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Exact Allocative and Technical Inefficiency Using the Normalized Quadratic Cost System

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Abstract

In this paper we have built a theoretical model using a normalized quadratic cost system to obtain expressions for actual input demand and cost as functions of three components: frontier, allocative and technical inefficiency. We have used the shadow prices approach in the line of exact decomposition that allows us to solve Greene's problem. Using the normalized quadratic cost system has permitted to isolate not only allocative inefficiency but also the technical one, as simple functions of both parameters and variables. The model allows us to obtain individual and time varying technical and allocative inefficiency measures when a panel data is available. This model has been applied to cargo handling in Spanish ports.

Keywords: Normalized Quadratic Cost System, Allocative Inefficiency, Technical Inefficiency, the Greene problem, Exact Decomposition, Ports

JEL classification: D21, C33, L92

Introduction

The estimation of efficiency through the stochastic cost frontier approach does not permit the correct decomposition of inefficiency into its technical and allocative components. This occurs because the error component that captures the allocative inefficiency and that which represents total inefficiency is not statistically independent and, therefore, parameter estimates are inconsistent (Greene, 1980). This is known in the literature as “Greene’s problem” (Bauer, 1990), and induces distortions on the technical characteristics and on the identification of the productivity components when these are analysed through stochastic costs frontier models.

The attempts to solve Greene’s problem based on trans-logarithmic stochastic frontiers, specify the relations between the error components of the cost frontier and the share equations. This approach to Greene’s problem has two disadvantages. First of all, the impact of inefficiency on shares and costs cannot depend on the levels of outputs and/or on the prices of inputs. Secondly, errors in the chosen specification bring a bias into the estimations obtained.

The shadow price approach is an alternative procedure where inefficiency is captured through a set of parameters that are jointly estimated with those characterizing technology. On one hand, input oriented technical inefficiency has been modelled by Atkinson and Cornwell (1994a) by scaling the input vector in the cost function through a parameter that is the inverse of the input oriented technical efficiency index proposed by Farrell (1957). Besides, they modelled the output oriented technical inefficiency using a parameter that indicates how the observed outputs are deviated from maximum output levels. On the other hand, the strategy followed for the analysis of allocative inefficiency has been the definition of unobserved input prices, named shadow prices, for which the observed input combination would be optimal. Along this line, Atkinson and Cornwell (1994b) estimated cost inefficiency in a panel data context using a translog cost system.

Following the approach designed to analyse allocative inefficiency, Kumbhakar (1997) proposed a model that incorporates both technical and allocative inefficiency in a translog cost system using input oriented technical inefficiency. In this way, he deduced the exact relation between allocative inefficiency, and its impact on costs and the share equations of

each input, in a theoretically consistent fashion for a trans-logarithmic specification. Greene's problem is solved theoretically using the shadow input prices approach. The empirical applications of this model are scarce, and they all use the translog-based equations system. Among these, Maietta (2000) introduces technical inefficiency as fixed individual effects and allocative inefficiency as parameters that are functions of time trend and individual dummy variables. Recently, Kumbhakar and Tsionas (2005) have estimated this model using the Bayesian approach in a translog cost system.

The main objective of this paper is to build a model to show separately, in the line of exact decomposition, the three components of actual cost and input quantities: frontier, allocative inefficiency impact and technical inefficiency effect, using the shadow price approach applied to the normalized quadratic cost function (NQCF).

This model presents three novelties regarding previous work. First, we have modelled allocative and technical inefficiency exactly for a normalized quadratic cost function. This function presents advantages regarding the usual translogarithmic specification. Thus, deviations of optimal input demands and cost are simple functions of allocative and technical inefficiency. Besides, this specification belongs to the family of flexible functions and has the advantage of being defined for zero output levels, a relevant property when dealing with multioutput activities because observations usually include zeroes for some firms' products during some periods, and because the analysis of economies of scope requires the valuation of outputs in zero. The second novelty is to model separately technical inefficiency as function of the model parameters, input prices and outputs. This exercise is done both for inputs demands and cost function. Thus, the Greene problem has been solved using a normalized quadratic cost function.

Thirdly, to examine empirically the properties of the theoretical model, it is applied to the analysis of cargo handling operations in Spanish ports during the nineties. This is a particularly attractive exercise as this activity, a key component in the logistic chain, has been described as traditionally inefficient in the specialized literature, as a consequence of the control exercised by local labour monopolies in practically all ports around the world. To the best of our knowledge, there is no published analysis of cargo handling activities in ports including both technical and allocative inefficiency.

The rest of the paper is structured as follows. Section 1 describes the procedure based on the shadow cost function approach under a quadratic specification in order to model the allocative inefficiency first, and then extended to include the technical inefficiency. Besides, we explain the procedure to obtain a specific firm and time varying measurement of technical and allocative inefficiency. In Section 2 the application to cargo handling in Spanish ports is presented. Finally, in Section 3 the main conclusions of this work are presented.

1. The model

1.1 Farrell's efficiency index

Farrell (1957) proposed an input oriented cost efficiency measure defined as the ratio between the minimum production cost for a given output level and the actual expense, i.e.

$$0 \leq CE = \frac{C^*}{C^a} = \frac{C(W, Q, t)}{C^a} \leq 1, \quad (1)$$

where $C^* = C(W, Q, t) = WX^*$ represents the minimum expenditure necessary to produce output vector $Q = (Q_1, \dots, Q_n)$ for a given input price vector $W = (W_1, \dots, W_m)$, and $X^* = (X_1^*, \dots, X_m^*)$ is the vector of optimal input demands. $C^a = WX$ represents actual expenditure. This cost efficiency measure indicates the minimum proportion of the observed expenditure that is sufficient to produce Q at the observed input prices.

Besides definition (1), Farrell (1957) proposed its decomposition as the product of a technical efficiency index and an allocative efficiency index. The former is defined as the ratio between the technically efficient cost (C_{te}) and the observed expenditure (C^a), i.e.

$$TE = \frac{C_{te}}{C^a} \quad (2)$$

The allocative efficiency index is defined as the ratio between the minimum cost and the technically efficient cost, i.e.

$$AE = \frac{C^*(W, Q, t)}{C_{te}}, \quad (3)$$

such that Farrell's index in equation (1) happens to be

$$CE = TE \times AE \quad (4)$$

This means that quantifying both types of inefficiencies separately requires the estimation of both the minimum cost (free of inefficiencies) and the cost with the technical inefficiency only.

1.2- Allocative inefficiency through the shadow cost function

This section brings in the effect of allocative inefficiency assuming that the agent behaves in a technically efficient manner, denoted by the *te* sub index.

Let $X_{te} = (X_{1,te}, \dots, X_{m,te})$ be a vector of m inputs and W the corresponding price vector, with W^S as the shadow price vector for which the combination of actual inputs is allocatively efficient. Therefore, the marginal productivity ratio is equal to the shadow prices ratio for each pair of inputs, i.e.

$$\frac{f_i(X)}{f_k(X)} = \frac{W_i^S}{W_k^S} \quad (5)$$

where $f_i(X_{te})$ is the marginal product of input i . Toda (1976), Atkinson and Halvorsen (1984, 1990) and Eakin (1993) suggest that relative shadow prices be defined as a multiplicative parametric correction of relative actual prices, that is

$$\frac{f_i(X)}{f_k(X)} = \frac{W_i}{W_k} \varepsilon_i \quad (6)$$

where ε_i represents a specific parameter that indicates how the relative actual prices between input i and k deviates from the relative shadow price ratio (obviously $\varepsilon_i \geq 0$, $i \neq k$). Thus, the measures of estimated allocative efficiency are in relation to input k , although, as Atkinson and Cornwell (1994a) point out, the choice of the reference input has no effect on the log likelihood.

If $\varepsilon_i = 1$, then the relative shadow price is the same as its relative actual price and, therefore, the chosen combination of production factors is allocatively efficient. On the other hand, if $\varepsilon_i < 1$, then the relative shadow price is lower than its relative actual price, and then the actual demand of input i exceeds the efficient quantity. On the contrary, if $\varepsilon_i > 1$, the producer has chosen a quantity that is below the efficient quantity.

As pointed out by Kumbhakar (1992), it is important to include temporal variability when defining the parameters that model the distortions caused by the allocative inefficiency. As cost reductions in time can be caused both by technical change and by variations in allocative efficiency, this procedure avoids possible bias in the measure of technical change. If the number of periods observed for each firm is sufficiently large, it would be possible to estimate consistently the parameter that measures such price distortion along with its corresponding variability (Atkinson and Cornwell, 1994b). Along this line, and using panel data, Maietta (2000) estimates parameters for allocative inefficiency as a function of individual dummies and a time trend.

Therefore, the parameters that link market prices with their corresponding shadow prices are specified here accounting for variations in time and firms. The parameters ε_{ift} must be non negative, so we impose this condition in the following way

$$\varepsilon_{ift} = \left(1 + \eta_{if} + \eta_{ift}\right)^2 = 1 + \Omega_{ift}; i \neq k \quad (7)$$

where Ω_{ift} is the deviation of the relative price of input i from the relative shadow price at period t of firm f . By definition, $-1 \leq \Omega_{it} < \infty$. In what follows, the time and firm sub-index will be suppressed to simplify notation.

With this reformulation of the conditions for minimising costs in terms of shadow prices, the technically efficient combination of inputs, X_{te} , is the solution to a problem of cost minimisation that considers shadow prices, W^S , which differs from the solution chosen by the producer, where the prices that explain his decision are market prices, W . Formally, the shadow cost function corresponds to

$$C_{te}(W^S, Q) = \min_x \left\{ W^S X_{te} / F(X_{te}, Q) = 0 \right\} = \sum_{i=1}^m W_i^S X_{i,te}(W^S, Q) \quad (8)$$

where $C_{te}(W^S, Q)$ is the minimum cost to obtain the output vector $Q=(Q_1, \dots, Q_n)$ when the price vector is W^S , and $F(X_{te}, Q)=0$ represents an optimum use of technology.

Let us define C_{te} as the expenditure under technical efficiency, which is the sum of the actual expenses on each technically efficient input. Using the definition of shadow prices given by (6) and (7), we obtain

$$C_{te} = \sum_{i=1}^m W_i X_{i,te}(W^S, Q, t) = C_{te}(W^S, Q, t) - \sum_{i \neq k} \Omega_i W_i X_{i,te}(W^S, Q, t) \quad (9)$$

Expression (9) decomposes C_{te} into two terms. The first is the shadow cost function $C(W^S, Q, t)$ that indicates the minimum cost of producing the vector of outputs (Q), given the input shadow price vector (W^S). The second term represents the difference between C_{te} and the minimum cost assessed with shadow prices. This second component will be zero only if there is allocative efficiency.

To continue with the analysis, we have to adopt a concrete functional form that, in our case, is the normalized quadratic shadow cost function (NQSCF). Let the NQSCF be

$$c_{te}(w^S, Q, t) = \alpha_0 + \sum_{i \neq k} \alpha_i w_i^S + \sum_{k=1}^n \alpha_k Q_k + \frac{1}{2} \sum_{i \neq k} \sum_{j \neq k} \alpha_{ij} w_i^S w_j^S + \frac{1}{2} \sum_{r=1}^n \sum_{l=1}^n \alpha_{rl} Q_r Q_l + \sum_{i \neq k} \sum_{r=1}^n \alpha_{ir} w_i^S Q_r + \sum_{i \neq k} \alpha_{it} w_i^S t + \sum_{r=1}^n \alpha_{rt} Q_r t + \alpha_t t + \alpha_{tt} t^2 \quad (10)$$

where $c_{te} = C_{te} / W_k$ and $w^S = (W_1^S / W_k, \dots, W_m^S / W_k)$. The price of input k has been used both to build the relative prices and to normalize the cost function to impose homogeneity of degree one. The normalized variables are expressed in lower-case letters.

In (10), along with the variables defined above, we have added a time trend as a *proxy* variable representing technical change that interacts with output levels and the prices of inputs to account for possible biases.

We have to remember that the quantity of input i that is efficient for the shadow prices can be obtained by applying Shephard's lemma to the shadow cost function defined in (10), i.e.

$$X_{i,te}(w^S, Q, t) = \frac{\partial c_{te}(w^S, Q, t)}{\partial w_i^S} = \alpha_i + \frac{1}{2} \sum_{j \neq k} \alpha_{ij} (1 + \Omega_j) w_j + \sum_{r=1}^n \alpha_{ir} Q_r + \alpha_{it} t \quad (11)$$

where w_j is the normalized actual price of input j .

Expression (11) can be decomposed into two terms,

$$X_{i,te}(w^S, Q, t) = X_i^*(w, Q, t) + X_i^{al}(w, \Omega) \quad (12)$$

The first component of (12) indicates the optimum level of input i , given the actual prices, i.e.

$$X_i^*(w, Q, t) = \alpha_i + \frac{1}{2} \sum_{j \neq k} \alpha_{ij} w_j + \sum_{r=1}^n \alpha_{ir} Q_r + \alpha_{it} t \quad (13)$$

The second component of (12) represents the impact of the allocative inefficiency on the demand for input i , that is

$$X_i^{al}(w, \Omega) = \frac{1}{2} \sum_{j \neq k} \Omega_j \alpha_{ij} w_j \quad (14)$$

Expression (14) shows the effect of allocative inefficiency on input demand i in a theoretically exact fashion. The effect is expressed as a function of the observed input prices, the parameters that determine the relationship between inputs (complements or substitutes) and the pattern of allocative inefficiency. The sign of (14) indicates whether allocative inefficiency leads to an excessive use or a saving of input i in comparison with efficient levels.

The decomposition of the actual technically efficient cost in equation (9) has been expressed in terms of shadow prices that are unknown to the researcher. Nonetheless, as shadow prices can be expressed as a parametric correction of prices, we can transform expression (9) in terms of actual input prices. For that, introducing (7) into (6), and using (10), we obtain the following expression for the normalized version of (9)

$$\begin{aligned}
c_{ie} = c_{ie}(w^S, Q, t) - \sum_{i \neq k} \Omega_i w_i X_{i,te}(w^S, Q, t) = \alpha_0 + \sum_{i \neq k} \alpha_i (1 + \Omega_i) w_i + \sum_{r=1}^n \alpha_r Q_r + \\
\frac{1}{2} \sum_{i \neq k} \sum_{j \neq k} \alpha_{ij} (1 + \Omega_i)(1 + \Omega_j) w_i w_j + \frac{1}{2} \sum_{r=1}^n \sum_{l=1}^n \alpha_{rl} Q_r Q_l + \sum_{i \neq k} \sum_{r=1}^n \alpha_{ir} (1 + \Omega_i) w_i Q_r + \\
\sum_{i \neq k} \alpha_{it} (1 + \Omega_i) w_i t + \sum_{r=1}^n \alpha_{rt} Q_r t + \alpha_t t + \alpha_{tt} t^2 - \sum_{i \neq k} \Omega_i w_i X_{i,te}(w^S, Q, t)
\end{aligned} \tag{15}$$

This expression can be decomposed as

$$c_{ie} = c^*(w, Q, t) + c^{al}(w, Q, t, X) \tag{16}$$

where the cost frontier, depending on actual input prices, is

$$\begin{aligned}
c^*(w, Q, t) = \alpha_0 + \sum_{i \neq k} \alpha_i w_i + \sum_{r=1}^n \alpha_r Q_r + \frac{1}{2} \sum_{i \neq k} \sum_{j \neq k} \alpha_{ij} w_i w_j + \\
\frac{1}{2} \sum_{r=1}^n \sum_{l=1}^n \alpha_{rl} Q_r Q_l + \sum_{i \neq k} \sum_{r=1}^n \alpha_{ir} w_i Q_r + \sum_{i \neq k} \alpha_{it} w_i t + \sum_{r=1}^n \alpha_{rt} Q_r t + \alpha_t t + \alpha_{tt} t^2
\end{aligned} \tag{17}$$

while the impact of allocative inefficiency on costs, c^{al} , is

$$\begin{aligned}
c^{al} = \sum_{i \neq k} \alpha_i \Omega_i w_i + \sum_{i \neq k} \sum_{j \neq k} \alpha_{ij} \Omega_j w_i w_j + \frac{1}{2} \sum_{i \neq k} \sum_{j \neq k} \alpha_{ij} \Omega_i \Omega_j w_i w_j + \\
\sum_{i \neq k} \sum_{r=1}^n \alpha_{ik} \Omega_i w_i Q_r + \sum_{i \neq k} \alpha_{it} \Omega_i w_i t - \sum_{i \neq k} \Omega_i w_i X_{i,te}(w^S, Q, t)
\end{aligned} \tag{18}$$

Introducing in (18) the expression for $X_{i,te}$ obtained in (11) we get

$$c^{al}(w, \Omega) = \frac{1}{2} \sum_{j \neq k} \sum_{i \neq k} \alpha_{ij} \Omega_i w_i w_j = \sum_{j \neq k} w_j X_{j,te}^{al} \tag{19}$$

The first equality shows the impact of allocative inefficiency on costs. The second shows that it could have been obtained from input demands, $X_{j,te}^{al}$. Expression (19) shows the exact relationship between costs and allocative inefficiency, and is consistent with economic theory.

1.3 Modelling technical inefficiency

Now we will incorporate Farrell's (1957) technical inefficiency measure, which is input oriented. Let ϕ be the parameter that measures the proportional deviation of actual input used X_i^a from the technically efficient input values, i.e.

$$X_i^a = \phi X_{i,te}(w^S, Q, t); i = 1, \dots, m, \quad (20)$$

where $\phi \geq 1$ and $\phi = 1$ means technically efficient input usage. Note that ϕ can be specified including both time and individual variability, which is not explicitly written to simplify notation.

Introducing (12) in (20), adding and subtracting $X_i^*(w, Q, t) + X_i^{al}$, we get

$$X_i^a = X_i^*(w, Q, t) + X_i^{al}(w, \Omega) + (\phi - 1) [X_i^*(w, Q, t) + X_i^{al}(w, \Omega)] \quad (21)$$

where the first term is the optimal demand for input i , obtained as in (13), the second is the impact of the allocative inefficiency on demand of input i obtained in (14), and the last term is the effect of technical inefficiency on factor i , X_i^{tech} . This latter can be written taking into account (13) and (14), as

$$X_i^{tech} = (\phi - 1) \left[\alpha_i + \frac{1}{2} \sum_{j \neq k} \alpha_{ij} (1 + \Omega_j) w_j + \frac{1}{2} \sum_{r=1}^n \alpha_r Q_r + \alpha_{it} t \right] \quad (22)$$

Expression (22) links the exact impact of technical inefficiency on input demands with observed input prices and output levels, and with the parameters representing allocative and technical inefficiency, Ω and ϕ respectively.

In the same way, observed expenditure c^a can be expressed as

$$c^a = \phi c_{te} \quad (23)$$

which makes it explicit that actual costs are directly proportional to the measure of technical inefficiency. Let us substitute the value of c_{te} from (16) into (23), adding and subtracting $c^*(w, Q, t) + c^{al}(w, \Omega)$. This yields

$$c^a = \phi \left(c^*(w, Q, t) + c^{al}(w, \Omega) \right) = c^*(w, Q, t) + c^{al}(w, \Omega) + (\phi - 1) \left[c^*(w, Q, t) + c^{al}(w, \Omega) \right] \quad (24)$$

The first component in expression (24) represents the cost frontier obtained in (17), the second is allocative inefficiency cost obtained in (19) and the last is technical inefficiency cost. This technical inefficiency, c^{tech} , can be expressed using equations (17) and (19) as

$$c^{tech} = (\phi - 1) \left[\alpha_0 + \sum_{i \neq k} \alpha_i w_i + \sum_{r=1}^n \alpha_r Q_r + \frac{1}{2} \sum_{r=1}^n \sum_{l=1}^n \alpha_{rl} Q_r Q_l + \sum_{i \neq k} \sum_{r=1}^n \alpha_{ir} w_i Q_r + \sum_{i \neq k}^m \alpha_{it} w_i t + \sum_{r=1}^n \alpha_{rt} Q_r t + \alpha_t t + \alpha_{tt} t^2 + \frac{1}{2} \sum_{i \neq k} \sum_{j \neq k} \alpha_{ij} (1 + \Omega_i) w_i w_j \right] \quad (25)$$

This is a novelty, as the models based upon the translog cost system have obtained an exact expression for the allocative inefficiency on costs and input shares only. Using the NQCF permits the estimation of the effect of technical inefficiency on costs and input demands as well. Equation (25) shows that the effect of technical inefficiency on costs is related with the exogenous variables, the parameters that describe technology and the parameters that measure the price distortions caused by allocative inefficiency. This means that it is possible to quantify the influence of both outputs and factor prices on the cost of technical inefficiency.

From (17), (19) and (25) Farrell's Indices can be calculated for each observation as the ratios between C^* and C_{te} and between this last and the observed cost C^a .

2.3 A procedure to estimate and calculate technical and allocative inefficiency

Here we present a procedure to estimate the parameters that characterize technical and allocative inefficiency, from which the measure of technical inefficiency and Farrell's indices can be calculated.

The set of equations to be estimated are all the ratios between X_i and X_j , taking into account that this ratio eliminates technical inefficiency keeping allocative inefficiency only. These are

$$\frac{X_i^a}{X_j^a} = \frac{X_{i,te}}{X_{j,te}} = \frac{\alpha_i + \frac{1}{2} \sum_{j \neq k} \alpha_{ij} (1 + \Omega_{jt}) w_j + \sum_{r=1}^n \alpha_{ir} Q_r + \alpha_{it} t}{\alpha_j + \frac{1}{2} \sum_{i \neq k} \alpha_{ji} (1 + \Omega_{it}) w_i + \sum_{r=1}^n \alpha_{jr} Q_r + \alpha_{jt} t}, \quad i, j \neq k \quad (26)$$

In order to obtain an explicit expression for the complete model, let us write the demand for the input whose price will be used to normalize the cost function, X_K , which is

$$X_k^a = \frac{C^a - \sum_{i \neq k} W_i X_i^a}{W_k} = \frac{C^a}{W_k} - \sum_{i \neq k} \frac{W_i}{W_k} X_i^a = c^a - \sum_{i \neq k} w_i X_i^a = \phi(c_{te} - \sum_{i \neq k} w_i X_{i,te}^a) \quad (27)$$

where the first term is the (normalized) observed cost and the second is the (normalized) expense on the rest of the inputs.

Now we write the ratio between X_k and a generic input X_j . For this we use the expression of c_{te} as indicated in the first term of (15). Then, taking into account in (10) that the parameters that measure the distortion between the relative shadow and market prices has been defined in (7), and using (11) we obtain

$$\frac{X_k^a}{X_j^a} = \frac{X_{k,te}}{X_{j,te}} = \frac{c_{te} - \sum_{i \neq k} w_i X_{i,te}}{X_{j,te}} = \frac{c_{te}(w^S, Q, t) - \sum_{i \neq k} (1 + \Omega_i) w_i X_{i,te}}{X_{j,te}} =$$

(28)

$$\frac{\alpha_0 + \sum_{r=1}^n \alpha_r Q_r + \frac{1}{2} \sum_{r=1}^n \sum_{l=1}^n \alpha_{rl} Q_r Q_l + \sum_{r=1}^n \alpha_{rt} Q_r t + \alpha_t t + \alpha_{tt} t^2}{\alpha_j + \frac{1}{2} \sum_{i \neq k} \alpha_{ji} (1 + \Omega_i) w_i + \sum_{r=1}^n \alpha_{jr} Q_r + \alpha_{jt} t}$$

Adding the classical additive disturbance terms in each of the input demand function ratios defined in (27) and (28), we can estimate this system by using an iterative nonlinear seemingly unrelated regression techniques.

The estimation of equation (27) and (28) yields the parameters that characterize the cost function and the shadow prices. From this, the adjusted optimal input demands, $\hat{X}_i^*$ ($i \neq k$), can be calculated using equation (13). The effects of allocative inefficiency on input demands, $\hat{X}_i^{al}$, can be estimated from equation (14).

Estimating the optimal amount of the k th input and the effect of allocative inefficiency requires a more complex procedure. First, the technically efficient only adjusted input value, $\hat{X}_{k,te}$, can be calculated from equation (27). Second, the cost function parameters α_{ki} associated to the k th input relation with other input prices can be calculated following the procedure described in Appendix 1. Combining these with equation (14), the effect of allocative inefficiency on input k , $\hat{X}_k^{al}$, can be calculated. Finally, optimal input demand

can be obtained as the difference, i.e. $\hat{X}_k^* = \hat{X}_{k,te} - \hat{X}_k^{al}$. Una vez obtenido el efecto de la ineficiencia asignativa sobre todas las demandas se calcula $\hat{C}^{al}$.

From the adjusted values $\hat{X}_{i,te}$ and $\hat{X}_{k,te}$, $\hat{X}_i^*$ and $\hat{X}_k^*$, we can calculate $\hat{C}_{te}$ and $\hat{C}^*$ whose ratio yields Farrell's allocative efficiency index for each observation, from which time and firm variability can be calculated. Note that it is data variability what permits that the calculated allocative efficiency index varies across agents and periods.

Finally, Farrell's input oriented technical efficiency index is obtained as the ratio between the technically efficient cost, C_{te} , and the observed cost C^a . In this case, the measure of technical efficiency has not been estimated through an additional parameter but calculated from the cost function, taking advantage of the elimination of the technical inefficiency effects by means of the input demand ratios. Again, it is data variability what permits that Farrell's indices vary across observations.

2. An application to cargo handling in Spanish ports

2.1 The Spanish cargo handling sector

Cargo handling involves all the movements of freight from arrival to the port to its location within the ship and vice versa, including loading and unloading, transshipment, reception and dispatch. Accordingly, it is the most important activity within a port. Different types of cranes, specialized labour and different types of vehicles are the most relevant production factors. Cargo handling has been usually a regulated activity and the need for specialized labour has generated groups of workers with monopolistic characteristics: the stevedores.

In the Spanish case, it is felt that until the eighties the law stimulated a disproportionate increase of workers, wages unrelated to productivity, and bad practices regarding the organization of work, like oversized teams and restricted schedules, causing low labour productivity and large prices for port services, diminishing Spanish ports competitiveness.

Two Royal Decrees from 1986-87 begun a legislative reform within cargo handling activities in Spain, later reinforced by the agreements reached among the ports administration, the firms and the unions in 1993 and 1997. This new legal framework establishes that in every port regarded of general interest, a Sociedad Estatal de Estiba y Desestiba (SEED) should be formed as a sociedad anónima with the State owning more than 50% of the assets in order to control decision making in an activity of public interest. Private firms can subscribe the remaining assets. Their participation depends on objective criteria as the fixed workers payroll, investment in equipment, volume of freight handled and payment for port infrastructure. Port workers doing cargo handling have to register with the SEED, who manages their assignment according to firms demands and following certain rotation rules. A detailed description of labour relations and conditions in port cargo handling activities can be found in Rodríguez Ramos (1997).

The reform that begun in the eighties aimed at more flexibility in the design of work teams and in the schedules, abandoning centralized regulation (state level) and permitting each firm to decide on team size and configuration within predetermined safety levels. Work periods can be increased to fulfil demand requirements, including night and weekend shifts. Wages and contracts are port specific collective agreements. As a result, the payroll has diminished and the design of work teams is now decentralized. Nevertheless, the opening of the activity to other firms, something which will led to more competitive prices, has been

non existent; and, in any case, in a subsidiary way and with the same wage level if the work would have been done by the SEED workers. Given the said characteristics, we believe it is an adequate sector to test the model developed in section 1.

2.2 Data

Cargo handling activities involve essentially two factors: labour and cranes. Data sources are the State Annual Reports on Ports, the Annual Report of each port and a questionnaire that we had drawn up and presented to the *SEEDs*. The data from the Annual Reports of the Ports of the State have been used to get the quantities of cargo moved by each port and year included in the sample. The outputs analysed in this study were defined according to how the merchandise is handled, which, in turn, will determine what kind of operation is needed to load or unload it. Thus, we can distinguish between general container cargo (*MGC*), non-containerised general cargo (*MGNC*) and solid bulk cargoes that are handled without special facilities (*GSSI*).

The Annual Reports of each port and the information received from crane operators in ports have given us the hours worked by cranes and have permitted the calculation total expenses on this item. The other data source, namely the questionnaire sent to all *SEEDs*, gave us important information on the labour factor, basically concerned with labour costs and hours worked by stevedores.

The costs we will explain encompass the expenditure in labour (L) and the expenditure in cranes (K) associated with the handling operations for the aforementioned cargo flows. To build input price indicators, we have the total expenditure on each input and a physical measure of the input used, in this case, the number of hours worked by stevedores and the number of hours of crane use.

The ports included in this study are as follows: Algeciras, Alicante, Bilbao, Cadiz, Cartagena, Castellon, Gijon, Huelva, Corunna, Malaga, Majorca, Alcudia, Motril, Pontevedra, Tenerife, Santander, Seville, Valencia and Vigo. However, as some *SEEDs* were created during the study period, the number of observations for each port varies. The above mentioned sources were used to build a data panel with 158 yearly observations for the period from 1990 to 1998.

2.3 Empirical Results

As we will consider only the two main inputs, the model to be estimated is the ratio between capital (cranes) and labour demand equations, with the price of the former being used to normalize, such that its demand should be specified as indicated in (27). Since the input demand functions are homogeneous of degree zero in ε_i , one of them is unidentified. We normalize ε_K to be unity and estimate ε_L .

Although the general model permits the estimation of price distortion parameters that vary across firms and periods, the limited number of periods has prevented to do so in this application. However, as stated in 1.3, an allocative inefficiency measure for each observation can be obtained.

Therefore, the parameters that link market prices with their corresponding shadow prices are specified here accounting for variations in time that also imposes non-negativity of price distortions, i.e.

$$\varepsilon_{Lt} = (1 + \eta_L + \eta_{Lt}t)^2 = 1 + \Omega_{Lt} \quad (29)$$

where Ω_{Lt} is the deviation of the actual relative price from the relative shadow price at period t .

Introducing labour demand from (11) and input price distortions from (29) into (28), the econometric model to be estimated in this application is finally obtained as the ratio between the demands for capital and labour, i.e.

$$\frac{X_K^a}{X_L^a} = \frac{\alpha_0 + \sum_{r=1}^n \alpha_r Q_r + \frac{1}{2} \sum_{r=1}^n \sum_{l=1}^n \alpha_{rl} Q_r Q_l + \sum_{r=1}^n \alpha_{rt} Q_r t + \alpha_t t + \alpha_{tt} t^2}{\alpha_L + \alpha_{LL} (1 + \Omega_L) w_L + \sum_{r=1}^n \alpha_{Lr} Q_r + \alpha_{Lt} t} \quad (30)$$

Adding a standard error term, equation (30) was estimated using non-linear least squares. Results are shown in Table 1. The fit is $R^2=0.95$ and the Likelihood ratio test shows significance of the set of parameters.

Table 1: Results of estimation

We tested three hypotheses related with allocative inefficiency: a) total absence ($\eta_L = \eta_{Lt} = 0$), b) absence of the permanent component ($\eta_L = 0$), and c) absence of temporal variability ($\eta_{Lt} = 0$). These restrictions have been analysed using the Wald test. The value obtained for hypothesis a) is 12.77, larger than the critical value of the χ^2 with 2 degrees of freedom at the 1 percent level of significance, which means that absence of allocative inefficiency is rejected. The values for the test in cases b) and c) are 63.52 and 3.96 respectively. In the first case, the Wald statistic is larger than the critical value of χ^2 with 1 degree of freedom at the 1 percent level of significance, which rejects the hypothesis of absence of a permanent component. In case c), the Wald statistic is larger than the critical value of χ^2 with 1 degree of freedom at the 5 percent level of significance. This result shows that distortion prices vary over time.

The shadow cost function corresponds to a well-behaved production function only if it is monotonically increasing in shadow input prices and output quantities, and concave and linear homogeneous in shadow input prices. Monotonicity is checked by determining if the calculated values of the input demands and cost are positive, which occurs for all observations. Concavity is checked by determining if the principal minors of the Hessian matrix have the correct alternating signs. In this application, the Hessian matrix is a negative semidefinite matrix and therefore concavity in shadow input prices is satisfied. As the NQCF fulfils homogeneity of degree one in prices by construction, the shadow cost function presents all the theoretical properties and can be regarded as an adequate representation of the productive structure of cargo handling activities in Spanish ports.

With the estimated values of η_L and η_{Lt} we calculated the series for ε_{Lt} using equation (29), which resulted to be less than unity for the whole period. This means that during the 1990-1998 period labour was over utilized regarding capital in cargo handling activities. The average value of ε_L is 0.842, which indicates that the capital-labour mix chosen within this sector was based upon relative prices that were 84.2% of the actual ones. The evolution in time of ε_L shows a continuous decline, which means that the distortion previously described grew within the period, worsening the choice of input combinations.

Following the procedure described in section 1.3, we estimated the effects of allocative inefficiency on the demand for labour and capital and on costs, for each observation. The values of the parameters α_{LK} and α_{KK} are 4.09 and -4.72 respectively.

Table 2 shows the average values of allocative inefficiency by port. These results confirm previous intuition regarding a larger than efficient utilization of labour while it would be

advisable to use more crane-hours. In average, labour was used 14.4% more than what is efficient; and the use of crane-hours was 13.2% less than optimal.

Table 2. Effects of allocative inefficiency

We have also calculated Farrell's (1957) efficiency indices for each observation. As explained earlier, the allocative index is given by the ratio between the optimum and the technically efficient cost, C^*/C_{te} , while the technical index is obtained as the ratio C_{te}/C^a where C^a is the actually observed cost. Both can be calculated for each firm and period. Finally, their product yields Farrell's cost efficiency index. Table 3 shows the average of the three indices for each port.

Table 3. Farrell's Efficiency Indices.

The average of the allocative efficiency index shows that the inadequate choice of labour and capital in Spanish ports meant an extra cost of 6.9%. On the other hand, technical inefficiency provoked an average extra cost of 8.6%. Finally, average total inefficiency is 11.7%. No clear relation with port characteristics can be detected, which very likely indicates differences in management capacity.

4. Conclusions

In this paper we have built a theoretical model using a normalized quadratic cost system to decompose the effects of allocative and technical inefficiency on input demands and cost. For this proposal, we have used the shadow prices approach in the line of exact decomposition that allows us to solve Greene's problem. We have obtained expressions for actual input demand and cost as functions of three components: frontier, allocative inefficiency and technical inefficiency. In this way, both allocative and technical

inefficiency effects depend on the exogenous variables in the cost model, on the parameters that characterise technology, and on the parameters that deal with both types of inefficiency. Using the normalized quadratic cost system has permitted to isolate not only allocative inefficiency but also the technical one, as simple functions of both parameters and variables. The general decomposition formulated in the theoretical model coupled with the estimation procedure and the corresponding calculations, allows to obtain individual and time varying technical and allocative inefficiency measures when a panel data is available.

The model has been applied to cargo handling activities in Spanish ports during the period of 1990-2000, obtaining the effects of both technical and allocative inefficiency on cost and input demands. The results show that, in average, labour was used 14.6% more than what is efficient; and the use of crane-hours was 13.9% less than optimal. The average of the allocative efficiency index shows that the inadequate choice of labour and capital in Spanish ports meant an extra cost of 6.9%. Besides, the distortion produced by the allocative inefficiency increases over time, which shows that the reforms policy has not been successful with regard to this question. On the other hand, technical inefficiency provoked an average extra cost of 8.6%. Finally, average total inefficiency is 11.7%. No clear relation with port characteristics can be detected, which very likely indicates differences in management capacity.

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Appendix 1

Let X_i be factor demand i ($i=1,\dots,m-1$)

$$X_i = X_i(W, Q, t) \tag{1a}$$

where W is the normalized prices vector, Q is the products vector and t is trend. If W_k is the normalizing price, then

$$\frac{\partial X_i}{\partial W_k} = \sum_{j \neq k} \frac{\partial X_i}{\partial (W_j/W_k)} \frac{\partial (W_j/W_k)}{\partial W_k} = \sum_{j \neq k} \frac{\partial X_i}{\partial (W_j/W_m)} \left(-\frac{W_j}{W_k^2}\right) = \frac{\partial X_k}{\partial W_i} \quad (2a)$$

Factor demand i from the NQCF is

$$X_i(W, Q, t) = \alpha_i + \frac{1}{2} \sum_{j \neq k} \alpha_{ij} W_j + \sum_{r=1}^n \alpha_{ir} Q_r + \alpha_{it} t \quad (3a)$$

Applying (2a) yields:

$$\frac{\partial X_i}{\partial W_k} = -\frac{1}{2W_k^2} \sum_{j \neq k} \alpha_{ij} W_j = \frac{\partial X_k}{\partial W_i} \quad (i \neq k) \quad (4a)$$

As factor demands are homogeneous of degree zero in relative factor prices, applying Euler's theorem yields:

$$\sum_{j=1}^m \frac{\partial X_k}{\partial W_j} W_j = 0 \quad (5a)$$

Solving for $\frac{\partial X_k}{\partial W_k}$ in (5a) and combining with (4a) we get:

$$\frac{\partial X_k}{\partial W_k} = \frac{1}{2W_k^3} \sum_{i \neq k} \sum_{j \neq k} \alpha_{ij} W_i W_j \quad (6a)$$

which permits the calculation of cross-price effects.

Table 1: Results of estimation

Parameter	Estimation	T-student
α_0	0.038	29.46
α_L	0.258	11.78
α_{LL}	-26.31	-3.89
α_{MGC}	0.117	6.41
α_{MGNC}	0.429	7.08
α_{GSSI}	0.107	5.63
α_T	-0.006	-4.28
α_{MGCMGC}	0.019	2.16
$\alpha_{MGNCMGNC}$	-0.074	-5.42
$\alpha_{GSSIGSSI}$	0.257	7.02
$\alpha_{MGCMGNC}$	-0.659	-3.89
$\alpha_{MGNCGSSI}$	-0.373	-3.33
$\alpha_{MGCGSSI}$	0.102	2.94
α_{MGCPL}	0.141	8.04
α_{MGNCPL}	0.977	6.84
α_{GSSIPL}	0.110	3.78
α_{TT}	0.002	0.43
α_{TPL}	-0.086	-7.26
α_{TMGC}	-0.002	-0.27
α_{TMGNC}	-0.012	-0.53

α_{TGSSI}	-0.019	-3.25
$\alpha_{CONSTANT}$	0.038	29.47
η_L	-0.029	-2.14
η_{Lt}	0.0002	1.58

Table 2. Effects of allocative inefficiency

Port	Over utilization of labor (%)	Under utilization of capital (%)
Algeciras	2.6	4.2
Alicante	9.1	16.3
Bilbao	3.8	3.7
Cádiz	12.5	20.6
Cartagena	21.8	18.2
Castellón	15.5	13.4
Gijón	22.1	19.3
Huelva	16.7	16.2
La Coruña	11.9	11.1
Málaga	17.47	17.8
P.Mallorca	19.5	10.9
Alcudia	21.9	19.5
Motril	21.3	11.8
Pontevedra	18.1	19.8
S/C Tenerife	11.2	20.0
Santander	9.5	7.2
Sevilla	20.7	12.4
Valencia	3.8	5.4
Vigo	17.4	16.9

Mean	14.6	13.9
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Table 3. Farrell's Efficiency Indices.

Port	Allocative Efficiency Index	Technical Efficiency Index	Cost Efficiency Index
Algeciras	0.995	0.965	0.960
Alicante	0.893	0.894	0.888
Bilbao	0.980	0.922	0.914
Cádiz	0.869	0.898	0.796
Cartagena	0.929	0.941	0.875
Castellón	0.955	0.948	0.919
Gijón	0.902	0.928	0.887
Huelva	0.933	0.970	0.906
La Coruña	0.979	0.924	0.908
Málaga	0.846	0.936	0.860
P.Mallorca	0.905	0.869	0.859
Alcudia	0.931	0.942	0.925
Motril	0.880	0.888	0.781
Pontevedra	0.944	0.918	0.913
S/C Tenerife	0.953	0.821	0.790
Santander	0.950	0.757	0.749
Sevilla	0.901	0.936	0.901
Valencia	0.981	0.948	0.931
Vigo	0.945	0.956	0.903
Mean	0.931	0.914	0.883