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EEE 88

December, 2000

http://www.fedea.es/hojas/publicado.html
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November 30, 2000

Abstract

One of the features of the large overlapping generations model pioneered by Auerbach and Kotlikoff (1987) is that individuals with different experience levels are perfect substitutes in production. This paper replaces this assumption with a labor market characterized by imperfect substitutability between less and more experienced workers. By comparing the quantitative properties of both cases in a calibrated model for Spain, it is found that in the model economy with imperfect substitution, the effects of aging on the financial viability of the pension system are less severe than in the standard model economy with perfect substitutability.

Keywords: Cohort Size Effects, Demographic Change, Social Security. JEL classification: E62; H55; J11.

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1 Introduction

The purpose of this paper is to examine the relationship between the aging of the population and the future prospects of the social security system in Spain. Although this topic has received much attention over the last 10 years mainly due to the sharp expected increase in the share of retired individuals over the working population (see Figure 1), the existing studies have abstracted from the interaction between the age composition of the population and the life-cycle profile of earnings. Since the pattern of pension benefits and contributions are strongly age dependent, this assumption can have important effects on the evolution of the percentage of GDP spent on pensions. For this reason in this paper I study the following question: How do the properties of the standard large overlapping generations model compare to the properties of a model that accounts for the existence of cohort size effects? I study this issue by calibrating a computable overlapping generations model to match some key features of the Spanish economy in 1995. Then I use the demographic projections for this economy from Eurostat Demographic Statistics 1996 to analyze what should be the adjustment in the tax rate needed to keep balanced the pension system in several model economies that have different assumptions about the degree of substitution of workers with
different levels of work experience.

This paper is not new in addressing the effects of the aging process upon the social security systems. The potential economic effects caused by the individuals that belong to the baby-boom generation as they enter retirement has motivated an increasing concern about the sustainability of the pension systems in developed countries. In this sense, the recent research effort on social security has mainly concentrated on the efficiency of the current pay-as-you-go pension system (e.g. Imrohoroglu et al. (1995) and Boldrin et al. (1999)), the design of a feasible reform to a funded system (e.g. Arjona (2000a) and Huang et al. (1996)) and the fiscal adjustments that prevents from privatization (De Nardi et al. (1999) and Arjona (2000b) and Montero (1999) for the Spanish economy). These studies are characterized by the perfect substitutability of workers with different levels of work experience, namely they abstract from the possible effect that an increase in the number of older workers relative to the number of young less experienced workers could have on the relative labor earnings of these workers.

In sharp contrast with this assumption, there are many empirical studies for the U.S. economy (e.g. Murphy and Welch (1992), Katz and Murphy (1992),
Freeman (1979), Welch (1979) and Berger (1985)) that have found that the age-earnings profile of workers appears to be significantly affected by the age composition of the workforce. In words of Freeman (1979), "apparently because younger and older male workers are imperfect substitutes in production, changes in the number of young male workers relative to older male workers substantially influence the ratio of the earnings of younger men to the earnings of older men". For the specific case of the Spanish economy the lack of data on wages by age has not allowed researchers to test these effects although there is some empirical evidence (Eguia and Echevarria (1999)) concerning the relative higher unemployment rate experienced by those individuals that belong to the baby-boom generation. In this sense, it is worth noting that in highly regulated labor markets one should expect the adjustment of the labor market to be done through a change in quantities (employment) instead of prices (wages) following a change in the relative supply of workers with different levels of work experience. Since both cases imply that the individuals of the baby-boom generation have on average lower labor earnings than individuals belonging to more scarce cohorts, we think that the assumption of perfect competitive labor markets with imperfect substitutability between young and old workers is a good starting point to tackle the question at hand.
Despite the potential implications of these cohort size effects for a variety of macroeconomic issues, there are not many studies that have attempted to introduce these effects in macroeconomic models. Some exemptions are the seminal work of Lam (1989) that studied the effects of changes in age structure on life-cycle wage profiles in stable populations. And more recently, Kremer and Thomson (1998) have studied the implications of the imperfect substitution between young and old workers for the speed of convergence of per capita output between countries and found that the existence of imperfect substitutability creates a kind of adjustment cost in human capital because total output depends positively on each generation's human capital but negatively on the change in human capital between generations.

Although we think that the assumption of perfect substitution between young and old workers may be useful depending on the question at hand, in the context of an aging population to legitimately abstract from the interaction between the age structure of the population and the life-cycle profile of labor earnings, the consequences of the aging of the baby-boom generation should be quantitatively similar in a model economy that abstracts from cohort size effects an one that it does not. In this paper I address this question by studying the effects of demographic projections from 1995 to 2050 on the
...nances of the social security system. I compare the equilibrium allocations in two model economies that only differ from each other in the degree of substitution of workers with different experience levels. Results indicate that there are quantitatively relevant differences between both economies. For instance it is found that if the rule used to compute pension benefits is left untouched, in the standard model the percentage of GDP spent on pensions will increase from 12.3% in 1995 to 26.1% in 2040. In contrast, in the model economy with cohort size effects this percentage will increase from 12.3% in 1995 to 19.2% in 2040. Similarly, under the perfect substitution case the social security tax rate has to be increased from 18.94% in 1995 to 45.26% in 2045 while with imperfect substitutability across age groups the tax rate should be increased from 18.94% in 1995 to 33.79% in 2045.

The rest of the paper is organized as follows. Section 2 describes the model economies I investigate. Section 3 describes how the model is parameterized to be a realistic description of the Spanish economy in 1995. Section 4 presents the main results of the paper. Section 5 studies the sensitivity of the results to different modelling strategies and finally Section 6 concludes.
2 The Model

2.1 Demographics

The economy is populated by agents that live a maximum of $I$ periods. Upon arrival at the age of $I_A$, an agent starts taking decisions. Each individual is endowed with 1 unit of time that can be allocated to work or leisure up to age $I_{Ri}$. After this age agents retire.

Each agent faces an age dependent probability of surviving between age $i$ and age $i+1$ at $t$ denoted by $s_{i;t}$. Then the unconditional probability of reaching age $i$ for an individual that has age $v$ at $t$ is

$$\frac{1}{q_i,v;t} = \prod_{j=v+1}^{i} s_{j;i:t;1}$$  \hspace{1cm} (1)

with $\frac{1}{q_i:v;t} = 1$. Let $^1_i;t$ be the share of age-$i$ individuals over the total population at time $t$. The law of motion of the age structure of the population is

$$^1_{i+1;t+1} = \frac{S_{i+1;i;t}}{1 + n_t} + p_{i+1;t+1}$$  \hspace{1cm} (2)

where $p_{i+1;t+1}$ is the age specific immigration rate and $n_t$ is the population
growth rate. Finally, the next period share of newly born agents \(1_{t+1}^{1}\) is given by

\[
1_{t+1}^{1} = \prod_{i=2}^{\infty} 1_{i,t+1}^{\infty}
\]  

(3)

### 2.2 Preferences

At each point in time agents are assumed to maximize lifetime utility. Hence the problem of the typical agent that at \(t\) has age \(i = v(1 + \nu)\) is to choose consumption and leisure \(l_{i,t} = 1; h_{i,t}\) to solve the problem

\[
\text{Max} \sum_{i=v}^{\infty} X^{i} U(c_{i,t} + i; h_{i,t} + i \nu)
\]  

subject to the following period-by-period constraint

\[
a_{i+1,t+1}^{1} s_{i,t} = (1 + r_{t}(1 + \nu))a_{i,t} + y_{i,t} i c_{i,t}
\]  

(5)

\[
0 \quad a_{i+1,t+1}^{1}; a_{1,t} = 0; a_{i+1,t} = 0;
\]  

(6)

The discount parameter is \(\bar{\nu}\), and is assumed to be the same for all agents. Borrowing is not possible and agents accumulate asset holdings to smooth
consumption over time. $r_t$ is the interest rate net of depreciation, $a_{i+1:t+1}$ denotes next period asset holdings, $y_{i:t}$ is labor income net of taxes plus transfers and $\xi$ is a proportional income tax. Notice that the formulation of the budget constraint of the agent implies the existence of annuity markets. The only reason to assume this is to avoid the issue of what to do with the assets accumulated by those agents who die.

Let $e_i$ be the efficiency index, $\xi_{SS;t}$ the social security proportional tax, $d_{i:t}$ the social security benefits that are zero if $i < L_R$ and $d_{i:t}$ otherwise. Finally $w_{i:t}$ denotes real wages, that are indexed by age to account for the case of imperfect substitutability of labor of different age groups. These considerations allow us to define the labor income net of taxes plus transfers as

$$y_{i:t} = w_{i:t}e_i h_{i:t}(1 - i - \xi_{SS;t}) + d_{i:t}. \quad (7)$$

2.3 Production Technology

Production in period $t$ is given by a standard constant returns to scale production function that converts capital $K_t$ and labor $N_t$ into output. The technology $A_t$ improves over time at a constant rate because of labor augmenting technological change, $A_{t+1} = (1 + \theta)A_t$. Hence,
\[ Y_t = F(K_t; A_t N_t) = K_t^{\hat{\beta}} (A_t N_t)^{\lambda_t} \]  \hspace{1cm} (8)

with

\[ N_t = g(L_t; H_t); \]  \hspace{1cm} (9)

where \( L_t \) and \( H_t \) denotes less and more experienced workers respectively and the function \( g \) has continuous second derivatives and it is increasing and concave in labor inputs. Finally, firms rent labor and capital at given wages and net interest rate to maximize

\[ F(K_t; A_t N_t) \mid (r_t + \hat{\delta})K_t \mid w_t L_t \mid w_h H_t \]  \hspace{1cm} (10)

where \( \hat{\delta} \) is the depreciation rate for capital.

2.4 Government

The government levies a proportional social security tax on labor income \( \xi_{ss,t} \) to finance a benefit \( d_{it} \) per retiree. This system is assumed to be self-financed, i.e.
where benefits are computed applying a legal replacement rate to an average of past earnings up to a maximum pension limit. Hence in age $I_R$ benefits are given by,

$$d_{i,t} = \min(P_{\text{max}}, \frac{\text{rep}}{1 + \cdot} w_{\text{av}})$$

where $\cdot$, $\text{rep}$, $w_{\text{av}}$ and $P_{\text{max}}$ are the productivity growth, the legal replacement rate, some average of past earnings and the maximum pension benefit respectively. From $I_R + 1$ to $I$, the pension benefit is normalized by productivity growth $(1 + \cdot)$, since new pensions are greater than old ones, i.e.

$$d_{i,t} = \frac{d_{i + 1,t}}{1 + \cdot}$$

The government also levies a proportional tax on capital and labor income $\cdot$ to finance per capita government consumption $G_t$ such that

$$\sum_{i=I_A}^{I} (r_{i,t}a_{i,t} + w_{i,t}h_{i,t}e_{i}) \cdot = G_t$$
2.5 The Equilibrium

In this economy a Competitive Equilibrium is a list of sequences of quantities $c_{i;t}$, $h_{i;t}$, $a_{i;t}$, $d_{i;t}$, $L_{t}$, $N_{t}$, $K_{t}$, prices $w_{i;t}$, $w_{h;t}$, $r_{t}$, social security tax rates $\xi_{ss;t}$ and an income tax rate such that, at each point in time $t$:

1) Firms maximize profits setting wages and the interest rate equal to marginal products,

$$w_{i;t} = F_{L}(K_{t};L_{t};H_{t})$$  \hspace{1cm} (15)

$$w_{h;t} = F_{H}(K_{t};L_{t};H_{t})$$  \hspace{1cm} (16)

$$r_{t} = F_{K}(K_{t};L_{t};H_{t}) \quad \pm$$  \hspace{1cm} (17)

2) Agents maximize lifetime utility subject to the period budget constraints taking wages, the interest rate, taxes, social security benefits, survival probabilities and the age structure of the population as given,

3) The age structure of the population $f_{1;i;t}$ is generated by the aggregate law of motion (1), (2) and (3), given initial conditions $^{1}_{i;0}$. 

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4) market clearing conditions for capital and each type of labor,

\[ K_t = \sum_{i=1}^{1_A} i; a_i; t \]  \hspace{1cm} (18) \\

\[ H_t = \sum_{i=1}^{1_E} i; e_i; t; h_i; t \]  \hspace{1cm} (19) \\

\[ L_t = \sum_{i=1}^{1_A} i; e_i; t; h_i; t \]  \hspace{1cm} (20)

where \( I_E \) denotes the age at which an individual starts being considered as an experienced worker.

5) Finally, the budget constraint of the government is satisfied period by period.

Hence with these conditions the goods market clears every period,

\[ F(K_t; L_t; H_t) + (1 - \delta)K_t = K_{t+1} + G_t + \sum_{i} X i; c_i; t; \]  \hspace{1cm} (21)
3 Calibration

3.1 Demographic Parameters

Agents reach adulthood at 20 and live up to age 95, after which death is certain. Each model period corresponds to 5 years. The initial steady state is characterized by the age structure of the population and the age survival probabilities in 1995 in Spain. The procedure to propagate the economy after this year uses the law of motion of the population characterized by equations (2) and (3). We use the database Eurostat Demographic Statistics 1996 to obtain the observed age structure of the population, the mortality rates and the net migration rates in 1995 and the estimates of these variables from 2000 to 2050. These estimates assume that in Spain, the average life expectancy at birth will increase from 77.2 years in 1995 to 82.7 years in 2050. On the other hand, the total fertility rate per woman is predicted to increase slightly from 1.2 in 1995 to 1.5 in 2020 and then remains constant until 2050. Finally, in order to compute the final steady state, after 2050 I fix the net migration rate and the survival probabilities of that year and let the age structure of the population run until it reaches a stationary structure characterized by a population growth rate equal to zero.
### 3.2 Preference Parameters

The period utility function is of the constant relative risk-aversion class

\[
u(c; l) = \frac{(c^{1/\mu} l)^{1/\theta}}{1 l^{1/\theta}}
\]

where the inverse of the elasticity of substitution \( \theta \) and the share of consumption \( \mu \) has been set such that the average time spent working is around 1/3 and the intertemporal elasticity of substitution is consistent with the empirical estimates reviewed in Auberbach and Kotlikoff (1987). Hence we use \( \theta = 2 \) and \( \mu = 0.33 \). Finally, the discount rate parameter is equal to the point estimate of Hurd (1989) which accounts for mortality, \( \bar{\gamma} = 1.011 \). Notice that the effective discount rate is the product of \( \bar{\gamma} \) and the unconditional survival probability, being this less than one.

### 3.3 Government

In this model, the first role played by the government is to run a pay-as-you-go social security system that consists of a legal rule used to compute
pension benefits as a function of past labor earnings. In particular, upon retirement an individual’s pension is computed applying a replacement rate of 100% over the average of earnings of the last 8 years before retirement. The pension system in Spain also includes a maximum pension level equal to 1.4 times the average earnings in the economy but in the present model economy it was not binding. Finally, the social security tax rate $\xi_{ss}$ is set endogenously to cover the pension burden so that the pension’s system is self financed. Secondly, the government levies an income tax rate on capital and labor income to finance a given level of government consumption. In particular, we use a value of $\xi$ that matches the average ratio of government consumption over output from 1970 to 1994 being equal to $G/Y = 12.8\%$. The income tax rate that satisfies this condition is $\xi = 17.1\%$.

3.4 Efficiency unit profile and Technology Parameters

3.4.1 Efficiency unit profile

The age specific labor productivities $e_i$ are compiled using the European Household Panel (1994) which offers data on gross hourly wages for workers above the age 19. The endowment of efficiency units is determined by dividing each cohort’s average wage by the average of the sample and then by
smoothing the wage pro..le with a polynomial of degree two.

3.4.2 Technology Parameters

The labor share parameter is \( \bar{\gamma} = 0.65 \) following the estimates of Puch and Licandro (1997) for the Spanish economy. The productivity growth has been set to \( \bar{\gamma} = 1.5\% \) in annual terms which is the average growth of per-capita consumption over the period 1960-1995, and the depreciation parameter is set to match the average ratio of gross investment over output over the period 1970-1994, \( I/Y = 25.18\% \). This yields a value of \( \bar{\gamma} = 8.2\% \) in annual terms.

A decision concerning the aggregation of the labor input across different age groups has to be made. The empirical studies of the effects of changes in the relative number of workers by age on age earnings pro..le have usually used the constant elasticity of substitution form and the translogarithmic form. For our purposes the CES functional form is very convenient because it has only one elasticity of substitution across workers with different levels of experience \( (1=\bar{\gamma}) \) and it is flexible enough to account for the perfect substitutability case \( (\bar{\gamma} = 0) \) and the imperfect substitutability model economy when \( \bar{\gamma} > 0 \).

An additional decision has to be made concerning the way in which indi-
individuals with different experience levels are sorted into different groups. Our approach follows the standard practice in the labor literature that usually sorts the population into two experience groups. The first includes those individuals with less than 25 years of experience, i.e. those who are between 20 and 44 years old. The second group contains those individuals with age more that 45 and that stay in the labor market until the retirement age 65. As part of the sensitivity analysis I consider alternative ways to sort the working population into experience groups. Given these considerations, the aggregate labor input is

\[
N_t = B \left( \theta L_t^{\frac{1}{\gamma}} + (1 - \theta) H_t^{\frac{1}{\gamma}} \right)^{\frac{1}{1-\gamma}}
\]  

(22)

where \( L_t \) and \( H_t \) are the labor supply in efficiency units of workers with less and more than 25 years of working experience respectively, and \( B \) is a parameter that measures the efficiency of aggregate labor. The general procedure to set the values of the inverse of the elasticity of substitution \( \gamma \), the parameter \( B \) and the share parameter \( \theta \) is as follows.
3.4.3 Perfect substitutability

In this model economy, a change in the relative supply of experienced workers does not translate into changes in the relative wages on individuals by age. Consequently, this is the case where \( \frac{1}{2} = 0 \). In addition, the value that governs the overall efficiency of labor input is set to a normalized value of \( B = 1 \). Finally, the value of the share parameter \( \circ \) is set such that the age-profile of earnings in the model economy which consists of a product of the market wage \( w_i \) and the efficiency index \( e_i \) resembles the smoothed profile of earnings in the data. However, notice that since by construction the age-specific profile of efficiency units \( e_i \) already captures this target, the share parameter \( \circ \) has to be set such that \( \frac{w_h}{w_i} = 1 \). Since the relative wage is given by

\[
\frac{w_h}{w_i} = \circ \frac{H^i}{L^i} \frac{1}{\frac{1}{2}}
\]

then, when \( \frac{1}{2} = 0 \), \( \frac{w_h}{w_i} = 1 \) if \( \circ = 0.5 \).

3.4.4 Imperfect substitutability

Murphy and Welch (1992), among others, have studied the existence of imperfect substitutability among workers with different levels of experience and
education. Their estimates of the elasticities of complementarity imply values of the $\frac{1}{2}$ parameter between 0.5 and 2. In this paper we use $\frac{1}{2} = 1.2$ as our benchmark case for the case of imperfect substitution although in the sensitivity analysis I check the robustness of the results with a much lower $\frac{1}{2}$ parameter. Finally, the share parameter $\omega$ is set (as before) such that $\frac{w_h}{w_l} = 1$, yielding $\omega = 0.7403$, and the parameter that governs the overall efficiency of the labor input $B$ is set so that the level of wages equals the level of spot wages in the benchmark model economy with perfect substitutability between young and old workers, so that both model economies are comparable. This yields $B = 0.897$.

3.5 Computation Method

The computational procedure used to solve for the transitional dynamics of the model follows Auerbach and Kotlikoff (1987). Notice that since the economy undergoes a transition in which conditions change over time and economic agents are assumed to take into account future prices in determining their behavior, it is necessary to solve simultaneously for equilibrium in all transition years. In order to implement the computational procedure I assume that the final steady state is reached in 200 model periods, and I
have checked that it was not binding. The main steps for solving this system of 200 equations and 200 unknowns are the following.

A) Given initial conditions $K_1$, $\mathbf{f}a_{i;A} \mathbf{g}_{i=1}^1$ and the path that follows the age structure of the population $\mathbf{f}1_{i;A} \mathbf{g}_{i=1}^{t=90}$, provide a guess for the path of the capital stock $\mathbf{f}K_{t} \mathbf{g}_{t=1}^{t=90}$ and the age profile of work effort $\mathbf{f}h_{i;A} \mathbf{g}_{i=1}^{t=90}$.

2. Using $\mathbf{f}h_{i;A} \mathbf{g}_{i=1}^{t=90}$ compute labor input $\mathbf{f}L_{t} \mathbf{g}_{t=1}^{t=90}$ and $\mathbf{f}H_{t} \mathbf{g}_{t=1}^{t=90}$.

2. Using $\mathbf{f}K_{t} \mathbf{g}_{t=1}^{t=90}$, $\mathbf{f}L_{t} \mathbf{g}_{t=1}^{t=90}$, $\mathbf{f}H_{t} \mathbf{g}_{t=1}^{t=90}$ and the marginal productivity conditions, compute $\mathbf{f}r_{t} \mathbf{g}_{t=1}^{t=90}$ and wages by type $\mathbf{f}w_{L;A} \mathbf{g}$ and $\mathbf{f}w_{H;A} \mathbf{g}$.

2. Using $\mathbf{f}h_{i;A} \mathbf{g}_{i=1}^{t=90}$, $\mathbf{f}w_{L;A} \mathbf{g}$ and $\mathbf{f}w_{H;A} \mathbf{g}$, compute pension benefits to which agents qualify $\mathbf{f}d_{i;A} \mathbf{g}_{i=1}^{t=90}$ and the necessary social security tax $\mathbf{f}ss_{t} \mathbf{g}_{t=1}^{t=90}$ to keep balanced the system.

2. Use $\mathbf{f}r_{t} \mathbf{g}$, wages, transfers, and labor effort $\mathbf{f}h_{i;A} \mathbf{g}_{i=1}^{t=90}$ to solve the consumer problem in asset holdings $\mathbf{f}a_{i;A} \mathbf{g}_{i=2}^{t=90}$.

2. Use $\mathbf{f}a_{i;A} \mathbf{g}_{i=2}^{t=90}$ to compute the implied capital stock $\mathbf{f}K_{t} \mathbf{g}_{t=2}^{t=90}$ by aggregating the asset holdings across ages for each $t$; and use the implied age profile of consumption to compute a new guess of the age profile...
of work effort \( q_{i}^{-} = 1, q_{A} = 90, t = 1 \) by means of the intratemporal marginal condition.

B) If the implied \( f f h_{i} t g_{i}^{t} = 90, f f h_{i} t g_{A}^{t} = 90 \) are equal to the guesses of step A) the algorithm is stopped. If not, update the guess and go back to step A).

4 Findings

Before describing the findings of the paper, I first explain what do I mean by the baby-boom generation. The term baby-boom generation in Spain refers to those individuals that were born between 1960 and 1978. In that period, the average number of children per woman was around 2.8. In contrast, the average number of children averaged 2.5 from 1950 to 1959 and 1.5 from 1980 to 1995. This recent fertility pattern has motivated an increasing concern about the sustainability of the pension system as the dependency ratio is expected to increase dramatically with the aging of the baby-boom generation. In particular, the number of retirees over the working population (old dependency ratio) is expected to increase from 0.230 in 1995 to 0.428 and
0.549 in 2035 and 2045 respectively. The effects of this ongoing process is first analyzed in the standard model economy characterized by the perfect substitutability of workers with different levels of experience.

4.0.1 Perfect substitutability model

In the above-mentioned demographic context, if the rule by which the pensions are calculated stays unaffected, the increment in the dependency ratio will necessarily produce an increase of social security tax rate. This process is partially compensated by the fact that as the baby-boom generation gets older, the labor input becomes relatively more scarce and consequently the capital-labor ratio increases (see Figure 3). As the capital-labor ratio grows, new wages are greater than old ones and consequently the pension burden is more easily sustainable.

(The INSERT FIGURES 3, 4 and 5 AROUND HERE)

The overall effect, shown in Figures 4 and 5, is an increase in the share of GDP spent on pensions from 12.3% in 1995 to 19.2% and 26.1% in 2030 and 2040 respectively, and an increase in the social security tax rate from 18.9% in 1995 to 40.2% by 2040. Notice also that associated with this sharp jump of the social security tax rate is the reduction of the average work effort by
16% from 1995 to 2040 due to the higher labor supply distortions faced by agents.

4.0.2 Imperfect substitutability model

The main feature of this model economy is that, in contrast to the previous case, the changes in the relative supply of work with different levels of experience induce changes in the relative wage by age. Before analyzing the effects of this mechanism on the social security tax rate, it is useful to describe the dynamics of the relative supply of less and more experienced workers. Notice firstly that in 1995 the baby-boom generation are between 15 and 34 years old and consequently they are mostly working as less experienced workers. By 2010 some baby-boomers (those born by 1960) start working as experienced workers and some members of the baby-bust generation enter the labor force pushing down the relative wage of old workers. This is so because the members of the baby-bust generation that enter the labor force have a smaller size than the members of the baby-boom cohort that start working in the group of more experienced workers. This process lasts until 2025 when all the baby-boomers are working as more skilled workers. Fi-
nally, by 2030 the baby-boom generation enter retirement and the relative wage of older workers slightly increases (see Figure 6). By 2045 the entire baby-boom generation is retired.

The consideration of the dynamics of relative wages changes dramatically the pattern of pension benefits as compared to the standard model economy. The pension benefit, as it stands in the Spanish economy, is computed applying a 100% to the average earnings of the last 8 years before retirement. We have already seen that as the baby-boom generation starts working as experienced workers, the wage of those workers declines in response to the increase in the relative supply of more skilled people. As this process happens from 2000, it affects to all workers that start retiring from this date even if they do not belong to the baby-boom cohort. This explains why the less pronounced increase of the share of GDP spent on pensions in the initial periods of the transition. As more members of the baby-boom generation belong to the more experienced category the relative wage of this category declines until it reaches its lowest value in 2025. From this date onwards the baby-boomers retire and the pension level to which they qualify for is lower than in the benchmark case since they had lower labor earnings before retirement.
We can also observe that the rise in the capital-labor ratio in the model economy with cohort size effects is more pronounced than in the standard model economy. The explanation of this behavior lies on the evolution of the ratio between the wage of more and less experienced workers. As previously explained, as the baby boomers get older they will observe a decline in their wages. Hence as these individuals forecast a lower level of social security benefits, they will tend to save more over the working career to provide for old-age consumption. Associated with the higher capital-labor ratio, the marginal productivity of aggregate labor and consequently taxable income is also relatively higher in the model economy with cohort size effects. This process is exacerbated by the less pronounced decline of average working time from 2030 to 2050 in this economy (see Table 2).

In summary, findings indicate that in contrast to the standard model economy where the social security tax rate increases from 18.94% in 1995 to 45.26% in 2045 respectively, in the model economy with cohort size effects the increase in labor taxation is less severe. In particular it should be increased from 18.94% in 1995 to 33.79% in 2045. In addition, in the model economy with cohort size effects the Pensions/GDP ratio would grow from 12.3% in 1995 to 19.2% in 2040, implying a substantial difference between this model economy

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and the standard one without cohort size effects.

4.1 A Decomposition Analysis

The purpose of this section is to perform a decomposition analysis in order to have a clear understanding of what is driving the different behaviour between both model economies. Hence, we first compute the difference between the social security tax rate in the benchmark model economy and the model economy with cohort size effects. Secondly, we take the social security budget and ask what is the tax rate that balances this budget assuming that the only departure from the standard economy is respectively the pension benefits $d_{i;t}$ and taxable labor income $w_{i;t}e_{i;t}^h_{i;t}$ of the model with cohort size effects. Finally, we compute the percentage that represents this last measure over the total difference between both model economies, ending up with an idea of how much of the difference between both models can be accounted for by the evolution of the pension benefits and taxable labor income in the cohort size economy. The results of this exercise for selected years is shown in Table 3 and indicate that most of the different behaviour between the model economies is driven by the lower labor earnings that the members of the baby-boom generation experience before retirement, since these years are
critical for the determination of the pension level.

5 Sensitivity Analysis

I now examine whether the findings on the effects of introducing imperfect substitutability in an overlapping generations model are robust to alternative assumptions about the degree of substitution of workers with different levels of experience and the number of experience groups used to sort the working population. This is important since in my model, the main mechanisms are operating through the effect of the aging of the baby-boom generation on the relative wages across cohorts.

5.1 Two experience groups and $\frac{1}{2} = 0.6$

In this case, the larger elasticity of substitution across labor inputs implies that the relative wages across experience groups react less to changes in the age composition of the population. Consequently, as we increase the elasticity of substitution (decrease $\frac{1}{2}$ one should expect the results being more in line with the standard model with perfect substitutability ($\frac{1}{2} = 0$). In particular, I consider a high elasticity case with $\frac{1}{2} = 0.6$ (and the implied $\phi = 0.628$).
and $B = 0.938)$ and...nd that by the period 2030-2050 the social security tax rate should be increased between 4 and 7 percentage points less than in the standard model with perfect substitutability.

5.2 Three experience groups and $\frac{1}{2} = 0.6$

In this case the working population is sorted into three groups. The first group $L_t$ contains those individuals with less than 14 years of experience, i.e. those aged between 20 and 34. The second group $M_t$ includes those individuals with more than 15 years of experience but less than 30. And the third one $H_t$ refers to those workers with more than 30 years of work experience. Given these considerations the aggregate labor input is

$$N_t = B \left( \theta_L L_t^{1/\alpha} + \theta_M M_t^{1/\alpha} + (1 - \theta_L - \theta_M) H_t^{1/\alpha} \right) L_t^{1/\alpha - 1}.$$  

(24)

The procedure to calibrate this model economy follow the lines explained in section 3. In particular, the perfect substitution model economy is characterized by $\frac{1}{2} = 0; \theta_L = 0.3333; \theta_M = 0.3333$ and $B = 1$. And in the model economy which accounts for the existence of cohort size effects the corresponding parameters are $\frac{1}{2} = 0.6; \theta_L = 0.379; \theta_M = 0.392$ and $B = 0.965$. Here again the results show that both model economies perform differently.
in terms of tax rate needed to keep balanced the social security system. For instance in the model economy with cohort size effects the tax rate has to increase by 3.6 and 8.5 percentage points less than the standard model in 2030 and 2045 respectively.

6 Concluding Remarks

This paper has extended the standard large overlapping generations model to allow for the interaction between changes in the age composition of the workforce and the shape of life-cycle earnings. We have found that the effect of aging on the sustainability of the social security system depends critically on the degree of substitution of labor at different ages. In particular, for the empirically plausible parameter space used in this paper we find that the adjustment of the tax rate needed to left untouched the pension system in Spain is less severe in a model that accounts for the existence of cohort size effects than in a model that it does not. However, despite the potential implications of the present paper for the political discussion about the desirability and the magnitude of the reform of the existing pay-as-you-go systems in developed countries, one should bear in mind that in the present
analysis it is crucial to have a good estimation of the degree of substitution between young and old workers. And for this to be the case, more empirical research is needed for countries other than the US. Finally, it is convenient to note that although this paper has not allowed for any source of heterogeneity across individuals other than age and asset holdings, the effects of considering other sources like education would go in the same direction pointed out by this paper. This would be the case due to the fact that empirical studies have found that the elasticity of substitution of workers with different levels of experience is lower the higher the educational attainment of individuals. Consequently, it is likely that the better educated a society is, the more difficult is to substitute old and young workers. However, these issues and its aggregate implications are out of the scope of this paper and are left for future research.

7 Acknowledgements

Financial support from a DGESIC grant PB98-1058-C03 is gratefully acknowledged. I have received very helpful comments from M. Boldrin, L. Bovemberg, Javier Diaz, J.J. Dolado, O. Licandro, J.F. Jimeno, Carlos Ur-
rutia and seminar participants at Universidad Carlos III de Madrid, the OCFEB (Erasmus University of Rotterdam), FEDEA and the Universidade Nova de Lisboa. All errors are my own.

References


### Table 1: Features of the Model Economies (in %)

<table>
<thead>
<tr>
<th>$\lambda_{ss}$</th>
<th>Pensions/Y</th>
<th>K/Y (not in %)</th>
<th>r</th>
<th>G/Y</th>
<th>I/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0$</td>
<td>18.9</td>
<td>12.3</td>
<td>3.1</td>
<td>3%</td>
<td>12.8</td>
</tr>
<tr>
<td>$\gamma = 1.2$</td>
<td>18.9</td>
<td>12.3</td>
<td>3.1</td>
<td>3%</td>
<td>12.8</td>
</tr>
</tbody>
</table>
Table 2: Average Work Effort (selected years)

<table>
<thead>
<tr>
<th>Year</th>
<th>( \frac{1}{n} = 0 )</th>
<th>( \frac{1}{n} = 1:2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>0.357</td>
<td>0.358</td>
</tr>
<tr>
<td>2010</td>
<td>0.352</td>
<td>0.357</td>
</tr>
<tr>
<td>2020</td>
<td>0.348</td>
<td>0.343</td>
</tr>
<tr>
<td>2030</td>
<td>0.335</td>
<td>0.331</td>
</tr>
<tr>
<td>2035</td>
<td>0.323</td>
<td>0.332</td>
</tr>
<tr>
<td>2045</td>
<td>0.290</td>
<td>0.322</td>
</tr>
<tr>
<td>2050</td>
<td>0.302</td>
<td>0.315</td>
</tr>
</tbody>
</table>

Table 3: Decomposition Analysis (selected years)

<table>
<thead>
<tr>
<th>Year</th>
<th>Pension benefits</th>
<th>Taxable labor income</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>34.9%</td>
<td>65.1%</td>
</tr>
<tr>
<td>2020</td>
<td>89.6%</td>
<td>10.4%</td>
</tr>
<tr>
<td>2030</td>
<td>98.2%</td>
<td>1.8%</td>
</tr>
<tr>
<td>2050</td>
<td>52.4%</td>
<td>47.6%</td>
</tr>
</tbody>
</table>
Figure 1: Old Dependency Ratio in Spain. Source: Eurostat Demographic Statistics 1996.
Figure 2: Dynamics of age groups (in % of total population). +:(0-19), *:(20-44), o:(45-64), x:(65-rest).
Figure 3: Capital-labor ratio in the model economies.

Figure 4: Social security tax rate in the model economies.
Figure 5: Percentage of GDP spent on Pensions in the model economies.
Figure 6: Ratio of wages of the model economy with imperfect substitution over the wages of the standard economy by age.