balance sheet of their managing entities.

The ABS is based on the concept of net worth, which is positive when assets exceed liabilities. Public pension systems have a negative net worth when liabilities exceed assets, indicating insolvency. In contrast, negative net worth in the private sector would indicate bankruptcy.

Following the ideas of Ball et al. (2024), the concept of net worth holds significant importance for the ABS. But public debates on pension systems often focus on pension entitlements rather than on the financial health of the system. This is because net worth encompasses all of the system's assets and liabilities, rather than a single, albeit important,

⁴ This principle recognizes transactions or events when they occur, irrespective of when the cash is paid or received. It seeks to match the costs incurred during a particular accounting period with the benefits earned, and the revenues with the goods or services provided. It involves a system of double-entry accounting.

⁵ This principle recognizes transactions when cash is paid or received, but these do not necessarily coincide in time with the services provided or benefits received.

⁶ Intergenerational equity expresses the position that no generation should profit from governmental services at the expense of another generation, and sustainability should seek to meet the needs of the present without compromising the ability of future generations to meet their own needs (Mann et al., 2019)

liability. Two pension systems with identical levels of liabilities but differing levels of assets do not have the same solvency position. Therefore, net worth is a better indicator of sustainability, solvency, and resilience as it captures all assets and liabilities.

As Figure 3 shows, the simplest way of determining the change in net worth is to compare the system's assets and liabilities on two consecutive valuation dates.

The size of the insets in Figure 3 is irrelevant; they are for illustration purposes only.

If $\delta NW_t^S > 0 \Rightarrow P_t^S$, the change in net worth is positive, i.e. the sum of the items on the left-hand side is greater in value than the sum of the items on the right. This means that the system has actuarial profits for the period.

If $\delta NW_t^S < 0 \Rightarrow L_t^S$, the sum of the items on the right-hand side is greater in value than the sum of the items on the left. This means that the net worth has decreased, and the system has actuarial losses for the period.

If $\delta NW_t^S = 0$, the system's net worth is unchanged.

As can be seen in Figures 2 and 3, the changes in net worth can be broken down into four main items: 1.- the change in the fund/financial asset 2.- the change in the contribution asset, 3.- the change in pension liability, and 4.- the change in financial liabilities.

We use the supplementary table's structure and main figures to determine the pension liability on two consecutive valuation dates and the associated changes. The nine steps to be followed to reach the ABS, the IS and the information on the solvency of the system from Table 29:

1.-From Table 29 on pension liabilities at a specific date, an analysis is made of the pension liabilities included and whether the assumptions used are appropriate (i.e. not biased), that there is consistency between them and that they respond to the principle of verifiable data and facts at the valuation date. Best practice for Defined benefit (DB) schemes (funded and unfunded) in actuarial valuations considers that assumptions are unbiased if they are neither imprudent nor excessively conservative. Actuarial assumptions are mutually compatible if they reflect the economic relationships between factors such as inflation, increases in salaries and discount rates. To carry out this analysis, reliable historical information must be available on the variation in GDP, salaries, income from contributions, revaluation of pensions, etc., and be contrasted with that used in the calculations.

2.- The adjustments/exclusions that may arise in the valuation of pension liabilities for both contributors and pensioners are made. The exclusions are mainly for two reasons: because there is no data available to value their equivalent in the asset part (there is no information on the assigned contribution rates, contribution bases, including the number of contributors attached to that pension scheme) or simply because there is no interest in that part of the system's liabilities.

The adjustments are made to adapt the pension liability to the most reasonable calculation assumptions; basically, considering the change between the new discount rate and the original value used for estimating pension entitlements, and the change between the new indexation rate of pensions in payment and the original value.

3.- Very often, pension systems resort to indebtedness in order to be able to meet the commitments derived from pensions in payment. Financial liabilities are liabilities resulting from an accumulation of treasury deficits. In some countries, for instance in the US (BOT, 2023), the pension system is not allowed to borrow under any circumstances,

whereas in others, such as Spain, the system increases its financial debt to pay for scheduled benefits. In Table 29 this information does not appear, but in the ABS of the system it should appear because it is a financial liability of the pension system that needs to be recognized.

4.- To conclude with the liability side, it is worth examining if the pension system has some mechanism to protect pension benefits, given that developed countries apply different security mechanisms in regulations to protect pension benefits. The information to complete this part of the liability side can usually be found in the general State budgets, especially the item relating to the income from sponsor contributions for supplementary benefits (SBs) which is an important element of formula 20 (Appendix 2).

5.- Once the liability side has been completed, it is time to determine the value of the system's contribution asset. The value of this contribution flow (i.e. the contribution asset) is estimated by multiplying the system's current contribution revenue for the period for all the contributory contingencies included in the system by the time expected to elapse between the payment of the contributions and receipt of the pensions (turnover duration).

6.- The asset side could include a buffer fund or reserve fund, which is the value of the stock of financial assets owned by the system. The fund's purpose is to compensate for temporary deficits in pension contributions in relation to pension disbursements. This asset could result from an accumulation of treasury surpluses or simply for the sake of liquidity resulting from an extraordinary contribution made by the sponsor (sponsor support).

7.- The scheme's net worth is computed. The net worth is a balancing item that emerges from valuing assets and liabilities (including sponsor support if any) according to the accounting principles used on the date of the balance sheet. Net worth can be positive, negative or zero, and is equal to the accumulated surplus plus the profit or, where appropriate, minus the loss for the period. It can also be equal to the accumulated shortfall plus the actuarial losses or, where applicable, minus the profit for the period. A higher net worth, reflected in a stronger ABS, offers several advantages. These include greater flexibility to respond to emergencies such as Covid-19, a quicker recovery after an economic shock, increased certainty on the capacity to pay promised benefits, and it avoids passing on the burden of restoring fiscal soundness to future generations.

8.- The scheme's income statement is compiled (the changes in the scheme's net worth). Under a present value model, assets and liabilities are measured at present value at each reporting date. More importantly, changes in present value are reported in each period as income or expenses and are included on the income statement.

9.- The system's main solvency indicators are shown, i.e. the system's solvency ratio (SI_t^S), a common measure used to assess the system's financial health, and the required growth rate (G_t^*).

Once the ABS and the IS are compiled, the stakeholders have useful information for decision making purposes, firstly that have a figure for public pension promises, and secondly and more importantly, a reliable estimation exists of the likelihood of receiving their future (promised) benefits.

2.2.- Data

Our starting point is the information published on the Instituto Nacional de Estadística (INE) website regarding pension obligations for the years 2015, 2018 and 2021, as entered on Table 29.

Table 1 presents the information referring exclusively to the Social Security system for these three years. This represents the lion's share of the liability. It accounts for approximately 91% of the total. This liability includes retirement pensions, widow's and orphan's pensions, benefits to family members and pensions for permanent disability. The state's Special Civil Service System (Clases Pasivas) is excluded because there is not enough information to properly calculate its contribution asset. We have omitted those rows in which all cells had zero values.

Table 1: Pension table (base scenario). Years 2015-2018-2021. Social Security. Spain. Unit: EUR million.				
Row No.	Item	2015	2018	2021
1	Pension entitlements at 1 January 2015/2018/2021	2,661,993	3,674,044	5,385,025
2	Increase due to social contributions	227,846	254,272	334,297
2.1	Employers current social contributions	67,592	78,306	86,761
2.3	Households current social contributions	27,154	29,004	32,135
2.4	Household social contribution supplements	133,100	146,962	215,401
3	Other (actuarial) changes in pension entitlements in social security pension schemes	18,394	117,979	8,552
4	Reductions due to pension benefit payments	-116,006	-129,098	-146,025
5 (2+3-4)	Changes in pension entitlements due to social contributions and pension benefits	130,234	243,153	196,824
10	Pension entitlements at 31 December 2015/2018/2021	2,792,227	3,917,197	5,581,849
Pension entitlements in closing balance sheet (% of GDP 2015/2018/2021)		263%	326%	457%
Source: Own elaboration based on INE (2023, 2020 and 2018)				

Figure 4 shows the evolution of unfunded pension entitlements (social security plus civil servants' schemes) in the closing balance sheet for the end of 2015, 2018 and 2021 for some of the EU countries for which data are available. It is worth noting that Spain is at the top of the ranking in 2021, well ahead of countries such as Austria, Italy, France and Portugal. For the period 2015-2021, Spain has increased its pension liabilities as a percentage of GDP by 74%, which is the highest in the EU. Only Hungary raised its pension liabilities by 47% over the same period.

The significant rise in Spanish pension entitlements between 2018 and 2021 is largely attributable to a shift in the underlying assumption regarding the indexation of pensions in payment, as illustrated in Table 3 below.

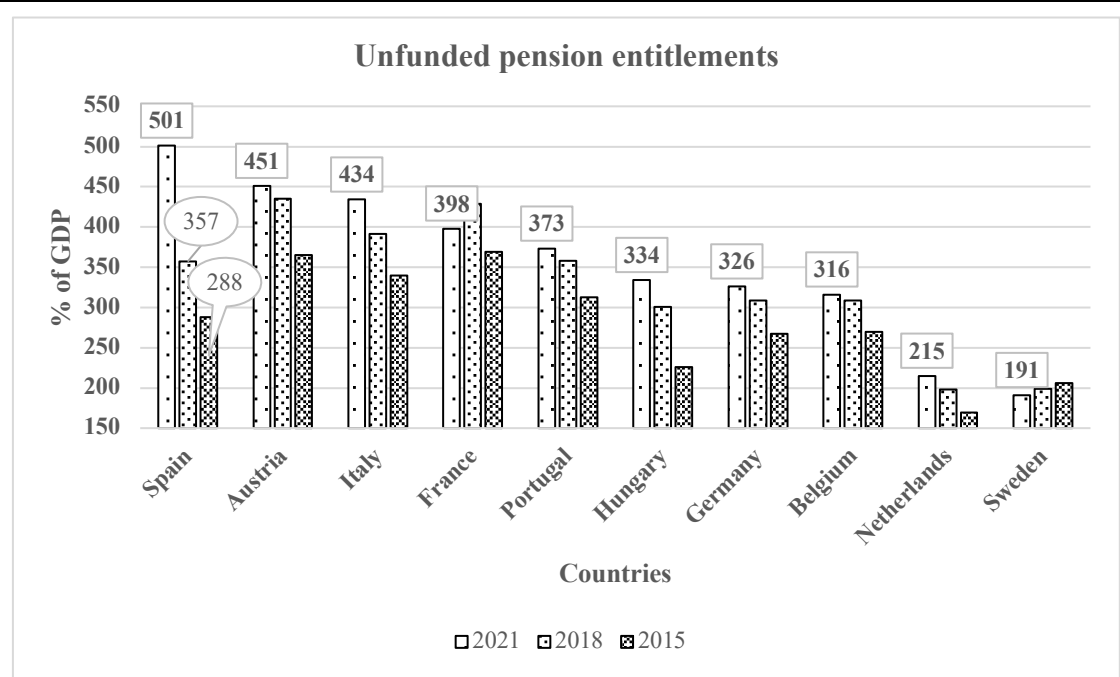
Figure 5 displays the evolution of the old-age dependency ratio (population 65+ years over the population of 20 to 64 years) for the years 2015, 2018 and 2021.

Contrary to what might appear at first sight, there does not appear to be a very clear relationship between the degree of ageing of the population (measured by the old-age dependency ratio) and the pension liabilities reported in Table 29; in fact, Austria and Spain, which have very high pension liabilities relative to GDP, are the two countries at the bottom of the ranking on this indicator. The remaining data and information needed for the compilation of the ABS and IS come from the following sources:

The evolution of the annual variation of GDP in Spain (datosmacro.com), the historical indexation of pensions in payment, the social security system's financial liabilities, pension contributions, sponsor contributions for non-contributory rights and pension disbursements (MTES, 2024 a,b). The MISSM (2022) provides the financial situation of

the Social Security Reserve Fund as at 31 December 2014, 2015, 2017, 2018, 2020 and 2021. As will be seen later, the Reserve Fund is supposed to be zero, given that the system is borrowing to pay benefits. The real balance of the reserve fund at 31-12-2021 is -79.874 million euros, however for purely political reasons the fictitious reserve fund - popularly known as the "pension piggy bank" - still exists.

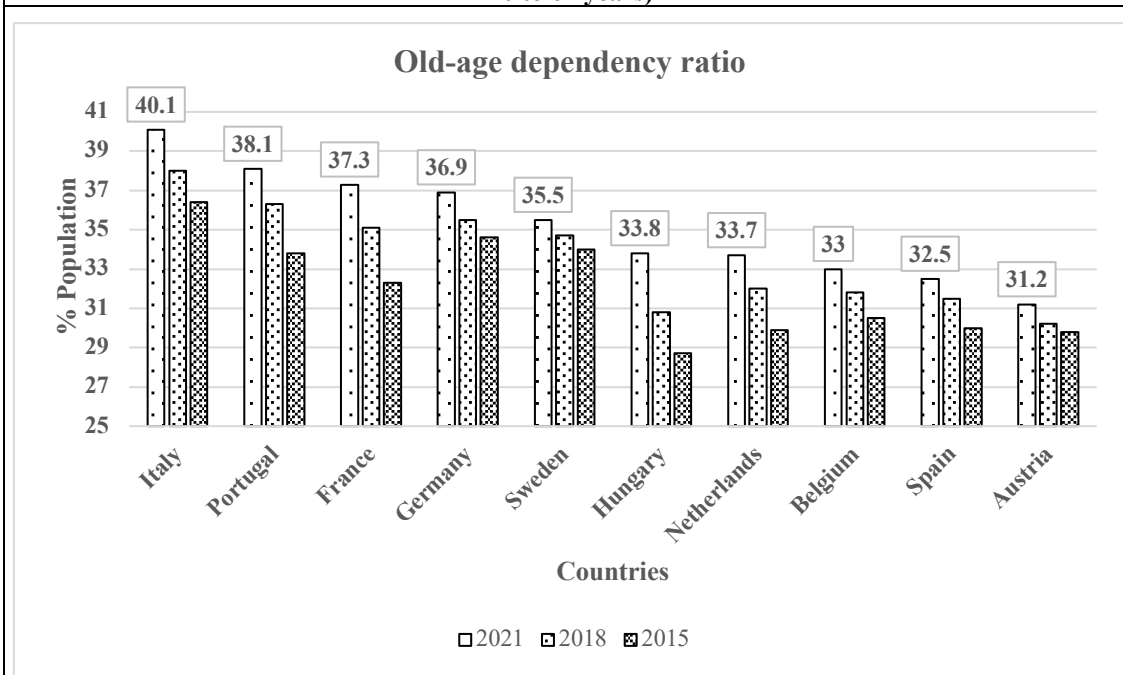
Figure 4: Evolution of the unfunded pension entitlements in closing balance sheet.



Source: Own elaboration based on:

https://ec.europa.eu/eurostat/databrowser/view/nasa_10_pens1/default/table?lang=en&category=na10.nasa_10_pens

Figure 5: Evolution of the old-age dependency ratio (population 65+ years over the population of 20 to 64 years)



Source: Own elaboration based on:

https://ec.europa.eu/eurostat/databrowser/view/demo_pjanind/default/table

The main data used to calculate the turnover duration of the system come from the Continuous Sample of Working Lives (CSWL); Muestra Continua de Vidas Laborales in Spanish. The CSWL contains the administrative data on working lives that form the basis of this sample, which is drawn from Spanish social security records and consists of anonymized microdata with detailed information on individuals.

Table 2: TD in years broken down by contingencies (Spain) and for retirement (Sweden). Period from 2014 to 2021

Year/Items	TDs System	TDr Retirement	TDsr Survivorship	TDds Disability	TDsw Sweden
2014	27.13	29.02	28.53	20.00	31.44
2015	27.05	28.89	28.55	19.86	30.38
2016	27.03	28.79	28.69	19.79	30.14
2017	27.09	28.77	28.92	19.81	29.86
2018	27.17	28.74	29.26	19.84	29.63
2019	27.15	28.67	29.40	19.70	29.77
2020	26.98	28.46	29.22	19.53	30.09
2021	26.95	28.39	29.24	19.46	30.39

Source: Own elaboration based on Continuous Sample of Working Lives (CSWL) and TSPS (2020-23).

As indicated in Section 2.1 and in Appendix 2, the system's turnover duration (TD) is a relevant element for calculating the contribution asset. According to data shown in Table 2, its value for Spain (TDs) was very stable over the period represented. Rounding to the nearest whole number, the TDs result for the period 2014-2021 is always 27 years; this provides stability in the calculation of the so-called contribution asset and in the number

of years used to calculate the appropriate discount rate for the actuarial valuation of pension entitlements.

If only the retirement contingency (TDr) is considered, then the value would be between 1.5 and 2 years higher. Such a value is comparable to that provided by the Swedish system (TDsw) (TSPS, 2019-2023), which only includes the retirement contingency. In Sweden, the value of the turnover duration has been calculated since 2001, when the Notional Defined Contribution (NDC) system was introduced. Turnover duration expresses how long it takes, on average, from the payment of the first SEK (Swedish kronor) earnings into the system to the payment of a pension based on the pension credit at the time the pension credit was earned. It thus reflects the difference in age between the average contributor and the average pensioner, if economic, demographic and legal conditions remain constant.

In essence, the inverse of turnover duration can be regarded as the discount rate applied in order to calculate the present value of a perpetual income of an amount equal to one year's contribution income.

It can be seen that the differences between the two countries tend to fluctuate around two years⁷. The value of the TD for the disability contingency (TDd) is also shown. It is much lower than for the retirement contingency because, once disability beneficiaries reach the ordinary retirement age, these pensioners are reclassified as retirement pensioners. Finally, the result for the TDsr for the survival contingency can also be described as very stable since, after rounding to the nearest integer, the result is always 29 years in the period shown in Table 2.

3.-RESULTS

The results reported in this section are based on the methodology described in Section 2.1, with the data presented in Section 2.2 and with the modification of some basic assumptions that are considered to be clearly biased according to the data and verifiable facts in the compilation of the ABS and IS.

As indicated in Section 2.1, best practice for DB schemes (funded and unfunded) in actuarial valuations considers that assumptions are unbiased if they are neither imprudent nor excessively conservative. Actuarial assumptions are mutually compatible if they reflect the economic relationships between factors such as inflation, increases in rates of salary and discount rates.

An SS scheme should measure its defined benefit obligations on a basis that reflects its estimations regarding future changes in the benefits payable. Historical data or other reliable evidence are used to determine the future indexing rate. For example, it is usual to assume that pensions in payment will be updated in line with future changes in general price or general salary levels.

Along with the use of dynamic mortality tables⁸, the main assumptions applied by INE to calculate the pension entitlements at 31 December 2015/2018/2021 are shown in Table 3:

⁷ The explanation for this is complex and clearly goes beyond the scope of this paper.

⁸ According to the "Swedish" open group (SOG) methodology, this assumption should be modified, but we are unable to do so because we cannot access the data used. It has been demonstrated, Arnold *et al.* (2019), that institutions that use mortality tables including future mortality evolution will most likely need to make more important adjustments (positive or negative) to their liabilities than institutions using periodic (static) tables whenever a new table is released.

Table 3: Main assumptions in % for the period 2014-2021									
Items	INE			Garvey et al. (2023)				This paper	
	2014/15	2017/18	2020/21	2014	2015	2017	2018	2020	2021
Nominal discount rate	5.00	4.00	4.00	4.30	4.25	4.15	4.14	3.77	3.91
Nominal indexation rate	0.25		2.00	1.00				2.00	
Inflation rate	2.00								
Real discount rate	2.94	1.96	1.96	2.25	2.20	2.11	2.10	1.73	1.88
Real indexation rate	-1.72		0.00	-0.98				0.00	
Nominal discount rate for Contribution asset	n.a.			3.69	3.70	3.69	3.68	3.71	3.71
Real discount rate for Contribution asset	n.a.			1.65	1.66	1.66	1.65	1.67	1.68
Source: Own elaboration									

It is worth keeping in mind that the use of mortality tables for the general population in the case of Spain may result in an underestimation of the liability to pensioners, given that the life expectancy of retirement pensioners at age 65 is approximately three years higher than that of the general population (Pérez-Salamero et al., 2022; Vidal-Meliá et al., 2023). Eurostat (2020a) proposes that in the event of a divergence in mortality rates between pension scheme members and the general population, scheme-specific mortality data should be used if available. In addition, both in the disability scheme and in the scheme for retired persons passing from the disability scheme (pension reclassification), the mortality tables that have been used by INE (2023), are those estimated by the Ministry of Labour, Migrations and Social Security, but we have not been able to find these mortality tables to verify their accuracy.

As Table 3 indicates, Table 29 for Spain was calculated with a nominal discount rate of 5% for 2014-2015 and 4% for 2017-2021, an inflation rate of 2% for all years, and pensions in payment indexed at 0.25% (IRP)⁹ for the period 2014-2018. According to the principle of verifiability of facts and transactions the assumption about the indexation of pensions in payment was not in line with the real inflation rate at the date the valuation is performed. This has led to an underestimation of the pension liability (as will be seen in Table 4), as previously noted by Garvey et al. (2023) for the period 2014-2018.

In its 2023 report, the INE modified the assumption regarding the indexation of pensions in payment. Pensions entitlements are now calculated under the assumption that the revaluation will align with the expected inflation rate of 2%. This represents a significant shift from the previous assumption.

The real discount rate considered to be the best estimate assumption¹⁰ is also different (1.88% and 1.73% for 2021 and 2020 respectively) to the rate applied (1.96%) by INE (2023). To ensure verifiable facts and transactions, the discount rate used is based on the

⁹ The IRP (Índice de Revalorización de Pensiones) was calculated based on the projected financial balance of the pension system, with a floor (0.25%) and a cap (CPIe + 0.5%).

¹⁰ An assumption that reflects anticipated experience with no provision for the risk of adverse deviation. The selection of an appropriate discount rate depends on the purpose and objectives of the calculations for which it is being used. Different objectives may require different discount rates.

real rate of average annual GDP growth¹¹ for the 27 years prior to the compilation of the ABS, which aligns with the TD result (see Table 2). This method for determining the annual discount rate is transparent, simple, and provides a relatively stable value, reducing the volatility of solvency indicators. Eurostat (2020a) recommends applying a stable discount rate in order to avoid the disturbance resulting from frequent changes.

It should be noted that in the UK, the discount rate used to calculate unfunded public service pension contributions is based on expected GDP growth (O'Brien and Zaranko, 2023). However, this goes against the principle of using verifiable facts and transactions at the time of compiling the ABS. Additionally, basing the discount rate on a projection introduces the possibility of forecast errors¹² and other uncertainties¹³. Morikawa (2022) finds that the upward bias in economic growth forecasts often leads to an optimistic bias in fiscal balance and debt projections.

It is a stylized fact that the accuracy of forecasts decreases as the length of the horizon increases. Projections tend to perform reasonably well at relatively short horizons (mainly within the same year), but as the horizon extends to 1.5 or two years, the accuracy of the forecasts deteriorate rapidly (Kammer, 2023; Celasun et al., 2021).

In order to calculate the contribution asset, it is necessary to use the turnover duration, the inverse of which is shown in Table 3 as the nominal discount rate for the contribution asset.

Table 4 is an extension of Table 29 for Spain (Table 1), presenting the system's income statement and the resulting simplified ABS for 2021. Table 5 displays the main data and indicators from Table 4.

The first thing that comes to our attention is the significant amount of actuarial profits (in green) in the income statement for the 2021 financial year: 159,589 million euros. This is approximately 13.06% of GDP for 2021 (Row 3 in Table 5). It is worth recalling that the system's annual profit or loss is the difference between the increase in assets and the increase in liabilities during that period. The above figure is therefore a direct consequence of the system's increase in liabilities being much lower, 45,989 million euros, than the estimated increase in the system's assets, which was 205,578 million euros. The System's actuarial profit in 2021 is mainly due to the item "Changes in entitlements due to revaluations and other", which reflects the effect of the increase in the discount rate from 3.77% to 3.91%. This amount (-172,706 million euros) significantly reduces the system's increase in liabilities.

The increase in the system's assets in 2021 is due to contributions, including state contributions or income, totalling 205,578 million euros. However, the system's financial assets, i.e. the reserve fund, maintained its value. As of 31st December 2021, the Reserve

¹¹ Following the classic Swedish ABS (TSPS, 2023), the aim is to calculate the value of commitments to contributors and pensioners within the system, rather than the cost of transferring those commitments to a third party. The interest rate used for discounting pension liabilities is based on the rate of overall economic growth (Settergren and Mikula, 2005; Gronchi and Nisticò, 2008; HMT, 2011; O'Brien and Zaranko, 2023).

¹² The literature has identified four conceptual categories through which forecasting errors may be manifested: lack of information, structural volatility, exogenous shocks, and forecaster capacity and bias (Gatti et al., 2024).

¹³ Timmermann (2007) found that forecasts for real GDP growth tend to systematically overpredict, with predicted growth exceeding actual growth on average. This bias is most significant in next-year forecasts. The tendency for overprediction of growth performance persists over time. Similarly, Morikawa (2022) found that economic growth and inflation forecasts for the next ten years are upwardly biased, and that academic researchers specializing in macroeconomics and economic growth have a greater upward bias than those in other fields.

Fund is deposited in full in the account opened at the Bank of Spain, with no investments in financial assets.

Despite the positive figure, the actuarial benefits in the financial year 2021, the opening ABS at 1-1-2021 showed a huge, accumulated shortfall of 1,743,333 million euros. In other words, there was a notable imbalance between the system's liabilities and assets, with a solvency ratio value of 0.6951 (Row 2 in Table 5). The ABS at 31-12-2021 includes the system's transactions during the year, and therefore incorporates the actuarial profits. As a result, the solvency ratio slightly improved, reaching a value of 0.7252.

Currently, only 72.52% of the system's liabilities are backed up by assets, leaving 27.48% of commitments uncovered. This implies that the sponsor will eventually need to allocate additional funds to cover the accumulated deficit. Otherwise, the promises made to some of the participants, particularly young contributors, will be partially or completely unfulfilled.

Table 4: The ABS and IS for 2021. Social Security scheme. Spain. Unit: EUR million.			
Income Statement for 2021			
ASSETS 1-1-2021	3,974,449	5,717,783	LIABILITIES 1-1-2021
Financial assets	2,138	68,182	Financial liabilities
Contribution assets	3,972,311	5,649,601	Pension entitlements
Change in fund assets	0	13,830	Change in financial liabilities
Value of change in financial loans	13,830	32.159	Change in pension entitlements
Pension contributions	118,896	340,063	Social contributions
Sponsor contributions for NCRs	36,111	86,761	Employer contributions
Pension disbursements	-146,025	32,135	Household contributions
Net return on funded capital	0	221,167	Household social contribution supplements
Other outflows	-22,812	-281,223	Other (actuarial) changes in pension entitlements
Changes in contribution assets	205,578	146,025	Payment of pension benefits
Value of change in contribution revenue	210,191	-172,706	Changes in entitlements due to revaluations and other
Value of change in TD	-4,614	-172,706	Changes in entitlements due to the discount rate.
Total change in system assets	205,578	45,989	Total change in system liabilities
		159,589	ACTUARIAL PROFITS
Actuarial Balance Sheet at 31-12-2021			
ASSETS 31-12-2021	4,180,027	5,763,772	LIABILITIES 31-12-2021
Financial assets	2,138	82,012	Financial liabilities
Contribution assets	4,177,889	5,681,760	Pension entitlements
ACCUMULATED SHORTFALL	1,743,333		
		159,589	ACTUARIAL PROFITS
TOTAL DEBIT SIDE	5,923,361	5,923,361	TOTAL CREDIT SIDE
Source: Own elaboration.			
NB: The totals will not necessarily equal the sums of the rounded components			

It is important to note that the solvency indicator includes transfers made by the state to the system as part of the public contribution asset. However, if these transfers are excluded from the calculations, the primary solvency index (PSI_t^S) is obtained (Row 1 in Table 5). This presents a more accurate depiction of the sustainability of the system, and its values for 2021 (0.5564) and 2020 (0.5262) show the severity of the problem, especially considering that it started from a value of 0.7505 in 2014.

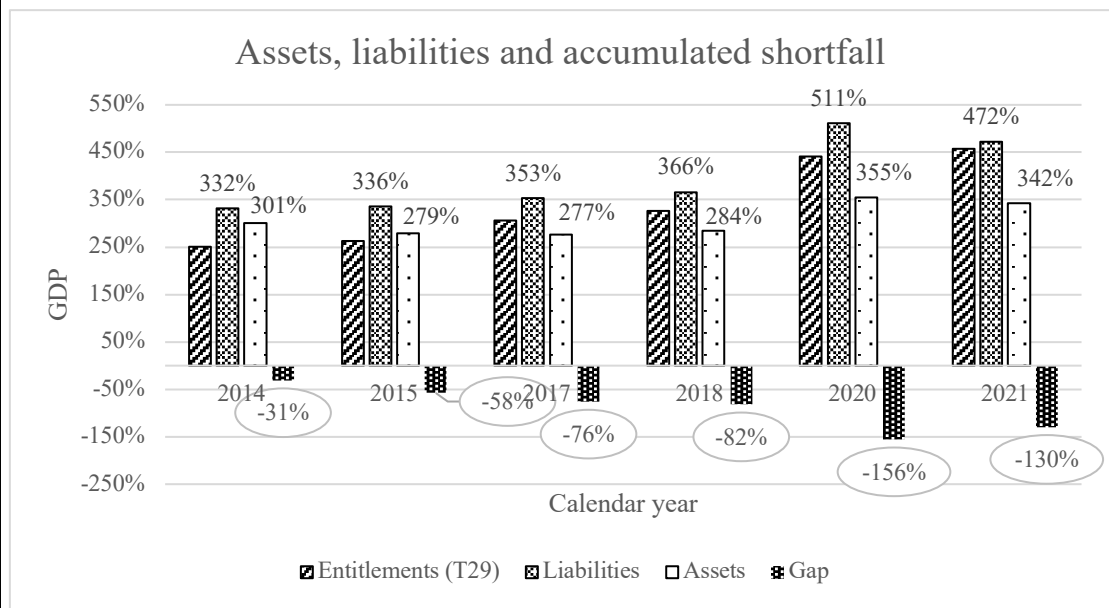
Row No.	Items	Units	2021	2020	2018	2017	2015	2014	
1	PSI_t^S	Between 0 and 1	0.5564	0.5262	0.6640	0.6764	0.7158	0.7505	
2	SI_t^S		0.7252	0.6951	0.7757	0.7839	0.8605	0.9062	
3	$\frac{L_t^S/P_t^S}{GDP_t}$	%	13.06	n.a.	-8.37	n.a.	-17.13	n.a.	
4	$\frac{Ad_t^S + L_t^S/P_t^S}{GDP_t}$		129.57	155.79	82.06	76.25	46.94	31.12	
5	$\frac{V_t^S}{GDP_t}$		471.56	510.97	365.92	352.90	336.45	331.69	
6	G_t^*		3,20	3,24	3.45	3.43	3.05	2.82	
7	Gap in G_t^*		1,32	1,50	1.35	1.32	0.85	0.57	
8	G_t^{**}		4,02	4,07	4.14	4.10	3.95	3.78	
9	Gap in G_t^{**}		2,14	2,34	2.03	2.00	1.74	1.52	
10	Duration		Years	20.99	21.12	18.52	18.05	16.44	16.32
11	Duration*			20.60	20.16	16.62	16.24	14.85	14.85
12	Convexity	Value	Positive						

Source: Own elaboration

Figure 6 presents the evolution of assets, liabilities, and accumulated deficit as a percentage of GDP over the analysed period. The Spanish pension system is experiencing a significant increase in its accumulated deficit (degree of insolvency). Liabilities grew at a higher annual cumulative rate (7.73%) than assets (4.35%) during the analysed period. To restore solvency, liabilities must grow at a much lower rate than assets. Unfortunately, in recent years, the opposite has been happening.

Figure 6: Evolution of the total assets, liabilities and accumulated shortfall (gap).

Period 2014-2021.



Source: Own elaboration based on Garvey et al. (2023)

The system's accumulated liabilities in relation to GDP show an extraordinary growth over the period 2014-2021, despite the scheme's reported actuarial benefits in 2021, from 3.32 times GDP in 2014 to 4.72 times GDP in 2021.

Figure 6 shows that the net system liability (gap) in 2021 was 130% of GDP, which is about 4.16 times higher than in 2014. In short, we can say that the Spanish social security system is in a "critical and declining state".

Finally, the required growth rate (G_t^*), which actuarially matches the pension system's liabilities with its assets at the valuation date, presents an extraordinarily high value of 3.20% (row 6 in Table 5) compared to the real discount rate¹⁴ used to calculate pension liabilities in 2021, which was 1.88% (Table 3). In other words, the discount rate (the future growth rate, ceteribus paribus) that should be applied is 70% (1,32% Gap in G_t^* , row 7 in Table 5) higher in real terms than the average rate recorded over the last 27 years.

If we were to exclude any contribution from the sponsor in the form of transfers to the system (row 9 in Table 5), the estimated growth gap would be 2.14%. This means that the Spanish economy would need to grow cumulatively in real terms by 4.02% every year. It is unlikely that the Spanish economy would grow steadily at a rate of at least 114% higher than the average recorded in recent years.

Table 5 also shows the estimated values for the duration of the original pension liability¹⁵. The duration value has increased from 16.32 years in 2014 to 20.99 in 2021, indicating an increased sensitivity of the liability value to changes in discount rates. The analysis shows that the convexity value is positive for all six years, indicating that the liability will experience larger increases in value as a result of a decline in discount rates rather than

¹⁴ This rate corresponds to the Spanish annual real average growth of GDP for the last 27 years (1995-2021).

¹⁵ The value for duration and convexity would be much more informative if they could be obtained separately for liabilities with contributors and pensioners; but the information to do so is not available.

decreases in value due to an increase in discount rates. The duration values for Spain in 2015, 2018 and 2021 are similar to the average values for all European countries¹⁶ with available data (18.69, 20.43 and 20.19, respectively). Similarly, the average convexity values for Spain are also close to the average values available for all countries analyzed, which are always positive.

As the duration is a direct function of the discount rate and an inverse function of the indexation rate of pensions in payment, we have included the values for the real duration (Duration*) based on our calculations of the pension liabilities using our best estimate assumptions. Our Duration* values are smaller than the original duration, resulting in less volatility in the estimated liabilities when the interest rate used to discount the liability is changed.

As the solvency indicators are dependent on the assumptions made, Table 6 analyses the potential impact on the system's solvency indicators at the end of 2021 if there were changes in both the relative value of pensions in payment (λ %) and the discount rate used (G %). The shaded cells show the value for our best estimate assumption (base scenario).

In the case of the basic solvency indicator, (SI_t^S), values equal to or above unity would only occur in scenarios of exceptionally high future economic growth and below-expected inflation revaluation of pensions in payment. However, such scenarios are unlikely to occur in practice.

With regard to the primary solvency indicator (PSI_t^S), it was found that in only one of the explored cases would the indicator reach a value above unity. This was contingent upon the assumption of a highly optimistic economic growth scenario and the indexation of pensions in payment at a rate far below the anticipated rate of inflation (RPI).

Finally, AIReF (2023) assumes an average annual GDP growth of 1.3% in real terms and 3.3% in nominal terms between 2027 and 2070. This is significantly lower than the best estimate value of 3.91% applied in our calculations for 2021.

The basic and primary solvency indicators would be 0.6443 and 0.4943 respectively under the AIReF scenario.

Table 6: Sensitivity analysis for the solvency indicators at the end of 2021					
Basic (SI_t^S)					
$G\% \downarrow ; \lambda\%$ →	0.5	1	1.5	2	2.5
3.3 (AIReF)	0.8916	0.7905	0.7100	0.6443	0.5898
3.5	0.9382	0.8271	0.7395	0.6687	0.6103
3.91	1.0516	0.9144	0.8081	0.7252	0.6567
4	1.0782	0.9346	0.8248	0.7380	0.6678
4.5	1.2652	1.0727	0.9310	0.8224	0.7365
5	1.5275	1.2566	1.0673	0.9275	0.8202
Primary (PSI_t^S)					
3.3 (AIReF)	0.6840	0.6064	0.5447	0.4943	0.4525
3.5	0.7198	0.6345	0.5673	0.5130	0.4682
3.91	0.8067	0.7015	0.6199	0.5564	0.5038
4	0.8272	0.7170	0.6327	0.5662	0.5123
4.5	0.9706	0.8229	0.7143	0.6309	0.5650
5	1.1718	0.9640	0.8188	0.7116	0.6292
Source: Own elaboration					

¹⁶ Data can be made available upon request to the authors.

4.-DISCUSSION

Timely and regular information is crucial for transparency and accountability. For instance, Table 29 for the year ended 31 December 2021 was published in December 2023. However, by the beginning of 2024, more than two years had passed since the last supplementary table on pension entitlements had been published. To make matters worse, Table 29 is only compiled every three years for data relating to year t-2. This indicates that the UE is not using Table 29 for any significant decision-making purposes. In essence, there is considerable doubt as to whether the UE is treating this matter with the seriousness it deserves.

The initial thought is a statement by Hans Hoogervorst, Chairman of the International Accounting Standards Board (IASB), on 20 June 2012, addressing the International Association for Accounting Education & Research (IAAER) conference in Amsterdam *“Around the world, governments give very incomplete information about the huge, unfunded social security liabilities they have incurred. Many executives in the private sector would end up in jail if they reported like Ministers of Finance, and rightly so”*.

After seeing the concerning solvency indicators presented in the previous section, at least six questions arise: In comparison to other countries that draw up actuarial balance sheets, is the pension system in Spain more or less favourable in terms of its solvency? How could the solvency of the system be restored? What measures would have to be taken if the public system were a funded defined-benefit pension plan? How would Spain's public debt be affected if the social security gap were taken into account? If the social security system were a private company what steps would be required to initiate insolvency proceedings? and What measures have been implemented in recent years to address the sustainability issues in the Spanish pension system?

1.- In comparison to other countries that draw up actuarial balance sheets, is the pension system in Spain more or less favourable in terms of its solvency?

Although the practice of making comparisons is often viewed negatively, we assert that in this case, it is a crucial tool for contextualizing the severity of the insolvency situation in the Spanish pension system. To achieve this objective, the ABSs of three countries (the USA, Canada, and Sweden) are presented with a structure similar to that developed for the case of Spain as of December 31, 2021.

The principal characteristics of the US, Canadian and Swedish public pension systems can be found in the references cited in their respective tables (7, 8, and 9) and a brief summary on the qualifying conditions, benefit calculation, variant careers or personal income tax and social security contributions can be consulted in OECD (2023a).

Table 7: OASDI Actuarial Balance Sheet at 31-12-2021 (billions of \$) (Under Intermediate Assumptions)			
ASSETS 31-12-2021	87,023	108,627	LIABILITIES 31-12-2021
Financial assets	2,852	1,154	Financial liabilities ¹⁷
Contribution assets	84,171	107,473	Pension liabilities
		-21,604	Net worth
TOTAL DEBIT SIDE	87,023	87,023	TOTAL CREDIT SIDE
Source: Own elaboration based on: Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds (BOT). (2022). 2022 Annual Report			

The federal Old-Age, Survivors, and Disability Insurance (OASDI) programme is the official name for Social Security in the United States. The OASDI tax deducted from people's paychecks funds this comprehensive federal benefits programme, which provides benefits to retirees and disabled people along with their spouses, children and survivors.

The open-group unfunded obligation¹⁸ for OASDI is \$21,604 billion in present value over the 75-year projection period through 2096. Under the intermediate assumptions, the projected hypothetical combined OASI and DI Trust Fund asset reserves (financial assets) become depleted and unable to pay scheduled benefits in full on a timely basis in 2035. At the time of the depletion of these combined reserves, continuing income to the combined trust funds would be sufficient to pay 80 percent of scheduled benefits. In short, the solvency indicator at 31 December 2021 has a value of 0.801.

Table 8: Base CPP Balance Sheet (Open Group Basis) (9.9% Legislated Contribution Rate, \$ billion) + Additional CPP Balance Sheet (Open Group Basis) (2.0%, 8.0% Legislated First and Second Additional Contribution Rates, \$ billion) at 31-12-2021 (Under Intermediate Assumptions)			
ASSETS 31-12-2021	4,497.10	4,379.50	LIABILITIES 31-12-2021
Financial assets	554.70	0.00	Financial liabilities
Contribution assets	3,942.40	4,379.50	Pension liabilities
		117.60	Net worth
TOTAL DEBIT SIDE	4,497.10	4,497.10	TOTAL CREDIT SIDE
Source: Own elaboration based on: Actuarial Report (31st) on the Canada Pension Plan			

The combined ABS for the base Canadian pension plan (CPP) and the additional CPP, prepared using an open group approach and the legislated contribution rates of each component, indicates a positive net worth, or assets in excess of liabilities. It may be posited that actual contributors and pensioners have a reasonable expectation regarding

¹⁷ The actuarial balance for a period includes the cost of maintaining a target fund equal to 100 percent of the following year's cost at the end of the period. Under present legislation, the OASI and DI Trust Funds are not permitted to borrow funds other than in the form of advance tax transfers, which are limited to expected taxes for the current calendar month (BOT, 2022).

¹⁸ The system's liabilities include the present value of all current beneficiaries and contributors to the social security schemes and the contributions and benefits of new affiliates under current rules. The options range from including only children not yet in the labour force to an infinite perspective. Usually, an arbitrary time period is chosen (75-95 years) and the methodology is applied over that period.

the fulfilment of their pension promises in view of the solvency indicator value being 1.027.

Table 9: Assets and Liabilities of the Swedish NDC System, at 31-12-2021 (billions of SEK)			
ASSETS 31-12-2021	11,191.94	9,990.99	LIABILITIES 31-12-2021
Financial assets	2,004.08	0.00	Financial liabilities
Contribution assets	9,187.86	9,990.99	Pension entitlements
		1,200.95	Net worth
TOTAL DEBIT SIDE	11,191.94	11,191.94	TOTAL CREDIT SIDE
Source: Own elaboration based on: The Swedish Pension System. (TSPS, 2022). Orange Rapport 2021.			

The Swedish public old-age pension system is comprised of two distinct components: an earnings-related notionally defined contribution (NDC) pay-as-you-go component and a fully funded, defined contribution (DC) pension component. Both components are based on lifetime earnings and individual accounts. Table 9 exclusively encompasses the NDC aspect of the pension system.

NDC balance ratio is 1.12, that is, assets exceed liabilities by 12 percent or SEK 1,200 billion. This indicator is the best for the countries analyzed, and it is not surprising given that Sweden is the European country in which pension entitlements are lower in 2015 than in 2021. Given that the solvency indicator is greater than 1.12 at 31-12-2021, a level that according to current Swedish legislation indicates possible distributable surpluses. However, no such proposal has been presented to the Swedish Parliament.

In summary, the solvency of the Spanish pension system is cause for concern, both at an individual level and in comparison, with other countries. This is not unexpected, given that the data in Table 29 indicated that Spain was the European leader in pension liabilities in 2021.

2.- How could the solvency of the system be restored?

Based on Table 3, it is possible to identify at least two possible actions and one future event that could raise the solvency ratio to 1. Obviously, a combination of these three actions/events would have a similar effect.

The initial action is based on the solvency indicators for 2021. As the liabilities in that year exceed the assets by approximately 27.48%, pensioners and contributors may receive reduced amounts proportional to the value of the acknowledged liability. In other words, only 72.52% of the scheduled benefits would be paid to contributors and pensioners.

A couple of pieces of information suggest reasons why public pensions in Spain are so high. The old-age pension replacement rate measures how effectively a pension system provides a retirement income to replace earnings, the main source of income before retirement. According to OECD (2023a), full-career male workers will have a replacement rate of 50.7% on average across OECD countries, with a high of 80% or more in Greece and Spain and a low of under 30% in Australia, Estonia, Ireland and Lithuania.

In Spain, only people aged 65 and over have experienced an increase in their average income from 2008 to present. Meanwhile, the average income of other age groups has continued to decline from 2009 to 2015. Pensioners have benefited most since the 2008 crisis (INE-ECV, 2023). In 2021, the pensions of new retirees from the general regime

(paid employees) reach €21,450 per year, which is already 14% higher than the most frequent salary in Spain, quantified by the INE at €18,503 per year.

The average pension in the system has increased by 24.61% in real terms from 2008 to 2021, primarily due to the substitution effect¹⁹ and the policy used for indexation of pensions in payment, while the average salary has increased by only 2.84% over the same period.

The second action corresponding to the sponsor (the State) would be to make an extraordinary contribution equivalent to 129.57% of GDP for that year. This would eliminate all losses from the system and result in a solvency ratio of 1. This figure is exceptionally high, but in line with the corresponding figure published for the United States.

In the US, the sponsor would have to contribute to restoring the sustainability of the US OASDI system with an extraordinary payment of 84.98% of GDP (BOT, 2023). It should be noted that the contribution rate is lower than Spain's, 12.40% compared to 28.30%. In addition, the US has a much higher ratio of contributors to pensioners, with 2.8 contributors per pensioner compared to 2.15 in Spain. Finally, the equivalent solvency indicator would be 80.11% (2022) compared with 72.52% (2021) in Spain.

There is no objective basis for the idea that the sponsor/state could make contributions of a certain size to fill part of the social security gap. In 2020, the country's deficit reached 10% of gross domestic product (GDP), while public debt reached its highest level in the last century, at 120% of GDP. Since then, the reactivation of the economy and significant growth in tax revenue have helped to improve the country's (traditional) fiscal metrics. However, projections indicate that Spain will continue to record persistently high structural deficits. There may be overall deficits in the future unless additional fiscal adjustments are made. (Lago-Peñas, 2023).

All of this is a lot easier said than done.

As discussed in the previous section, an economic miracle in the form of a much higher sustained economic growth (*ceteris paribus*) in comparison with that evidenced in recent years could partly alleviate the situation of this huge social security gap.

While it may not be immediately evident from the indicators presented in Table 3, a potential solution could be to increase the contribution rate applied to fund old-age, disability, and survivors' pensions. This increase in the contribution rate would be substantial, almost 38%, (from 28.30% to 39.03%), and should not result in additional entitlements for contributors. According to information provided by ISSA²⁰ (2024), Spain was already one of the EU countries with the highest contribution rate (28.30%), surpassed only by Portugal (34.8%), Italy (33%), and Hungary (29.5%). Such a significant increase in contribution rates would undermine the contributory nature of the system and intergenerational equity, and could have adverse effects on the economy, particularly on employment, growth, and competitiveness (IEE, 2023).

This very high increase in the contribution rate would, *ceteris paribus*, be aimed at alleviating the cash deficit of the system and the effect that the relationship between expected contributions and pension benefits 'yields' too high an implicit internal rate of

¹⁹ The substitution effect pertains to the rise in the average pension system resulting from the disparity between the benefits received by new entrants (recent retirees with higher benefits) and system withdrawals (usually elderly individuals with lower amounts).

²⁰ <https://www.issa.int/databases/country-profiles/contribution-rates>

return (IRR) for the average participant, to such an extent that this implicit IRR is incompatible with the sustainable rate of return of the system (i.e. the growth rate of real GDP), or, in other words, the cost of selling (pensions and acquired obligations to contributors) is much higher than the selling price (contributions) (Boado-Penas et al. 2008; Vidal-Meliá, 2014).

The age composition of the affiliates may influence the decision-making process, as younger and older employees and retirees may have differing interests. A higher proportion of retirees may result in more participants favouring higher contributions over no indexation or pension cuts, as the latter would disproportionately affect inactive participants compared to active participants who still have time to save. Contribution increases would only affect active participants. As discussed below in relation to the recent public pension reform in Spain (2020-2023), the tax burden on the young has been increased to maintain the generosity of pension benefits.

3.- What measures would have to be taken if the public system were a funded defined-benefit pension plan?

The social security gap, which refers to the accumulated shortfall situation, would require the sponsor to submit an immediate long-term recovery plan. The plan should aim to eliminate the shortfall within 10-15 years so that the scheme meets the minimum solvency index required by law. In this context, a recovery plan should outline the measures that the authorities governing the SS scheme wish to implement to improve its solvency situation.

Countries typically implement mechanisms to ensure the sustainability of defined benefit (DB) plans and to guarantee the benefits promised to members to some extent. One common approach is to establish minimum funding requirements, which typically require a recovery plan (with varying timeframes across countries) for underfunded plans. These recovery plans could imply additional contributions from employers (e.g. the United Kingdom) and penalties for the sponsors failing to make these deficit-repair contributions (e.g. up to KRW 10 million in Korea from 2022). Recovery plans could also include benefit cuts or adjustments. Some countries have a pension protection fund (e.g. the Guarantee Fund in Liechtenstein, the Pension Protection Fund in the United Kingdom, the Pension Benefit Guarantee Corporation in the United States), which can take over the liabilities of a DB plan and pay benefits to members when an employer goes bankrupt, or an underfunded plan is wound up. (OECD, 2023b).

Under current Spanish legislation, any pension plan deficit should be eliminated by extraordinary contributions from the sponsor (which in the public system is the State) within a period no greater than 5 years (or 10 years in exceptional cases).

If such a scenario were to occur, the Spanish government would need to make exceptional contributions. This could be achieved through debt issuance, tax increases, spending reductions, and potentially selling government assets. However, as Steindel (2020) suggests, these actions may increase the cost of doing business and reduce aggregate demand, which could depress economic activity. Even if no action is taken to address pension problems in the short-term, the perception that a government may be forced to make major fiscal adjustments to pay pension benefits in the future could discourage business activity and depress property values.

Similar regulations can be found in OECD countries, including Canada, the Netherlands, and the UK. In Canada, the plan sponsor must make additional periodic contributions, known as special payments, to eliminate these shortfalls over a prescribed period of time,

which can range from 5 to 15 years. In the Netherlands, pension funds are allowed a recovery period of up to twelve years to restore financial stability if the funding ratio falls below 105%. According to De Haan (2018), pension cuts are more likely when the funding ratio is very low, there is little time left for recovery, the pension fund is not a corporate pension fund, and its participants are still relatively young. Dutch pension funds prioritize contribution increases, followed by no indexation, and only resort to pension cuts as a last option.

For UK DB pension schemes (Deloitte, 2017), if the statutory funding objective is not met and this leads to a deficit in the pension fund, the employer and pension scheme trustee must agree on a recovery plan outlining the necessary steps to rectify the situation. In the past, the pension regulator has recommended that deficits be eliminated 'as quickly as the employer can reasonably afford to'. However, in July 2014, the pension regulator's new objective came into force. The objective is to minimise any adverse impact on the sustainable growth of an employer.

4.- If the social security system were a private company, what steps would be required to initiate insolvency proceedings?²¹

Considering the decline in the solvency position of the social security system, it is worth exploring the steps that a private company would take in a similar situation.

To initiate insolvency proceedings, it is recommended to apply for a declaration of insolvency during the 'common phase'. This phase involves gathering information on the debtor's economic and financial situation, conducting a thorough investigation to determine the amount and nature of claims, identifying assets and liabilities, and assessing the viability of the company or the debtor's ability to pay. Additionally, an insolvency administrator or trustee is appointed to manage the proceedings and protect the interests of creditors.

If it is possible to restructure the debt or implement measures that allow the company to continue, the debtor can submit a structural proposal to the creditors during the agreement phase. This proposal should include a detailed plan for repaying debts, restructuring financial commitments, or any other measures that may make the debtor's economic recovery feasible.

If there is no proposed arrangement, or if existing proposals are not approved or not complied with, the 'liquidation phase' will begin.

However, the debtor may request this phase at any time if it is not feasible to continue with the activity.

Finally, the appraisal phase is reached. Its aim is to assess the conduct of the insolvent debtor and the causes of the insolvency in order to determine whether there has been any negligent conduct that has led the company to default on its payment obligations.

Defective insolvency proceedings are those caused or aggravated by fraud or negligence on the part of the debtor or its administrators. Such proceedings may be declared for various reasons, including reducing or eliminating assets to lower creditors' expectations of collection, preparing double accounting statements, failing to file annual accounts with the corresponding registry in any of the three previous years, or submitting false documentation and/or information.

²¹ <https://e-justice.europa.eu/447/EN/insolvencybankruptcy?SPAIN&init=true&member=1>

In short, insolvency proceedings are considered irregular if they attribute the cause of the insolvency situation to the debtor or administrator, or if their actions have led to a deterioration of the situation.

The outlined procedure requires replacing the term 'debtor' with 'sponsor', 'creditor' with '(pensioners and contributors)', and 'insolvency administrator' with 'office of the chief actuary'; allowing us to visualize how this process could be further developed. Perhaps Mr. Hoogervorst had this entire process in mind when he made the statement we quoted at the beginning of this section, with which we couldn't agree more.

5.-How would Spain's public debt be affected if the social security gap were taken into account?

Non-debt liabilities, such as pension entitlements, make up a significant portion of the government's total liabilities. However, these liabilities are often ignored by debt-based fiscal metrics (Ball et al., 2024). Based on Governmental Accounting Standards Board Statements²² 67/68 (GASB 67/68), and International Accounting Standards (IAS 19), the concept of combining the social security gap with government would be a commonsense step in our opinion.

GASB 67/68 states that any net pension liability (NPL) should be recognized on financial statements. The NPL is the difference between the assets held by the Plan on the measurement date and the liability for future expected benefit payments earned by current members and pensioners up to the measurement date. The NPL is then allocated among the employers based on their share of the unfunded liability measured in the annual actuarial valuation report.

The IAS 19²³ – applied in the European Union since 2005 for listed companies – aims to include pension obligations in the sponsoring entity balance-sheet. Indeed, reporting entities are required to recognise the net liability (asset) in their statement of financial position, this net position being equal to the accumulated deficit (surplus) in the defined benefit plan (Biondi and Boisseau-Sierra, 2017)

In our case, the sponsor is the state, and the report is the ABS.

Therefore, by analogy, the net social security liability (gap) owed to current affiliates and pensioners could be treated similarly to government debt; given the fact that the net pension promises are a form of government debt with strong rights (Rauh, 2017; Anantharaman and Chuk, 2023; Ball et al., 2024).

Table 10 shows the evolution of the Government debt, social security gap and total as a percentage of GDP for the period 2014-2021. Although the government debt has increased moderately over this period (less than 12 percentage points), taking into account the presence of the COVID 19 pandemic, the picture would change radically if the SS gap were added. In this scenario, one could argue that the 'extended' government debt as a percentage of GDP would be 1.81 times higher in 2021 than in 2014.

²² GASB | Governmental Accounting Standards Board. An independent organization that establishes and improves standards of accounting and financial reporting and is recognized as the official source of generally accepted accounting principles (GAAP) for U.S. state and local governments.

²³ IAS 19 Employee Benefits (amended 2018) outlines the accounting requirements for employee benefits, including short-term benefits (e.g. wages and salaries, annual leave), post-employment benefits such as retirement benefits, other long-term benefits (e.g. long-term service leave) and termination benefits.

Table 10: Evolution of the Government debt, social security gap and total as a percentage of GDP. Period 2014-2021.			
Items	Government debt	SS gap	Total
2014	105.10	31.12%	136.22%
2015	103.30	57.65%	160.95%
2017	101.80	76.25%	178.05%
2018	100.40	82.06%	182.46%
2020	120.30	155.79%	276.09%
2021	116.80	129.57%	246.37%
Source: Own elaboration			

Why would it be interesting to present the pension gap together with government debt? It would clearly encourage both the EU and national governments to take pension imbalances much more seriously and to be much more careful about the pension promises they make. It could also have an impact on financial markets, as higher reported SS gaps could trigger credit rating downgrades, increase the cost of financing government debt and/or raise the specter of covenant breaches and renegotiation of pension promises.

In summary, there are no good options once the Spanish pension system has accumulated a significant shortfall.

6.-What measures have been implemented in recent years to address the sustainability issues in the Spanish pension system?

The recent public pension reform in Spain (2020-2023)²⁴ may not improve intergenerational equity as the main measures included seem to be taking the opposite direction.

Very briefly, the main modifications to the social security system are as follows:

The transfer of funds to the Social Security system has been increased to finance specific expenditures, as stated in Law 11/2020.

Law 21/2021 introduces changes to the pension system, including the indexation of pensions in line with the CPI and the elimination of the Sustainability Factor introduced in the 2013 reform. The law also aims to incentivise delayed retirement and increase transfers from the State.

The RDL of 13/2022 approves a new contribution system for the Self-Employed Workers' Scheme.

The period used to calculate the pension has been modified by RDL of 2/2023. This RDL introduces a choice for calculating the Regulatory Base by selecting the period to be taken into account. The options are to either (i) use the last 25 years as a reference or (ii) use the last 29 years as a reference and exclude 24 monthly contribution bases that the contributor deems appropriate.

The contribution rate linked to the intergenerational equity mechanism (IEM) is increased, and its validity is extended until 2050. This contribution will not be taken into account for benefit purposes and will be allocated to the social security reserve fund. The contribution amount will be 1.2%, with 1% paid by the employer and 0.2% paid by the

²⁴ Law 11/2020, Law 21/2021 and Royal Decree-Laws 13/2022 and 2/2023.

worker. In 2023, the contribution rate will be 0.6%, with 0.5% paid by the employer and 0.1% paid by the worker. The contribution rate will increase gradually until 2029, following the sliding scale outlined in the law.

A solidarity contribution is introduced on the maximum contribution bases. This increases the maximum contribution base above the increase in the CPI and the maximum pension, and imposes a surcharge on wages above the maximum base.

The solidarity contribution will be calculated by multiplying (i) the portion of compensation that falls between the maximum contribution base and the figure that is 10% higher than that base by 5.5%, (ii) the portion of compensation that falls between the figure that is between 10% and 50% higher than the maximum contribution base will be by 6%, and (iii) the portion of compensation above that percentage by 7%. It is important to note that this contribution is calculated based on a specific formula.

The solidarity contribution will come into effect on January 1, 2025 and will increase annually until 2045, when the final rate will be reached.

An automatic adjustment mechanism (AAM) has been introduced to adjust measures or increase social security contributions if pension expenditure deviates from a recommended reference path. It should be noted that the success of an AAM depends on its implementation in a system in equilibrium (Vidal-Meliá et al., 2009). However, this is not the case with the Spanish pension system, as demonstrated in the previous section.

It can be said that the reform carried out in the period 2020-2023 is poorly thought out because it is based on the fiction that the pension system is in some kind of equilibrium when, as we have seen, the system has a very vulnerable solvency situation with a very high negative net worth, which means that it is shifting the burden of restoring fiscal soundness to future generations.

The recent pension system reform, despite government propaganda, is unlikely to reduce the growing contributory deficit observed in recent years. It is important to note that this evaluation is objective and not influenced by personal opinions or biases. Several reports severely question the viability of the reforms carried out between 2021 and 2023.

According to the latest reports by García-Díaz (2023) and De la Fuente et al. (2023), the official projections of the Ministry of Social Security are optimistic but poorly documented. The report suggests that the projections are implausible because they rely on demographic and macroeconomic assumptions that are more favourable than those used by other institutions. Additionally, the report highlights that some of the estimates of the budgetary effects of recent reform measures are insufficiently documented and not very credible. These projections may underestimate the impact of the reform on the pension system's budget deficit, which could limit the spending room available for other policies.

Brindusa et al. (2023) predict that additional adjustments to pension reform will be required as early as 2025. The analysis of the main regulatory changes introduced since 2021, although subject to high uncertainty, suggests that new measures will be necessary from 2025 onwards to strengthen the financial sustainability of the reform.

AIReF (2023) predicts that the pension reform measures implemented between 2021 and 2023 will result in a 1.1% increase in the government deficit in 2050 and an additional 1% increase in 2070, leading to a government debt of 186% of GDP. However, the reform will also increase public revenues by 1.3% of GDP through measures such as raising additional social security contributions for those in the highest income brackets.

However, the increase will not be enough to cover the expenditure that will rise by 2.4 points due to measures such as automatic revaluation in line with the CPI.

OECD (2023a) agrees with AIREF's (2023) calculations on the additional cost of the reforms implemented in Spain, attributing it mainly to the reintroduction of pension indexation to coincide with inflation.

De la Fuente (2024) argues that the pension system's contributory character will be reduced, and social contributions will become a pure tax due to the insufficient reform carried out between 2021-2023. He also notes that the reform will not improve intergenerational equity as it continues to increase the tax burden on the young to maintain the generosity of pension benefits.

5.-CONCLUSIONS

The ABS displays the current state of the public pension system based on past decisions. It identifies the net pension liability, including any asset shortfall or social security gap, that will be passed on to future generations.

Information on cash flows is important. However, cash accounting alone does not provide policymakers with reliable information on the majority of pension system assets and liabilities. Without this information, effective management of the system is impossible.

The financial health of the Spanish pension system is weak and getting weaker, driven by incorrect pension policies (generously defined pension benefit promises, poor actuarial estimates of retiree behavior and longevity), lack of economic growth and demographic trends. If this problem is not addressed soon, it will increase the burden on future generations and put further strain on our political system. Understanding the true state of solvency of the Spanish pension system requires a full ABS driven by accrual accounting, or in other words, it is imperative to shift the management of pension liabilities from an 'out of sight, out of mind' approach to a more transparent approach that accurately reflects the net worth of the system, i.e. a 'tell it like it is' approach. The new approach results in higher pension wealth and, consequently, a higher net worth for the household sector. This reflects the inclusion of unfunded accrued promises, which were previously ignored. Now, they are treated as assets of households and liabilities for the public sector.

The answers to the questions discussed in the prior section have identified double standards in the measurement, recognition, and disclosure of pension liabilities and assets between the public and private sectors. It has also highlighted the differing responsibilities of managers and policymakers in situations of insolvency or mismanagement. We can conclude that the pension system in Spain is in urgent need of reform to ensure intergenerational equity, but the approach to social security policy in Spain also needs to be reformed. It can be said that the reform carried out in the period 2020-2023 is poorly thought out because it is based on the fiction that the pension system is in some kind of equilibrium when, as we have seen, the system has a very vulnerable solvency situation with a very high negative net worth, which means that it is shifting the burden of restoring fiscal soundness to future generations.

Spanish policymakers are failing to address the need for timely reform. Politicians often procrastinate when faced with difficult or unpleasant decisions, leading to delayed actions (Turner, 2017). Spanish history shows a tendency to do nothing until a crisis arises, particularly when it comes to dealing with Social Security reform. Procrastination by politicians may be due, at least in part, to the traditional political and economic concern over the impact of enacting Social Security reform on their chances for re-election.

Spanish policy makers may only take the social security gap seriously and stop procrastinating if the data shown in Table 4 and 10 becomes official and the EU imposes a recovery plan. This would highlight that the Spanish social security system is in a “critical and declining state”, which could increase awareness among voters and politicians about the need for reform and potentially generate more support for reform.

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APPENDIX 1: TECHNICAL APPROACH FOR ESTIMATING THE ACCRUED-TO-DATE PENSION ENTITLEMENTS INCLUDED IN THE TABLE 29

The analysis of the methodology used to estimate the accrued-to-date pension entitlements included in Table 29 (Eurostat, 2020a; Plamondon et al., 2002; Winklevoss, 1993) is outlined below.

It is crucial to acknowledge that the technical development presented in this appendix is predicated on the authors' convictions regarding the optimal methodology for calculating pension liabilities. This differs from the approach outlined in some technical documents that have been cited as the basis for pension calculations in various countries.

The development presented in this appendix is based on final earnings schemes. In such schemes, pensions are calculated on the basis of final earnings before retirement, as implied by the term. These final earnings may comprise earnings over the entire contribution career or over a shorter period. For notional defined contribution accounts (NDCs) and flat-rate pension schemes, the formulas to calculate the liability to contributors would be different.

In the event that individualised data on contributors and pensioners are not available at the reference date for calculating pension liabilities, which seems unlikely, particularly in more developed countries, the representative contributor approach should be used.

Two main steps are required to estimate accrued-to-date pension liabilities. In the initial step, average pension entitlements for different age and gender groups are calculated. In a second step, these group-specific pension rights are multiplied by the respective age- and gender-specific cohort sizes of the pension scheme members. Step 2 generates the final result, namely the total value of pension entitlements.

In order to accurately assess pension liabilities, it is essential to differentiate between the pension entitlements of current pensioners (liability to pensioners) and those of current contributors (liability to contributors). It is important not to forget that pensioners have already accumulated the full pension benefits to which they are entitled, whereas contributors have only partially accrued their future full pension benefits.

In order to develop the actuarial formulae for the assessment of pension liabilities, it is necessary to ascertain the following transition probabilities, in which no more than one transition in a year is assumed and we also assume that participants' lives could last $(\omega - x)$ years, where ω is the maximum lifespan and x is the current age of the individual:

$p_{(g,x+k)}^{aa,f}$ is the probability that an active aged $(x + k)$ of gender (g) in the base year (b) will reach age $(x + k + 1)$ being active in the year (f) .

In general, the values for (k) are to be found in the interval $[0, \omega - x - 1]$ and for the year (f) in the interval $[b + 1, b + \omega - x - k]$.

$p_{(g,x+k)}^{ad,f}$ is the probability that an active aged $(x + k)$ of gender (g) in the base year (b) will reach age $(x + k + 1)$ being disabled.

$q_{(g,x+k)}^{a,f}$ is the probability that an active aged $(x + k)$ of gender (g) in the base year (b) will die before reaching age $(x + k + 1)$.

$p_{(g,x+k)}^{dd,f}$ is the probability that a disabled person aged (x) of gender (g) in the base year (b) will reach age $(x + 1)$ in the same state²⁵ in the year (f) .

$q_{(g,x+k)}^{d,f}$ is the probability that a disabled person aged $(x + k)$ of gender (g) in the base year (b) will die during the year (f) .

Obviously, under the assumption of non-negative probabilities:

$$\begin{aligned} p_{(g,x+k)}^{aa,f} + p_{(g,x+k)}^{ad,f} + q_{(g,x+k)}^{a,f} &= 1 \\ p_{(g,x+k)}^{dd,f} + q_{(g,x+k)}^{d,f} &= 1 \end{aligned} \quad [1.]$$

²⁵ We do not take conversions or recoveries into account, i.e. conversion and recovery rates are null in our model.

The above probabilities are valid for contributors aged $x < x_r$; being x_r the normal retirement age. For all types of pensioners, the following probabilities also hold:

$$\begin{aligned} p_{(g,x+k)}^{a,f} + q_{(g,x+k)}^{a,f} &= 1 \\ p_{(g,x+k)}^{d,f} + q_{(g,x+k)}^{d,f} &= 1 \end{aligned} \quad [2.]$$

The yearly probability of dying for the general population for a given group aged $(x + k)$ of gender (g) in the year (f) can be calculated as a weighted average of the probabilities of dying for both collectives (active and disabled people), the weighting being the disability and active prevalence rates:

$$\overbrace{q_{(g,x+k)}^f}^{\text{General population}} = \underbrace{\vartheta_{(g,x+k)}^f}_{\text{Disability prevalence rate}} \cdot \overbrace{q_{(g,x+k)}^{dd,f}}^{\text{Disabled persons}} + \underbrace{\left(1 - \vartheta_{(g,x+k)}^f\right)}_{\text{Active prevalence rate}} \cdot \overbrace{q_{(g,x+k)}^{ad,f}}^{\text{Active persons}} \quad [3.]$$

Formula [3.] implies that the following probabilities also hold:

$$\overbrace{p_{(g,x+k)}^f}^{\text{General population}} = \frac{\overbrace{\vartheta_{(g,x+k)}^f}^{\text{Disability prevalence rate}}}{l_{(g,x+k)}^a + l_{(g,x+k)}^d} \cdot p_{(g,x+k)}^{dd,f} + \frac{\overbrace{\left(1 - \vartheta_{(g,x+k)}^f\right)}^{\text{Active prevalence rate}}}{l_{(g,x+k)}^a + l_{(g,x+k)}^d} \cdot p_{(g,x+k)}^{a,f} \quad [4.]$$

$$p_{(g,x+k)}^{a,f} = p_{(g,x+k)}^{aa,f} + p_{(g,x+k)}^{ad,f} = 1 - q_{(g,x+k)}^{a,f}$$

$$p_{(g,x+k)}^f + q_{(g,x+k)}^f = 1$$

where:

$\vartheta_{(g,x+k)}^f$: Disability prevalence rate for the group aged $(x + k)$ of gender (g) in the year (f) , which is the ratio between the number of disabled persons and the total population (active+disabled persons) aged $(x + k)$.

$p_{(g,x+k)}^{a,f}$: Probability that a person (active) aged $(x + k)$ of gender (g) will reach age $(x + k + 1)$ in the same state or as a disabled person.

It is well documented in the literature (Pitacco (2019), Ventura-Marco and Vidal-Meliá (2016)) that the mortality of disabled people contains an “extra-mortality” term and can be represented either as a specific mortality (using the appropriate numerical tables or parametric mortality laws) or via adjustments to the standard age pattern of mortality. The “extra-mortality” term is a challenging concept to model, and its overestimation could have significant implications. If the mortality of disabled individuals is underestimated, it would result in an underestimation of the disability liabilities, as disability benefits are considered “living” benefits. Conversely, if the mortality of dependent individuals is overestimated, it would lead to an overestimation of the disability liabilities.

STEP 1:

1) Liability to pensioners

1a) Liability to retirement pensioners

The group of pensioners at the reference date has already accrued the total of their pension entitlements. Consequently, the present value of the accrued-to-date pension entitlements is equal to the present value of all the pension payments that they would receive until the termination of their pension entitlements, which is generally when the individual dies.

The liability to retirement pensioners can be expressed as follows:

Expected benefits to be paid to the retirement pensioners

$$\begin{aligned}
 E_{(x,g,b)}^{Rp} &= \sum_{t=1}^{w-x-1} B_{(x+t,g,f)}^{Ar} \cdot {}_t p_{(g,x)}^f \cdot (1+r)^{-t} \\
 &= \underbrace{{}_x a_x^\lambda}_{\text{[5.]}} \\
 & \quad B_{(x,g,b)}^{Ar} \cdot \sum_{t=1}^{w-x-1} {}_t p_{(g,x)}^f \cdot F^t
 \end{aligned}$$

Being $E_{(x,g,b)}^{Rp}$ the average direct pension entitlement of a pensioner aged (x) and of gender (g) in the base year (b), $B_{(x+t,g,b+t)}^{Ar}$ is the average annual pension benefit accrued-to-date for the group of retirement pensioners aged ($x + t$) of gender (g) in a future year ($f = b + t$), ${}_t p_{(g,x)}^{b+t}$ is the probability of a retirement pensioner aged (x) of gender (g) surviving to age ($x + t$) in a future year ($f = b + t$), $F = \frac{1+\lambda}{1+r}$ is an indexation factor which depends on λ , the rate on indexation on pensions in payment, and r , the annual nominal discount rate and w is the maximum lifetime.

Pension entitlements represent the expected amount of pension payments accrued to date. In accordance with this concept, future survival rates are taken into account for the calculation of entitlements.

The element $B_{(x+t,g,f)}^{Ar}$ denotes the average annual pension level accrued-to-date. It is crucial to highlight that only future pension payments are considered in the estimation of pension entitlements. All payments made during or before the base year are excluded. Consequently, the control variable commences with the age ($x + 1$), which is one year after the base year, and concludes at age (w). In this context, as previously stated, the parameter w denotes the highest maximum lifetime considered in the calculations.

The parameter λ is used to denote the future annual adjustment factor, which reflects the indexation rules in a given country:

$$B_{(x+t,g,f)}^{Ar} = B_{(x,g,b)}^{Ar} \cdot (1 + \lambda)^t \quad \text{[6.]}$$

${}_x a_x^\lambda$ is the annuity factor, i.e. the present value of a lifetime annuity valued in the year (b) for the retirement pensioner aged (x) years of 1 monetary unit per year payable at the end of the year and growing at nominal rate (λ), and with a nominal interest rate equal to (r). Unless otherwise stated, all actuarial values are computed using these rates, and under the assumption that all payments are made regularly at the end of the year.

The annuity factor is calculated in accordance with the methodology employed in the construction of generational tables, which incorporate projected mortality trends. This methodology is not optimal for calculating the liability to pensioners. It would be more advantageous to use periodic tables for a specific cohort of pensioners. (Arnold et al., 2019).

In order to calculate entitlements, it may be advisable to include the indexation practice of the respective pension scheme in the estimation process. This may be achieved by considering those forms of indexation that are referred to as "price indexation" or "wage indexation".

Besides indexation rules, future pension levels of current retirees might also be altered due to future adjustments in the benefit formula. It would be prudent to consider the potential impact of future alterations to the benefit formula when projecting future pension benefits for current retirees.

Expected benefits to be paid to other beneficiaries (survivor benefits)

In the event of death, survivors' pensions may be payable to partners or children of old-age and disability pensioners. A widowhood pension is a pension granted to the spouses of disabled and retired workers and pensioners on their death. In general, the widowhood pension is compatible with the retirement pension (or permanent disability) to which one was entitled, provided that the maximum amount is not exceeded.

$$E_{(x,g,b)}^{wp} = B_{(x,g,b)}^{Ar} \cdot \gamma_w \cdot M_x \cdot \sum_{t=1}^{w-x} {}_t p_{(g^*,y)}^f \cdot (1 - {}_t p_{(g,x)}^f) \cdot F^t \quad [7.]$$

$$= B_{(x,g,b)}^{Ar} \cdot \gamma_w \cdot M_x \cdot ({}^\lambda a_y^a - {}^\lambda a_{xy}^a)$$

Where:

${}_t p_{(g^*,y)}^f$ is the probability of a representative individual aged (y), legal partner²⁶ of (x) (primary beneficiary) of gender (g^*) (initially, the opposite gender of g) surviving to age ($y + t$) in a future year ($f = b + t$)

The age of the retirement pensioner in the base year is denoted by ($x \geq x_r$) and that of the secondary beneficiary by (y); in this specific case – the annuitant (male) and the co-annuitant (female), but obviously other couple combinations are possible.

γ_w is the fraction of the benefit paid to the widow(er) or secondary beneficiary.

${}^\lambda a_y^a$ is the present value of a lifetime annuity valued in year (b), for the beneficiary aged (y) years of 1 monetary unit per year payable at the end of each year.

${}^\lambda a_{xy}^a$ is the present value of a lifetime annuity valued in year (b), while both partners of the couple remain alive of 1 monetary unit per year payable at the end of each year.

M_x is the probability that the beneficiary, aged (x) years, will have a spouse at the time of death (AWG, 2018; Plamondon et al., 2002).

In short, the general formula for calculating the liability to retirement pensioners can be expressed as follows:

$$E_{(x,g,b)}^{TRp} = \overbrace{E_{(x,g,b)}^{rp}}^{\text{Benefits to be paid to the retirement pensioner}} + \overbrace{E_{(x,g,b)}^{wp}}^{\text{other beneficiaries}} \quad [8.]$$

$$=$$

²⁶ Survivor's pensions may be limited to married people only or may be extended to common law partners.

$$B_{(x,g,b)}^{Ar} \cdot \overbrace{\left(\lambda a_x^a + \gamma_w \cdot M_x \cdot \left(\lambda a_y^a - \lambda a_{xy}^a \right) \right)}^{\lambda a_{(x/y)}^a}$$

$\lambda a_{(x/y)}^a$ is the present value of a life annuity with contingent survivor benefit. With this type of life annuity, a periodic payment at the end of the year is made to the primary beneficiary with an initial age of x years, which he/she receives until his/her death. From this moment his/her legal partner with initial age of y years, assuming she/he has survived until this date, will start to receive an amount calculated as a percentage (γ_w) of what the deceased primary beneficiary was receiving.

1b) Liability to disability pensioners

The methodology used in this instance is analogous to that used in the case of retirement pensioners. However, it is necessary to consider the specific mortality of this particular group of pensioners. Disabled people have a lower life expectancy than active people, but the difference in longevity tends to decrease notably as the individuals increase in age (Pitacco, 2019; Ventura-Marco and Vidal-Meliá, 2016; 2014; Plamondon et al. 2002).

In order to calculate the total liability to disability pensioners, it is necessary to apply the general formula, which can be expressed as follows:

$$E_{(x,g,b)}^{TDp} = \overbrace{E_{(x,g,b)}^{dp}}^{\text{the disability pensioner}} + \overbrace{E_{(x,g,b)}^{dwp}}^{\text{other beneficiaries}} \quad [9.]$$

$$= \overbrace{B_{(x,g,b)}^{Ad} \cdot \left(\lambda a_x^d + \gamma_w^d \cdot M_x \cdot \left(\lambda a_y^d - \lambda a_{xy}^{d,a} \right) \right)}^{\lambda a_{(x/y)}^{d,a}}$$

The elements of the above formula have the same meaning as those of retirement, with the exception that the superscript d indicates disability pensioners, which implies the use of the specific mortality tables for the disabled in the case of direct benefits.

It is crucial to highlight that the disability benefit is contingent upon the degree of disability ascribed to the pensioner (in Spain, there are three degrees: total, absolute and severe disability). It would have been possible to develop a separate formula for each of the recognised degrees of disability.

1c) Liability to survivorship pensioners and other types of pensioners (in favour of family members)

In the case of widowhood and orphanhood pension schemes and pensions in favour of family members, the entitlement is terminated in certain cases once a certain age has been reached. Therefore, it would be necessary to distinguish between the group of pensions in force at the reference date, which are of a lifelong nature (mainly widowhood, and special cases for orphanhood and in favour of family members), and the rest (temporary benefits).

It is evident that the most significant liability is the provision of lifetime annuities to widows and other dependents, $E_{(x,g,b)}^{sp}$:

$$E_{(x,g,b)}^{sp} = \left(\overbrace{B_{(x,g,b)}^{Aw}}^{\text{widowhood}} + \underbrace{B_{(x,g,b)}^{Ao}}_{\text{orphanhood}} + \overbrace{B_{(x,g,b)}^{Afm}}^{\text{family members}} \right) \cdot \lambda a_x^a \quad [10.]$$

Where:

$B_{(s,g,b)}^{Aw}$ is the average annual pension benefits for the group of widowhood pensioners aged (x) of gender (g) in the base year (b),

$B_{(s,g,b)}^{Ao}$ is the average annual pension benefits for the group of orphanhood pensioners aged (x) of gender (g) in the base year (b),

$B_{(s,g,b)}^{Afm}$ is the average annual pension benefits for the group of family member pensioners aged (x) of gender (g) in the base year (b)

It is important to consider the value of temporary pensions, $E_{(x,g,b)}^{sp*}$, in the event that the age of the beneficiary is below the maximum age for receiving the orphan's pension (x_0) or the family benefit (x_{fm}):

$$E_{(x,g,b)}^{sp*} = B_{(x,g,b)}^{Ao} \cdot \ddot{a}_{(x:x_0-x)}^a + B_{(x,g,b)}^{Afm} \cdot \ddot{a}_{(x:x_{fm}-x)}^a \quad [11.]$$

Where:

$\ddot{a}_{(x:x_0-x)}^a$ is the present value of a temporary annuity valued in year (b) for an orphanhood pensioner aged (x) years of 1 monetary unit per year. As indicated by the subscript, the payments cease at the end of ($x_0 - x$) years, or, if sooner, at the time of the pensioner's death.

$\ddot{a}_{(x:x_{fm}-x)}^a$ is the present value of a temporary annuity valued in year (b) for a family member pensioner aged (x) years of 1 monetary unit per year. As indicated by the subscript, the payments cease at the end of ($x_{fm} - x$) years, or, if sooner, at the time of the pensioner's death.

A further distinctive feature is that the death of the beneficiary does not entail the possibility of a total or partial reversal of benefits, with the exception of the case of total orphanhood.

2) Liability to contributors

2a) Liability to contributors by the retirement and survivor contingencies (after retirement)

We need to estimate, the average pension entitlements for a group of contributors aged (x), of gender (g), who is likely to retire²⁷ at the age of (x_r) in a future year ($f = b + x_r - x$):

$$E_{(x,g,x_r,f)}^{TRc} = B_{(x,g,x_r,b+x_r-x)}^{Ar} \cdot \overbrace{\left(\ddot{a}_{x_r}^a + \gamma_w \cdot M_{x_r} \cdot \left(\ddot{a}_y^a - \ddot{a}_{x_r y}^a \right) \right)}^{\ddot{a}_{(x_r/y)}^a} \quad [12.]$$

Where:

$B_{(x,g,x_r,b+x_r-x)}^{Ar}$ is the expected average annual pension benefits for the group of contributors aged (x), of gender (g), who is likely to retire at the age of (x_r) in a future

²⁷ It would have been beneficial to consider the heterogeneous retirement behaviour of individuals. Typically, retirement ages vary between males and females. Additionally, different retirement behaviour may be observed within a cohort.

year $f = (b + x_r - x)$. The determination of this element is based on the projected benefit obligation (PBO) approach, which incorporates salary increases over time.

The traditional definition of the normal retirement age is considered, that is, the first age at which retirement can occur without any reduction in the benefits calculated according to the retirement benefit formula.

In general, there are two approaches to calculating the initial benefit to which a current contributor is entitled (Eurostat, 2020a). One approach is based on the assumption that all contributors have a homogeneous contribution career (homCC). This approach differs from previous methods in that future pension levels for current contributors are not estimated using past contribution data. Instead, future retirement benefits are approximated based on current pension levels. This approach has the advantage of significantly limiting the input data required for estimations, allowing for partially disregarding data on past contributions.

In the absence of or in the case of limited contribution data, the homCC approach is recommended. Alternatively, the heterogeneous contribution careers (hetCC) approach may be considered. This approach requires comprehensive data on past contributions. The hetCC approach has the advantage of reflecting cohort-specific employment careers. Consequently, its application may result in more accurate estimations than the homCC approach.

The average liability for the group of contributors aged (x), of gender (g), who are likely to retire at the age of (x_r) in a future year $f = (b + x_r - x)$ valued in the base year (b) can be calculated as follows:

$$E_{(x,g,x_r,f,b)}^{TRC} = \frac{\delta_{(g,x,b)}^{x_r} \cdot \underset{x_r}{p}_{(g,x)}^{aa,f} \cdot (1+r)^{x-x_r} \cdot B_{(x,g,x_r,f,b)}^{Ar} \cdot \overbrace{\left(\underset{x_r}{a}_{x_r}^a + \gamma_w \cdot M_{x_r} \cdot \left(\underset{x_r}{a}_y^a - \underset{x_r}{a}_{x_r y}^a \right) \right)}^{\lambda a_{(x_r/y)}^a}}{[13.]}$$

$\delta_{(g,x,b)}^{x_r}$ is the accrual factor (prorate accrual factor), defined as the proportion of the total pension entitlement that has been accrued by a beneficiary of gender (g) and age (x) at the end of year (b), who is likely to retire at the age of (x_r) in a future year $f = (b + x_r - x)$. The quotient is calculated by dividing the number of years contributing to the pension until the reference year by the expected total number of years contributing until the acquisition of the condition of pensioner (or disabled person, in the case of a disability scheme).

$$\delta_{(g,x,b)}^{x_r} = \frac{x - x_e}{x_r - x_e} \quad [14.]$$

Being x_e the age of entry into the system.

$\underset{x_r}{p}_{(g,x)}^{aa,f}$ is the gender- g -specific probability that a contributor aged (x) years will reach age (x_r) being active in a future year ($b + x_r - x$). The contributor could reach age (x_r) in a state of disability in a future year ($b + x_r - x$). This includes the decreases due to death and disability associated within $[x, x_r - 1]$ ages at the interval time $[b, b + x_r - x - 1]$, with no possibility of a return to active life being considered.

However, where possible, heterogeneous retirement ages should be taken into account.

The presence of disability among active contributors precludes their eligibility for retirement benefits. Consequently, the associated costs are reduced. However, the extent of the reduction may be greater or less than the cost of disability, depending on the legislative benefits associated with the disability pension.

2b) Liability to contributors by disability and survivor contingencies (after retirement)

The average liability by disability for the group of contributors aged (x), of gender (g), who are likely to retire at the age of (x_r) in a future year $f = (b + x_r - x)$ valued in the base year (b) can be calculated as follows:

$$E_{(x,g,x_r,f,b)}^{Ds} = \delta_{(g,x,b)}^{x_r} \cdot \sum_{t=1}^{x_r-x} \overbrace{t-1 p_{(g,x)}^{aa,f} \cdot p_{(g,x+t-1)}^{ad,f}}^{} \cdot (1+r)^{-t} \cdot B_{(x+t,g,f)}^{cd} \cdot \ddot{a}_{(x+t/y+t)}^{\lambda,d,a} \quad [15.]$$

Where:

$B_{(x+t,g,f)}^{cd}$ is the expected average annual disability benefits a for the group of contributors aged ($x + t$) of gender (g) in a future year ($f = b + t$) if disability occurs during age ($x + t$).

$t-1 p_{(g,x)}^{aa,f}$ is the gender- g -specific probability that a contributor aged (x) years will reach age ($x + t - 1$) being active in a future year ($f = b + t - 1$).

$p_{(g,x+t-1)}^{ad,f}$ is the probability that a contributor of gender (g) aged ($x + t - 1$) will enter into a state of disability this year ($f = b + t - x$).

${}_t p_{(g,x)}^{ad,f}$ is the probability that a contributor of gender (g) aged (x), initially active, reaches age ($x + t$) in the disability state in a future year ($f = b + t$). It can occur because the individual remains active for the next ($n - 1$) years, becomes disabled at age ($x + n - 1$) and then lives as an invalid for the number of years remaining in the interval ($n < t$).

$\ddot{a}_{(x+t/y+t)}^{\lambda,d,a}$ = ($\ddot{a}_{x+t}^{\lambda,d} + \gamma_w^d \cdot M_x \cdot (\ddot{a}_{y+t}^{\lambda,a} - \ddot{a}_{(x+t,y+t)}^{\lambda,d,a})$) is the present value of 1 monetary unit per year payable in advance of a life annuity with contingent survivor benefit. With this type of life annuity, a periodic payment is made to the primary annuitant (disabled, with initial age of $x + t$ years), which he/she receives until his/her death. From this moment his legal partner (active with initial age of $y + t$ years), assuming she/he has survived until this date, will start to receive an amount calculated as a percentage (γ_w^d) of what the deceased annuitant was receiving.

$\ddot{a}_{x+t}^{\lambda,d}$ is the present value of a lifetime annuity for the disabled person aged ($x + t$) years of 1 monetary unit per year payable in advance and growing at a nominal rate (λ), and with a nominal interest rate equal to (r). This annuity is based on disabled life mortality.

$\ddot{a}_{(x+t,y+t)}^{\lambda,d,a}$ is the present value of a lifetime annuity while both members of the couple remain alive of 1 monetary unit payable in advance. The primary annuitant is disabled, with an initial age of $x + t$ years, and the legal partner is active with initial age of $y + t$ years.

As previously stated in the context of the liability to disability pensioners, it would have been possible to develop a separate formula for each of the recognised degrees of disability in the case of the liability to contributors.

2c) Liability to contributors by survivorship contingencies (before retirement)

Mortality among contributors eliminates the retirement benefit obligation; but mortality prior to retirement may create another form of pension obligation; for instance, the commencement of periodic payments to a surviving spouse for life.

The average liability by survivorship contingencies (before retirement) for the group of contributors aged (x), of gender (g), who are likely to retire at the age of (x_r) in a future year $f = (b + x_r - x)$ valued in the base year (b) can be calculated as follows:

$$E_{(x,g,x_r,f,b)}^{Sc} = \delta_{(g,x,b)}^{x_r} \cdot M_x \cdot \sum_{t=1}^{x_r-x-1} {}_t p_{(g,y)}^f \cdot \overbrace{({}_{t-1} p_{(g,x)}^{aa,f} - {}_t p_{(g,x)}^{aa,f})}^{t-1/q_{(g,x)}^{aa,f}} \cdot F^t \cdot B_{(x+t,g,b)}^{sd} \cdot {}_x \ddot{a}_{y+t}^a \quad [16.]$$

Where:

$B_{(x+t,g,b)}^{sd}$ is the expected average annual survivor's benefits for the group of contributors aged ($x + t$) of gender (g) in a future year ($f = b + t$) if the death occurs during age ($x + t - 1$).

$t-1/q_{(g,x)}^{aa,f}$ is the probability that an active contributor of initial age (x) will die in the course of year (t) or at age ($x + t - 1$) is the so-called deferred probability of death.

STEP 2:

Having described the simplified approach to calculating average pension entitlements for the several groups of pensioners and current contributors, it is now necessary to consider the number of pensioners ($LP_{(x+k,g,b)}^{\square}$) and contributors ($LC_{(x+k,g,b)}^{\square}$) by age and gender at the end of year b . This will enable the total pension liabilities of the system to be calculated.

The total liability to pensioners at the end of year b is calculated as follows:

$$E_{(b)}^P = \sum_{g=1}^2 \sum_{k=0}^{w-x} \left(\overbrace{E_{(x+k,g,b)}^{TRp} \cdot LP_{(x+k,g,b)}^{Rp}}^{\text{Retirement}} + \underbrace{E_{(x+k,g,b)}^{TDp} \cdot LP_{(x+k,g,b)}^{Dp}}_{\text{Disability}} + \overbrace{E_{(x+k,g,b)}^{Sp} \cdot LP_{(x+k,g,b)}^{Sp}}^{\text{Survivorship+}} \right) \quad [17.]$$

For $g = 1$, males and $g = 2$ females.

while the total liability to contributors at the end of year b is:

$$E_{(b)}^C = \sum_{g=1}^2 \sum_{k=0}^{x_r-x_e-1} \left(\overbrace{E_{(x_e+k,g,x_r,f,b)}^{TRc}}^{\text{Retirement}} + \underbrace{E_{(x_e+k,g,x_r,f,b)}^{Ds}}_{\text{Disability}} + \overbrace{E_{(x_e+k,g,x_r,f,b)}^{Sc}}^{\text{Survivorship+}} \right) \cdot LC_{(x_e+k,g,b)}^{\square} \quad [18.]$$

APPENDIX 2: MAIN FORMULAS USED TO TRANSFORM TABLE 29 INTO AN ABS AND COMPILE ITS ASSOCIATED INCOME STATEMENT (IS)

This Technical appendix shows the main formulas used to transform Table 29 into an ABS and compile its associated income statement (IS).

The change in net worth can (δNW_t^S) be detailed as follows:

$$\begin{aligned}
\underbrace{\delta NW_t^S = NW_t^S - NW_{t-1}^S}_{\text{Changes in net worth}} &= \delta A_t^S - \delta V_t^S = \left(\overbrace{A_t^S - A_{t-1}^S}^{\text{Assets}} \right) - \left(\overbrace{V_t^{TS} - V_{t-1}^{TS}}^{\text{Total Liabilities}} \right) \\
&= \\
&\quad \underbrace{\overbrace{\delta BF_t^S}_{\text{Fund}} + \overbrace{\delta TCA_t^S}_{\text{Contribution}}}_{\text{Assets}} - \underbrace{\overbrace{\delta V_t^S}_{\text{Total Liabilities}} - \overbrace{\delta FL_t^S}_{\text{Financial}}}_{\text{Pension}}
\end{aligned} \tag{19.]$$

1.- Change in the fund/financial asset (δBF_t^S):

$$\delta BF_t^S = C_t^S + \overline{NCR} SC_t^S - PT_t^S - OO_t^S + D_t^S + \delta FL_t^S \tag{20.]$$

where $\overline{NCR} SC_t^S$ is the income from sponsor contributions for non-contributory rights (NCRs), PT_t^S is total pension disbursements, OO_t^S is other outflows, D_t^S is the net return on funded capital, and δFL_t^S is the value of the change in financial loans.

2.-Change in the total contribution asset (δtCA_t^S):

$$\begin{aligned}
\delta TCA_t^S &= TCA_t^S - TCA_{t-1}^S = \overbrace{\delta C_t^S}^{\text{Revenue effect}} + \underbrace{\delta TD_t^S}_{\text{Turnover duration effect}} \\
&= \\
&\quad \overbrace{(TC_t^S - TC_{t-1}^S)}^{\delta TC_t^S} \cdot \frac{(TD_{t-1}^S + TD_t^S)}{2} + \underbrace{\frac{(TC_t^S + TC_{t-1}^S)}{2} \cdot (TD_t^S - TD_{t-1}^S)}_{\delta TD_t^S}
\end{aligned} \tag{21.]$$

where TCA_t^S is the total contribution asset (contribution asset plus public contribution asset, see formulas 28 and 29), TC_t^S is the total income from contributions (income from sponsor contributions for non-contributory rights plus employer and employee actual social contributions) and TD_t^S is the turnover duration (see formula 28).

3.-Change in pension liability (δV_t^S):

$$\delta V_t^S = \overbrace{C_t^{*S}}^{\text{Row 2}} - \underbrace{PT_t^S}_{\text{Row 4}} + \overbrace{\delta AC_t^S}_{\text{Row 3}} + \underbrace{\delta SS_t^S}_{\text{Row 7}} + \overbrace{\delta R_t^S}_{\text{Row 8}} + \underbrace{\delta Ov_t^S}_{\text{Row 9}} \tag{22.]$$

where C_t^{*S} is employer and employee actual social contributions plus the property income earned or imputed in the schemes, δAC_t^S is other (actuarial) changes of pension entitlements in social security pension schemes, δSS_t^S is the change in entitlements due to negotiated changes in the scheme's structure, δR_t^S is changes in entitlements due to revaluations, and δOv_t^S is changes in entitlements due to other changes in volume.

4.-Change in financial liabilities:

$$\delta FL_t^S = NL_t^S + PL_t^S \tag{23.]$$

where δFL_t^S is the value of the change in financial loans, NL_t^S is loans for the period, and PL_t^S is the payment of loans for the period.

5.-In the case of Spain, the (new) pension liability is estimated based on the values published by the INE (V_t^{T29}), for the base scenario and the alternatives scenarios 1 and 2, taking into account the change (Δdr) between the new discount rate (G) and the original value used for estimating pension entitlements (r), and the change (Δir) between the new indexation rate of pensions in payment (λ^*) and the original value (λ). It can be expressed as:

$$\hat{V}_t^S \Big|_{ir=\lambda^*\%} = \overbrace{V_t^{T29} + \Delta dr \cdot \frac{\partial V_t^{T29}}{\partial dr} \Big|_{dr=r\%}}^{\hat{V}_t^S \Big|_{dr=G\%}} + \Delta ir \cdot \frac{\partial \hat{V}_t^S \Big|_{dr=G\%}}{\partial ir} \Big|_{ir=\lambda\%} \quad [24.]$$

where $\Delta dr = G\% - r\%$ and $\Delta ir = \lambda^*\% - \lambda\%$.

A two-stage numerical derivation procedure and a sensitivity analysis are applied from a polynomial adjustment of the values published by the INE for each of the different scenarios taken from Table 29.

Our procedure is superior to Rauh's (2017) method of approximating the change in value of a liability when the discount interest rate changes. We calculate the change in value of the liability simultaneously for two parameters (the discount rate and the indexation rate of pensions in payment), rather than relying solely on the duration and convexity of the pension entitlements.

These are parameters that allow for an approximation of the change in value of a liability when the interest rate used to discount that liability is changed. Measures of the duration and convexity of pension benefits help to understand the impact of changes in discount rates on the present value of liabilities (Novy-Marx and Rauh, 2011).

Table 29 disclosures oblige countries to reveal pension liabilities ($V_{t,r}^S$) based on different discount rate assumptions: 1 percentage point higher ($V_{t,r+0.01}^S$) and 1 percentage point lower ($V_{t,r-0.01}^S$). The duration, (D_t^S) expressed in years, can be calculated as follows:

$$D_t^S = \frac{V_{t,r+0.01}^S - V_{t,r-0.01}^S}{2 \cdot V_{t,r}^S \cdot 0.01} \quad [25.]$$

The convexity, (Cvx_t^S) can be calculated as:

$$Cvx_t^S = \frac{V_{t,r+0.01}^S + V_{t,r-0.01}^S - 2 \cdot V_{t,r}^S}{V_{t,r}^S \cdot 0.01^2} \quad [26.]$$

Duration is not a perfect measure of a liability's change in value because it assumes a linear relationship when in reality it exhibits a sloped or 'convex' shape. A liability has positive convexity if its duration increases as the discount rate declines. A liability with positive convexity will experience larger increases in value due to a decline in discount rates rather than decreases in value due to an increase in discount rates.

To estimate the value of the liability, taking into account the change (Δdr) between the new discount rate (G) and the original value used for estimating pension entitlements (r), is calculated as follows:

$$\hat{V}_{t,G}^S = V_{t,r}^S \cdot (1 + (-D_t^S \cdot (\Delta dr) + \frac{1}{2} \cdot Cvx_t^S \cdot (\Delta dr)^2)) \quad [27.]$$

As previously mentioned, we must calculate the change in liability value for two parameters simultaneously. Therefore, we should use formula 24 instead of 27.

6.-The contribution asset (CA_t^S):

$$CA_t^S = TD_t^S \cdot C_t^S = (A_p^S - A_c^S) \cdot C_t^S \quad [28.]$$

where C_t^S is the income from ordinary contributions, TD_t^S is the turnover duration, A_p^S is the weighted average age of the pensioners, and A_c^S is the weighted average age of the contributors.

To preserve solvency, the asset counterpart that underwrites the liabilities caused by SBs²⁸ can appear as a special type of public contribution asset (PCA_t^S), financed by general government revenues.

It is also straightforward to obtain the value of the system's public contribution asset. It is the product of the system's turnover duration and the value of the public contributions made in the period by the sponsor in order to pay for the minimum pension benefits (MPBs) in force:

$$PCA_t^S = TD_t^S \cdot {}^{SB}SC_t^S = (A_p^S - A_c^S) \cdot {}^{SB}SC_t^S \quad [29.]$$

where ${}^{SB}SC_t^S$ is the income from sponsor contributions for SBs.

7.-The system's solvency indicator is the ratio between its assets and liabilities (SI_t^S):

$$SI_t^S = \frac{(BF_t^S + CA_t^S + PCA_t^S)}{\left(\frac{V_t^S = Pension\ obligations}{V_t^c + V_t^p + {}^{NCR}V_t^c + {}^{NCR}V_t^p + FL_t^S} \right)} \quad [30.]$$

Where, CA_t^S is the contribution asset, BF_t^S is the buffer fund, PCA_t^S is the public contribution asset, V_t^c is the liability to contributors, V_t^p is the liability to pensioners, ${}^{NCR}V_t^c$ is the liability to contributors due to non-contributory rights (NCR), ${}^{NCR}V_t^p$ is the liability to pensioners for NCRs, and FL_t^S is the amount of the financial loans.

It is denoted as primary (PSI_t^S) if the public contribution asset is excluded (PCA_t^S). At the valuation date the balance ratio value will be 1 if the net worth is zero or greater.

$$PSI_t^S = \frac{(BF_t^S + CA_t^S)}{(V_t^c + V_t^p + {}^{NCR}V_t^c + {}^{NCR}V_t^p + FL_t^S)} \quad [31.]$$

²⁸ The commitments deriving from supplementing benefits (SB) to achieve the minimum pension benefit (MPB) in force.

A solvency ratio of 1 indicates that a system has enough assets to meet all scheduled accrued benefits payable, while a ratio of 0.5 means that only half of the promised benefits can be covered.

8.-Another valuable indicator of the system's sustainability is the required growth rate (G_t^*), which actuarially matches the pension system's liabilities with its assets at the valuation date:

$$[BF_t^S + CA_t^S + PCA_t^S] \underset{\sim}{G}_t^* [V_t^c + V_t^p + {}^{NCR}V_t^c + {}^{NCR}V_t^p + FL_t^S] \quad [32.]$$

To put it another way, this is the expected discount rate necessary to achieve the system's solvency ($SI_t^S = 1$). The required growth rate is denoted as primary (G_t^{**}) if the public contribution asset is excluded from the calculation.

$$[BF_t^S + CA_t^S] \underset{\sim}{G}_t^{**} [V_t^c + V_t^p + {}^{NCR}V_t^c + {}^{NCR}V_t^p + FL_t^S] \quad [33.]$$